Communication and authority with a partially informed expert*

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Abstract

A sender-receiver game a la Crawford-Sobel is analyzed where the sender has expertise on some but not all the payoff-relevant factors. This residual uncertainty can either improve (even allow full revelation) or worsen the quality of transmitted information depending on a statistic called the effective bias. *For symmetrically distributed residual uncertainty or quadratic loss functions, (i) the quality of information transmission is independent of the riskiness of residual uncertainty, (ii) it may be sub-optimal to allocate authority to the informed player, (iii) despite players' preferences being arbitrarily close, it is impossible to assert that the receiver prefers delegation over authority or vice-versa.*

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1 Introduction

■ In many economic situations where an action affects the welfare of two parties, the formal decision rights belong to one, the Principal (**P**), whereas the other party, the expert (**A**, for Agent), has information relevant for determining the optimal action. Naturally, **P** seeks to elicit information from **A**. Crawford and Sobel (1982) (CS) is a seminal article that studies the relation between the quality of advice strategically offered by **A** and the degree to which her objectives are aligned with those of **P** assuming that neither actions nor advice is contractible, i.e., advice is *cheap talk*. The CS model or its variants have been used extensively to study a wide range of issues, e.g., merits of open vs. closed legislative rules, the politics of special interest groups, doctor-patient interactions, issues in corporate governance, and financial advice.[1] Since Dessein (2002), it has also been an important tool in Organizational Economics to study the merits of delegation of authority.

An assumption implicit in much of this literature is that the sender privately holds all payoff-relevant information, i.e., she is an expert at *everything*. We drop this assumption. We consider a scenario where the information relevant for **P**'s optimal decision is partly with **A** but the rest of it, that we call *residual uncertainty*, is outside **A**'s expertise. Financial manager or clients personnel can advise only in their respective domains such as market prospects but the information about the firm's overall budget situation gets known only after various other plans get in shape and the pending orders and expenditures are settled. Specific government departments often care about a single issue, the costs and benefits of which they are well informed, but the policy maker also weighs the opinion polls or the resolution of other uncertainties. The CEO of a multinational company seeks advice from a country division chief, who is presumably given to "empire building" motive locally, when the CEO's decision must also take into account the global conditions that the division chief might not know or care about. Thus the experts in such cases are *partially informed*, as opposed to the fully knowledgable experts often assumed in the cheap-talk/delegation literature.

This article presents a thorough analysis of strategic communication in the presence of resid-

[1] See Gilligan and Krehbiel (1989), Krishna and Morgan (2001a), Grossman and Helpman (2001), Köszegi (2006), Morgan and Stocken (2003), Benabou and Laroque (1992), and Harris and Raviv (2008). A significant theoretical literature on strategic information transmission also exists which extends the basic CS-model to allow for multi-dimensional type uncertainty, partial verifiability of actions, multiple experts reporting on the state, multiple principals etc. (Seidmann and Winter, 1997; Krishna and Morgan, 2001b; Krishna and Morgan, 2004; Battaglini, 2002; Levy and Razin, 2007; Ambrus and Takahashi, 2008; Li and Madarsz, 2008; Chakraborty and Harbaugh, 2010). Sobel (2013) offers a nice survey of this literature.

ual uncertainty and uses it to offer new insights on the issue of delegation vs. authority. Section 2 formally introduces the game of *Strategic Information Transmission Under Uncertainty* (the SITU game). Essentially, the canonical payoffs of the CS model used in the literature are augmented to allow for an additional source of uncertainty apart from the uncertainty about sender's type. This introduces randomness into **P**'s ultimate actions from **A**'s viewpoint. As a result, **A** faces countervailing incentives – her ex-post optimal action exceeds that of **P** or is lower depending on the realized residual uncertainty. Depending on how the residual uncertainty is distributed, this feature may either improve the quality of communication to the extent of making truthful reporting the best hedge for **A** or increase the loss to **P** from communication. The implications for delegation of authority follow accordingly.

More specifically, differences in the two players' objectives in the SITU game are captured through two parameters – the usual bias in the ex-post optimal actions of the two players when there is no residual uncertainty, denoted by b, and a parameter that captures the degree to which the two players differently weigh the residual uncertainty, denoted by w. The players' ex-post optimal actions coincide only when the value of the realized uncertainty is $\beta = -b/w$, which we term the *bias countervailing value*. These parameters, together with the distribution of the residual uncertainty, determine the extent of strategic information transmission by **A** (and the payoffs).

We begin with few assumptions on the distribution of residual uncertainty. We are able to show that the extent of strategic information transmission depends on what we call the *effective bias*, which is the usual bias b net of a correction that occurs due to the presence of the residual uncertainty. In particular, a fully revealing equilibrium exists if and only if the effective bias is zero (Proposition 1); with a non-zero effective bias, the equilibrium partitions the types in a manner isomorphic to constant bias version of Crawford and Sobel (1982) (Proposition 2). If one assumes, however, that either the residual uncertainty is symmetrically distributed or the expert's loss function is quadratic, the effective bias can be shown to be the sum of b and the mean of the residual uncertainty scaled by w. Section 3 uses this closed form solution for the effective bias to draw interesting insights on the impact of residual uncertainty on strategic information transmission and equilibrium payoffs. These include: (i) The riskiness of the residual uncertainty (as measured by its variance) has *no* impact on information transmission, only the mean does (Proposition 3); (ii) The extent of information transmission and **P**'s payoff increase as the mean of residual uncertainty gets closer to β (Proposition 4 and Fig. 1); (iii) A mean-preserving spread of the residual uncertainty, despite not changing the extent of information transmission, leaves **A** worse off and **P** indifferent (Proposition 5).

An important aspect of the internal organization of a firm concerns the allocation of authority to employees. Aghion and Tirole (1997) draw a distinction between formal authority, typically determined by organizational charts, job descriptions and statement of responsibilities, and "real authority" which belongs to the person whose view is eventually implemented. Dessein (2002) uses the CS model (essentially the SITU game absent residual uncertainty) to discuss the tradeoff faced by **P** in delegating his formal authority so that real authority now rests with the better informed **A**. Continuing with our theme that **A** cannot be the "know-all" and that additional information becomes available subsequently, our analysis of the SITU game allows Dessein's contribution on delegation of authority to be viewed from a broader perspective.[2]

For example, recall that absent residual uncertainty, because **A**'s preferences differ from those of **P**, retaining authority is costly for **P** with **A** strategically withholding information about the state. Delegating authority to the better informed **A** is costly for **P** as the latter has to endure the action that is best for **A**. Dessein (2002) has shown that when preferences of the two players are "close", delegation is a superior strategy for **P**. Proposition 10 (Section 4) shows that it is impossible to extend this result in a meaningful manner when there is residual uncertainty. As it turns out, whether delegation is preferred to authority is determined by the value of β – authority is a superior choice when $\beta \approx 0$ but is an inferior choice when it is large. As both large and small values of β are consistent with $b \approx 0$ and $w \approx 0$, it is impossible to conclude that delegation dominates authority or vice-versa even if two players have *arbitrarily close* preferences.

For much of Section 4, the discussion on the tradeoffs between delegation and authority, we (therefore) fix the preference parameters and ask how changes in the mean and variance of residual uncertainty affects the delegation vs. authority decision. Residual uncertainty affects the delegation payoff of **P** through its variance whereas the effective bias (which depends only on the mean) affects the payoff from authority. The mean can make the information more or less precise, but what matters is the magnitude of the mean relative to the size of variance.

[2] **A**'s type or private information may be thought of as soft, even subjective, knowledge that is often important for taking complicated managerial decisions. The residual uncertainty could, for instance, represent the knowledge relevant for the appropriate decision that is dispersed across the firm whose generation is unrelated to the activities of the two players. The premise that there could be such a dispersion of information across the organization is not new – for instance, Bolton and Dewatripont (1994) attribute to Chandler (1966) that the M-form firm is a direct response to "handling ever increasing flow of information".

Our findings are that (i) **P**'s optimal decision to delegate is *non-monotonic* in the mean of the residual uncertainty: for a fixed variance, **P** prefers to retain authority when the mean is close to and far from the bias countervailing value whereas for intermediate values delegation is preferred;[3] and (ii) For a given mean, there is a threshold level of risk (variance), below (above) which delegation (authority) is superior. Dessein (2002)'s result corresponds to the zero risk case. These results are presented as Propositions 7–9, some of which assume a quadratic loss function.

With the introduction of residual uncertainty, this article adds to a considerable theoretical literature on extensions of Crawford and Sobel (1982) noted in footnote 1. The SITU game presented here embeds two related models each focusing on seemingly two different applications of the information transmission problem: Harris and Raviv (2005), and Goltsman and Pavlov (2011). In the former, the authors study the interaction between a CEO and a divisional manager with differing information about the global and local variables that affect investment decisions. The latter studies strategic communication between a single sender (or expert) and multiple audiences with their respective biases. The different states of residual uncertainty in our model can be reinterpreted as our **P** being a composition of multiple audiences. From our Section 2 presentation, the relations between these two models and our SITU game will become clearer.

In the SITU game, **P** takes an action only after the resolution of the residual uncertainty. In Section 3 ('Uncertain principal and the SITU game'), we compare this with the case where **P** has to choose before that uncertainty is resolved. Under the assumption that either type uncertainty is uniform or preferences are quadratic, we are able to show that both players are better off in the SITU game. This value of information complements the work of Watson (1996), McGee (2009), Chen (2009) and de Barreda (2011), where too **P** gets an additional signal. In these models, however, the additional signals do not directly affect the vNM utility of **P**, rather they are useful due to their correlation with **A**'s type. Our discussion here concerns the value of information on a signal that is distributed independently of type uncertainty but affects the vNM utilities.[4]

[3] This conclusion may appear at odds with the one in the previous paragraph. Here, the variance is held fixed and mean changes. On the other hand, when the preference parameters change, the mean and the variance are simultaneously affected.

[4] Seidmann (1990) presents a number of examples where signals that affect payoffs are correlated with type. Each of these examples violate some assumption of CS to suggest that "there can be effective cheap talk even in the least promising circumstances". In the first two examples he removes the single-crossing property assumed in CS and introduces a condition labeled as (IA) under which all types of the sender agree on the relative ranking

The key contributions concerning delegation and communication to which our work most closely relates are Aghion and Tirole (1997), Ottaviani (2000), Dessein (2002), and Krahmer (2006). The common distinguishing feature here, as also in our model, is that the decision variable is not contractible.[5] In all these models the agent with the superior information is fully informed but may have possibly multi-dimensional information. In our model, the type uncertainty of the expert is still one-dimensional, just as in Dessein (2002). By varying the mean and variance of the residual uncertainty faced by our partially informed agent, we are able to offer a wider perspective to the results in Dessein (2002) in particular.

The remainder of this article is organized as follows. Section 2 presents the SITU game and contains the basic results for general distributions of residual uncertainty. Section 3 discusses the comparative statics of the mean and variance of residual uncertainty under a symmetry assumption. Section 4 discusses delegation of authority. Section 5 concludes. Unless explicitly mentioned in the body of the article, proofs of formal results appear in the Appendix.

2 The SITU Game

■ There are two players, **P** and **A**. The authority for choosing an action that affects the payoffs of both players rests with **P**. Their payoffs also depend on two independently distributed real-valued random variables whose realizations are denoted by θ and s.[6] The game unfolds with **A** privately observing her "type" θ and sending a message from a given message space $\mathcal{M}$. **P** observes both **A**'s recommendation and realization s. Then he chooses an action which ends the game. The ensuing vNM utility of **P** and **A** from an action $\xi \in \mathbb{R}$ in state (θ, s) are respectively

$$\begin{aligned} u_p(\xi, \theta, s) &= -\ell_p(|\,\xi - \theta - w_p s\,|) \qquad \text{and} \\ u_a(\xi, \theta, s) &= -\ell_a(|\,\xi - \theta - w_a s - b\,|), \end{aligned}$$

where ℓ_a and ℓ_p are strictly convex and twice continuously differentiable loss functions. w_a, w_p and b are fixed parameters assumed to be non-negative without loss of generality.

of *any* pair of possible decisions to be taken by the receiver. (This is also the case in Watson (1996).) In his Example 3, Seidmann considers non-scalar actions and the receiver's type is in fact common-knowledge. These are considerably different scenarios to the one considered in this article.

[5]A significant literature considers the merits of delegation in two-agent settings where the decision variable is contractible. These include Holmstrom (1984), Armstrong (1994), Melumad and Shibano (1991), Alonso and Matouschek (2007; 2008), Armstrong and Vickers (2010), Koessler and Martimort (2012), among others.

[6]The formal analysis extends easily if the additional uncertainty is denoted instead as a vector.

The uncertainty concerning θ is referred to as *type uncertainty* whereas uncertainty concerning s is *residual uncertainty*. Assume that the former is distributed continuously with a probability density function f on an interval $\Theta = [\theta_\ell, \theta_h]$. Let F denote its cumulative probability distribution. Residual uncertainty is distributed according to some probability distribution function G with mean μ_s and variance σ_s^2. The message space $\mathcal{M}$ is sufficiently rich to allow an onto function from itself to Θ. We refer to the above as the game of *strategic information transmission under uncertainty*, hereafter the SITU game. Observe that the rules of play for the SITU game are analogous to the game of strategic information transmission in Crawford and Sobel (1982).

Informational assumptions. Throughout, at the communication stage **A** is privately informed of θ, and at the decision stage **P** is informed of s except in the treatment of 'Uncertain principal and the SITU game' in Section 3. In the SITU game, because communication occurs prior to the realization of s, it is a moot point whether **A** also learns s eventually. Later in Section 4 where we discuss **P**'s ex-ante decision on whether to retain authority, our assumption is that **A** knows s except in parts of the discussion of 'Timing of communication and authority' in Section 4.

The residual uncertainty enters the payoffs additively with respect to the type uncertainty. This makes it possible to benchmark the results here with the rest of the literature as the above reduces to the canonical specification of the CS model studied in the literature by setting $w_a = w_p = 0$.[7] In this case, the ex-post optimal actions of **P** and **A** are separated by the constant b for every θ. The parameter b is often referred to as the "bias", an a priori measure of the extent of the agency costs within the relationship. We shall refer to this version of the SITU game as the *CS-game with constant bias* b.

[7] It is easy to adapt our analysis to certain other specifications where residual uncertainty is multiplicative. For instance, suppose $\ell_p(|\xi - s\theta|)$ and $\ell_a(\xi, \theta) = (\xi - \theta)^2$. In particular, on having full knowledge of (θ, s), **P** would choose $s\theta$ whereas **A** would chose θ. So there are now different level effects on the ex-post optimal choice with respect to θ and s. Nonetheless, our method can be readily adapted to deal with this case. Presently, we define the "effective bias" that determines the degree of information transmission. Following that logic, if **P** were to believe **A**'s reported θ, then **A** would choose to report her type as

$$\theta^*(\theta) \quad = \quad \text{argmin}_{\theta'} \int (\theta' s - \theta)^2 \, dG(s) \ = \frac{\mu_s}{\sigma_s^2 + \mu_s^2} \theta.$$

Now, with $b^* = \mu_s / (\sigma_s^2 + \mu_s^2)$, we recover an analogue of Proposition 1. Other qualitative results can also be recovered.

More generally, letting

$$w := (w_a - w_p),$$

the distance between the ex-post optimal actions of **A** and **P** (or *ex-post bias*) is $\Delta(s) = |b + ws|$. If $w = 0$ then, just as in the CS-game, the ex-post optimal action of **P** is always a distance $|b|$ away from that of **A**. However, if $w \neq 0$, their ex-post optimal choices are identical when the realization of residual uncertainty equals

$$\beta := -b/w.$$

In what follows, β will be referred to as the *bias countervailing value* (BCV). We shall see that the location of β vis-à-vis μ_s plays an important role in determining the extent of strategically transmitted information in the SITU game.

The above specification captures at least a couple of models in the literature. For instance, by setting $w_a = w_p = 1$, we obtain the model Harris and Raviv (2005) use to study the interaction between a CEO (**P**) and a divisional manager (**A**) in a firm. The profit-maximizing investment in a company depends on both a local knowledge parameter θ and a global parameter s. The divisional manager is privy to the local information whereas the CEO learns about the global parameter prior to making the investment. The ex-post optimal choice of the CEO is $y = \theta + s$, whereas the divisional manager, perhaps given her proclivity to empire building, wishes to invest $y + b$, in this case presumably $b > 0$. The model here is richer as it generates interesting strategic effects even when $b = 0$ but $w \neq 0$.

The special case of our model with $b = w_a = 0$ and $w_p = 1$ also relates to Goltsman and Pavlov (2011)'s and Farrell and Gibbons (1989)'s model of communication via public cheap-talk with one sender and two receivers.[8] The fact that **P** could take very different decisions depending on different realizations of s, when reporting about θ, **A** views as if her message is being directed to multiple audiences each of whom will then take an independent decision and **A**'s payoff becomes the average of the losses resulting from those individual decisions. The formal structure and analysis of our SITU game thus admits several economic applications noted in Goltsman-Pavlov and Farrell-Gibbons (e.g., a firm's claim to profitability affecting both bond rating and labor negotiations, or capital market and competitors or regulators; a prime minister

[8] Goltsman-Pavlov generalize the two-state, two-action model of Farrell-Gibbons to a continuum of states and actions.

or a bureaucrat communicating with different ministers in charge of various policy making).[9]

□ **Effective bias and equilibrium.** A pure strategy of **A** in the SITU game is any (measurable) function $\sigma_a : \Theta \longrightarrow \mathcal{M}$ and **P**'s strategy is a mapping $\sigma_p : \mathcal{M} \times \mathcal{S} \longrightarrow \mathbb{R}$. Without loss of generality, the analysis is restricted to pure strategies. The composition of a strategy of **A** with a strategy of **P** yields an *outcome function* $Y : \Theta \times \mathcal{S} \longrightarrow \mathbb{R}$ where

$$Y(\theta, s) = \sigma_p(\sigma_a(\theta), s).$$

$Y(\cdot, \cdot)$ is said to be an equilibrium outcome function (EOF) of the SITU game if it is the outcome function of some Perfect Bayesian equilibrium strategy profile (σ_a, σ_p) of the SITU game. For example, in a fully revealing equilibrium, the outcome function would simply be **P**'s ex-post optimal action, i.e., $Y(\theta, s) = \theta + w_p s$.

To provide a complete characterization of all EOFs, we now introduce the concept of *effective bias*. Define the function $\varphi : \mathbb{R} \longrightarrow \mathbb{R}$ where

$$\varphi(\xi) = \int \ell_a(|\xi - ws|)\, dG(s), \tag{1}$$

$$\text{and set} \qquad b_s := \operatorname{argmin}_{\xi} \varphi(\xi). \tag{2}$$

Note that φ inherits the strict convexity of $\ell_a(\cdot)$ which ensures that (2) is well-defined.

b_s admits an intuitive interpretation. Assume, for the time being, $b = 0$ so that a possible conflict of preferences arises only due to residual uncertainty. Fix the beliefs of **P** so that whenever he hears the report θ' from **A**, he believes it to be true. He would then take the action $\theta' + w_p s$ upon observing s. The expected loss of **A** of type θ, if she were to report her type to be θ', is $\varphi(\theta' - \theta)$. Therefore, a type θ would most prefer that **P** believes that she is of type $\theta + b_s$. In other words, b_s is the amount by which **A** optimally distorts her type.[10]

When ℓ_a is quadratic, b_s minimizes the sum of squared deviations and hence $b_s = w\mu_s$. On the other hand, if ℓ_a were linear, it minimizes the sum of absolute deviations, and hence

[9]By setting $w_p = 0$ and letting s be a binary-valued random variable, one might mistakenly think that the model here also embeds those in Gilligan and Krehbiel (1989) and Krishna and Morgan (2001a) on the design of legislative rules by imagining **P** to be the legislature facing two committee members corresponding to the two realizations of s. In the SITU game, realizations of s are mutually exclusive whereas in the legislative rules example one committee member has to consider the other report when choosing her own report. This difficulty does not arise in the converse interpretation of the model alluded to above to match Goltsman and Pavlov (2011), because there each audience chooses a different action.

[10]This statement should be read with the caveat that a type θ whose type is such that $b_s + \theta \notin \Theta$ would prefer to mimic type θ_ℓ if $b_s > 0$ and θ_h if $b_s < 0$.

$G(b_s) = 1/2$, i.e. it is the *median* of G. Observe however that if G also happens to be symmetric, then $b_s = w\mu_s$ holds. The following lemma confirms this equality for general loss functions.

Lemma 1 Suppose s is symmetrically distributed. Then

1. $b_s = w\mu_s$, and

2. $\xi \neq \xi'$ solves $\varphi(\xi) = \varphi(\xi')$ if and only if $|b_s - \xi| = |b_s - \xi'|$.

Naturally, when the distribution of states is asymmetric enough, the agent may not want an unbiased decision because she weighs the outcomes state by state and $b_s \neq w\mu_s$ is a possibility. If $b \neq 0$, then the notion of optimal distortion attributed to b_s would be corrected to

(Effective Bias) $$b^* = b + b_s.$$

It is now intuitive that b^* would play a similar role in the SITU game as the bias b does in the CS-game in determining the extent of strategic information transmission.[11]

Proposition 1 A fully revealing equilibrium exists if and only if $b^* = 0$.

Interestingly, existence of a fully revealing equilibrium is a possibility in a SITU game despite the fact that this is a single sender-single receiver pure communication game. This is unlike say in Battaglini (2002) which has multiple senders or in Kartik, Ottaviani, and Squintani (2007) where messages affect the payoffs directly and signaling is a possibility (see also Kartik, 2009). Koessler and Martimort (2012) provide a screening model (with full commitment) where decision variables are multi-dimensional. Their principal, with an upward bias in each dimension relative to the agent's ideal actions, optimally distorts the actions in opposite directions to create countervailing incentives to minimize his loss due to information asymmetry. In contrast the decision variables in our setup are non-contractible, but the basic intuitions for communication in the Koesller-Martimort model and our model are the same: translated in our case, different dimensions (of Koessler and Martimort) come from different realizations of the interim uncertainty. Unlike in the above literature, in our analysis the decision maker cannot commit ex-ante to any of the actions.

[11] In the discussion following their Proposition 2 concerning public communication to multiple audiences, Goltsman and Pavlov (2011) in fact recognize this possibility of full revelation in the special case where there are two audiences. They also remark on the lack of an obvious generalization. The idea of an effective bias and Proposition 1 below are precisely the required generalization.

Given the intuitive meaning of effective bias, that full revelation occurs only when $b^* = 0$ in the SITU game is only natural. However, whereas in the above models full revelation obtains *generically*, here properties of G must match the preferences parameters b, w_a, w_p appropriately to provide exact canceling of the weighing of the ex-post outcomes (for example, $\mu_s = \beta$ when Lemma 1 applies) for full revelation to occur.

When $b^* \neq 0$, full revelation is no longer possible. There are typically multiple (but finite number of) EOF, each one distinguished by a partitioning of Θ into finitely many sub-intervals, just as in a CS-game. To characterize the EOF, first define for any $a < a'$,

$$x(a, a') \quad = \quad \mathrm{argmin}_{\xi} \int_{a}^{a'} \ell_p(|\ \xi - \theta\ |) f(\theta) d\theta \tag{3}$$

to be the optimal action of $\mathbf{P}$ in the event he knows that θ lies in the interval $[a, a']$. Also let $\mathbf{a} = (a_0, a_1, \ldots, a_N)$ denote a typical partition of Θ into N sub-intervals where $\theta_\ell = a_0 < a_1 < \cdots < a_N = \theta_h$ are the end points of the sub-intervals.

Proposition 2 Consider a SITU game with $b^* \neq 0$.

1. In any equilibrium, there is a partition $\mathbf{a} = (a_0, a_1, \cdots, a_N)$ of Θ such that

$$\varphi(x(a_{i-1}, a_i) - a_i - b) \quad = \quad \varphi(x(a_i, a_{i+1}) - a_i - b) \tag{4}$$

   and

$$Y(\theta, s) \quad = \quad x(a_{i-1}, a_i) + w_p s \tag{5}$$

   for all $\theta \in [a_{i-1}, a_i]$, for all $s \in \mathcal{S}$ and for $i = 1, \ldots, N$.

2. There exists a finite integer N^* such that an equilibrium described in Part 1 exists if and only if $N \leq N^*$.

In words, just as in the CS-game, any equilibrium of the SITU game involves partitioning the state space Θ into some N sub-intervals. All types of $\mathbf{A}$ within a sub-interval pool to send the same message. In what follows, we will refer to such an equilibrium as the N-*equilibrium* and the corresponding partition as an *equilibrium partition.* We shall refer to N^*-equilibrium as the *most informative equilibrium* of the SITU game when $b^* \neq 0$.

Remark 1 It is useful to note that for $w = 0$, the equilibria of the SITU game are isomorphic to those of a CS-game of constant bias b. This is readily seen by observing that $\phi(\xi) \equiv \ell_a(|\xi|)$ when $w = 0$. Consequently, (4) reduces to the usual arbitrage conditions found in the CS-game of constant bias b. This equivalence with respect to the extent of information transmission does not translate to the payoffs – residual uncertainty does affect the payoffs. Harris and Raviv (2005) study delegation vs. authority for precisely this case.

3 Residual Uncertainty and Communication

■ This section studies the relation between residual uncertainty, the extent of strategic information transmission and the equilibrium payoffs, having fixed the preference parameters w_a, w_p and b. Following Remark 1, assume for the remainder of this article that $w \neq 0$, unless stated otherwise. Furthermore, although a slightly more general treatment is possible, throughout this section we assume Condition A.

Condition A. Either 1. ℓ_a is quadratic, or 2. The residual uncertainty is symmetrically distributed.[12]

Quadratic loss is a standard assumption in much of the cheap-talk literature following CS. Residual uncertainty would be symmetric if s is assumed to follow a normal distribution, as is often assumed in finance, or is uniform. If we think of the **P** as a policy maker and θ is the advice given by a close confidante or a special interest group, and s the *average* opinion elicited from opinion polls, then s would follow a Student's t-distribution, which too is symmetric. Therefore Condition A may be expected to hold in a variety of environments.

Proposition 3 Suppose Condition A holds in a SITU game. Then $b^* = w(\mu_s - \beta)$ and the extent of strategically transmitted information (i.e. an equilibrium information partition) depends only on μ_s, the mean of the distribution of residual uncertainty. It is independent of σ_s^2, the variance of residual uncertainty. Moreover,

1. $\mathbf{a}_N = (a_0, a_1, \ldots, a_N)$ is an N-equilibrium partition if and only if

$$a_i = \frac{x_i + x_{i+1}}{2} - w(\mu_s - \beta) \qquad \forall i = 1, \ldots, N-1, \tag{6}$$

where $x_i = x(a_{i-1}, a_i)$.

[12] That is, the random variable $X = (s - \mu_s)$ and $-X$ are identically distributed.

2. N-equilibrium is strictly Pareto-superior[13] to an $N-1$ equilibrium.

That the variance of residual uncertainty has no effect on information transmission is rather surprising. Formally, it is shown that the function φ is symmetric about $E[ws] = w\mu_s$ under Condition A. As φ is also convex, it follows that $b_s = w\mu_s$. Given this, the arbitrage condition (4) that makes the marginal type a_i indifferent to pooling with types $[a_{i-1}, a_i)$ and $(a_i, a_{i+1}]$ reduces to (6). Furthermore, as **P** makes a choice *after* observing s, given the information partition, his choice is independent of σ_s^2. Consequently the equilibrium partition of Θ determined by (6), or equivalently the extent of information transmission, is independent of the variance of residual uncertainty.

Let us now turn to the impact of residual uncertainty on the payoffs of the two players. Observe that the arbitrage conditions expressed in (6) are identical to those found in a CS-game of constant bias where the bias is $b + w\mu_s$. Furthermore, **P**'s decision is contingent on the realization of the residual uncertainty. We may therefore resort to the well-known comparative statics of the most informative equilibrium in a CS-game described in **?**. Accordingly, whether the residual uncertainty has a beneficial impact on **P**'s payoff depends on whether its mean reduces the effective bias $|b + w\mu_s|$. The following proposition makes this precise.

Proposition 4 Suppose Condition A holds. **P**'s ex-ante payoff with respect to μ_s increases in the corresponding most informative equilibrium as μ_s gets closer to BCV, namely β, unless it is a babbling equilibrium.[14]

Given the proof of Proposition 3, proof of Proposition 4 would be evident to readers conversant with Crawford and Sobel (1982) and is omitted.

Let $|b'|$ denote the critical value of the bias b in CS-game of constant bias such that meaningful communication occurs in the most informative equilibrium if and only if $|b| \leq |b'|$. Set $\beta' = -|b'|/w$. Proposition 4 is illustrated in Fig. 1 for quadratic loss functions, $w < 0$ and a typical value of b.

Recall Harris and Raviv (2005) model from Section 2: $w_a = w_p = 1$. As $w = 0$, changes in μ_s will have no impact on how **A** communicates (see Remark 1). Let us consider as a possibility a different scenario: $w = w_a - w_p < 0$, the CEO (**P**) places a greater importance on residual uncertainty than the Manager (**A**). Assuming $b > 0$ (empire building motive), now starting

[13] Throughout, assertions on Pareto comparisons are in terms of ex-ante payoffs.

[14] The equilibrium is babbling if all types pool.

from $\mu_s = 0$ an increase in μ_s will initially have a beneficial effect on information transmission (i.e., s counters the agency costs) with **A** reporting θ precisely at the bias countervailing value $\mu_s = -b/w$, then communication becomes gradually worse as μ_s increases further.

Although the variance σ_s^2 has no impact on **A**'s information partition, the variance *does* affect **A**'s ex-ante payoff. This is to be expected as **A** remains uncertain regarding s at the communication stage. Other things being the same, it is reasonable to expect that a higher variance lowers **A**'s payoff. On the other hand, if change in variance is accompanied with a change in the mean as well, it is difficult to determine whether the change benefits **A**. For, as we have seen, the change in μ_s may lead to more improved communication that may more than offset the increased uncertainty through an increase in the variance. A general result on the comparative statics with respect to changes in (μ_s, σ_s^2) does not seem possible (except perhaps in the case of quadratic loss functions). Under the stronger assumption on how the change in residual uncertainty occurs, we have the following result:

Proposition 5 Consider a change in the distribution of residual uncertainty from G to G' such that Condition A applies and $\mu_s' = \mu_s$ holds for the corresponding distributions. If G' second order stochastically dominates G, comparing the N-equilibrium of G and G', **A** is better off in the latter whereas **P** is indifferent.[15]

The intuition is as follows. Distributions G' and G have the same mean but for the former the density mass is more concentrated around the mean shifting in from the tails. **A**'s expected loss from communication does not depend on θ but depends only on s (see (1)). Because the loss function $\ell_a(.)$ is convex, a mean-preserving narrowing of the spread would imply lower expected loss for **A**. On the other hand, **P**'s expected loss remains unchanged due to $\mu_s = \mu_s'$ (Fig. 1).

☐ **Uncertain principal and the SITU game.** Until now, we have considered the implications of changes in residual uncertainty by fixing the information structure of the players. Another way to consider the impact of interim uncertainty is to benchmark the SITU game with a change that **P** takes the decision before observing s. Comparing **P**'s payoff in these two games would for instance tell us whether **P** has an incentive to acquire information about s.

We shall refer to this game where both players are uncertain of s as the SITU* game.

[15]Because $\mu_s' = \mu_s$, G admits an N-equilibrium if and only if G' does as well (see Proposition 3). To avoid mathematical technicalities that are costless with regard to economic intuition, the proof of Proposition 5 in the Appendix is for the case where the distribution of s is bounded above.

Proposition 6 Suppose Condition A holds.

1. The SITU game has a fully revealing equilibrium if and only if the SITU* game has a fully revealing equilibrium, i.e., if and only if $\mu_s = \beta$.

2. If either θ is uniformly distributed or ℓ_p is quadratic,

   (a) An equilibrium partition of the SITU* game is an equilibrium partition of the SITU game and vice-versa.

   (b) Both players are strictly better off in the SITU game relative to the SITU* game in their corresponding equilibria.

The above proposition can be understood as follows. Analogous to (1), define for $i = a, p$,

$$\varphi_i(\xi) = E_s[\ell_i(|\xi - w_i s|)].$$

SITU* is effectively a CS game in which **P**'s loss from ξ if θ occurs is now $\varphi_p(\xi - \theta)$ and that of **A** is $\varphi_a(\xi - \theta - b)$. If θ is commonly known, **P**'s optimal action is $\theta + b_p$ and that of **A** is $\theta + b_a - b$ where $b_i = \text{argmin}_\xi\ \varphi_i(\xi)$. From Lemma 1, $b_a = w_a\mu_s$ and $b_p = w_p\mu_s$. That is to say, the bias in their actions is $b + b_a - b_p = b + b_s$. As in a CS game, SITU* admits a fully revealing equilibrium if and only if this bias is zero, i.e. $\mu_s = \beta$. Proof of the remaining part is in the Appendix.

Recall that in general strategic environments, the additional information is not always desirable. The above proposition lists conditions where it is in the interest of both parties for **P** to wait for realization of s after communication over θ before making the choice. This exercise on the value of information about a factor that enters the payoff function complements other studies on value of information in a CS game such as Watson (1996), de Barreda (2011), Chen (2009) and McGee (2009). In their models, the additional signal is correlated with the type uncertainty and thus acquiring it gives **P** information about **A**'s type. This effect is absent in our model, whereas in those models the correlated signal does not directly enter the payoff function.[16]

[16] In Che and Kartik (2009), the two players are symmetric in all respects with regard to their ex-post preferences but differ in their prior beliefs of the true state. We may incorporate crude analogue of their setting $\ell_p = \ell_a$ and $w_a = w_p$ but assume that **P** and **A** respectively believe that s is distributed according to symmetric probability distribution with different means μ_a and μ_p. Then the "equilibrium partition" of the SITU and SITU* games, under the hypothesis of Proposition 6, coincides with the CS-game of constant bias $|w_p| \cdot |\mu_a - \mu_p|$.

4 Residual Uncertainty and Authority

■ Apart from eliciting information from **A** and making a choice, another possible course of action for **P** is to delegate decision making authority to **A**. Until now it has been sufficient to maintain that **P** is informed of s at the time of decision and **A** is uninformed of s at the time of reporting for all the results stated thus far to hold. From here onwards, we will make the assumption that s is a public signal available to *both* players only at the time of decision. This seems a natural benchmark to study as the heart of the question on whether to delegate authority is whether the player with the *superior* information must be given authority. Making the above assumption will ensure that at each stage, **A** has superior information.

In the absence of residual uncertainty, Dessein (2002) compares the expected utilities of the two players under delegation and authority in the CS-game. He shows that under certain conditions when preferences of **P** and **A** are "close", the former is in fact better off through delegation. Below ('Proximate preferences and delegation'), we see how this does not hold under residual uncertainty. We begin, however, by fixing the preference parameters b, w_a and w_b and study how the incentive to retain authority changes with respect to changes in the uncertainty.

That delegation may be an inferior choice for some parameter values of the residual uncertainty is of course immediate from Proposition 1. Whenever the effective bias is zero, the SITU game admits a fully revealing equilibrium. Because **P** decides after being informed of the residual uncertainty, this is the best possible outcome for **P** (provided the players coordinate on this equilibrium). Thus,

- By retaining authority **P** obtains his first-best utility, not just that it is better than delegation in an ex-ante sense.
- If **P** could *choose* when to seek advice from **A**, it is optimal for **P** to do so *before* the residual uncertainty is realized.

Further, the above observations do not depend on whether s is publicly observed (after communication) or **P** privately observes it.

In the remainder of this section, we shall focus on the case where a fully revealing equilibrium does *not* exist. For convenience, we introduce the notation

$$\mu_b := b + w\mu_s.$$

To evaluate the costs of delegation it is necessary to specify in greater detail **A**'s information regarding the residual uncertainty at the time of making her choice, should **P** delegate authority. Except when we analyze 'Timing of communication and authority', we assume that upon delegation **A** chooses her action after observing the realization of the residual uncertainty, just as **P** does in the SITU game. With this assumption, it is clear that if **P** were to delegate authority, **A** takes the action $y_a(\theta, s) = \theta + w_a s + b$ in state (θ, s). Hence, **P**'s payoff from delegation is $-E[\ell_p(|b + ws|)]$. Let $K := \min_{\xi \in \Theta} \ell_p''(\xi)/2$.[17] Taylor's expansion of ℓ_p about μ_b with the Lagrange form for the remainder term shows that **P**'s payoff from delegation is bounded above by

$$\bar{\pi}_D(\mu_s, \sigma_s^2) \quad = \quad -\ell_p(|\mu_b|) - Kw^2\sigma_s^2. \tag{7}$$

Once **P** delegates, he has no means of insuring against the variability of **A**'s choice which varies with the residual uncertainty. This is evident from the above bound on **P**'s payoff from delegation. If **P** keeps authority, as he takes an action after observing s, he is completely protected from the variability in the residual uncertainty. By keeping authority, however, **P** has to endure the loss of information regarding θ. In the event the fully pooling equilibrium is played so that no information about θ is conveyed, **P** optimally chooses $x(\theta_\ell, \theta_h)$ (defined in Eq. (3)) resulting in a payoff of

$$-\int_{\theta \in \Theta} \ell_p\left(|\theta - x(\theta_\ell, \theta_h)|\right)\, dF(\theta), \tag{8}$$

which is of course independent of the residual uncertainty. Moreover, in any informative N-equilibrium with $N > 1$, **P** is only better off (see Part 2, Proposition 3) relative to the babbling equilibrium payoff above. Comparing (7) with (8), we have the following simple and intuitive observation:

Proposition 7 In any SITU game, if either the mean or the variance of the residual uncertainty is sufficiently large, **P**'s payoff is higher in every equilibrium of the SITU game than under delegation.

Two points are worth noting. First, although the variance of the residual uncertainty is

[17] K is well-defined as ℓ_p is assumed to be twice continuously differentiable. Further, because we assumed that ℓ_p is strictly convex, $K > 0$.

irrelevant for the extent of information transmission (and **P**'s payoff), it does however affect the cost of delegation. The value of σ_s^2 plays a non-trivial role in the delegation decision. Second, it is worth noting that the above result does not depend on Condition A. If Condition A in fact holds, then **P**'s payoff in an N-equilibrium of the SITU game (by Proposition 3) is identical to his payoff in a CS-game with bias μ_b. The upper bound for payoff from delegation in a SITU game, presented in (7), is the payoff from delegation in a CS-game with a bias μ_b minus the positive term $Kw^2\sigma_s^2$. Thus, for a fixed μ_s, any result that shows the superiority of authority over delegation in a CS-game also carries over for a SITU game. The reader may refer to Proposition 4 in Dessein (2002) that presents one such set of conditions. We now turn to the more interesting case of residual uncertainty with low variance.

□ **Low variance and authority.** The sufficient condition provided in Proposition 7 should be expected. Indeed, **P**, by retaining authority, is able to completely hedge against the residual uncertainty whereas under delegation he is unable to do so. A higher mean and (being risk-averse) a higher variance in the ex-post bias then increase the cost of delegation in predictable ways leading **P** to prefer authority.

A salient feature of Proposition 7 is that it is agnostic as to which equilibrium of the SITU game is played, albeit under a stringent sufficient condition. As we relax this condition to allow for residual uncertainty with a low variance, the question of equilibrium selection comes into play. Because **P**'s payoff across N-equilibria is increasing in N, the *minimal value* of the σ_s^2 required for delegation to become a poor strategy, for a given μ_s, would be lower as we select a more informative equilibrium. In fact it turns out that this minimal value of variance falls *rapidly* as N increases. To appreciate this, consider for the moment (Proposition 9 considers more general preferences) the SITU game where **P** has a quadratic loss function. By borrowing some of the calculations regarding the linear-quadratic CS-game of a fixed bias in Section 4 of Crawford and Sobel (1982), we have the following proposition.

Let $N^* \equiv N^*_{\mu_s}$, the maximal number of elements in the equilibrium partition in the SITU game, given μ_s.

Proposition 8 Consider a SITU game in which $\ell_p(\xi) = \xi^2$, Condition A holds and θ is uniformly distributed over $[0, 1]$. Fix any (μ_s, σ_s^2). For any $N \leq N^*_{\mu_s}$,

1. **P** strictly prefers to retain authority in an N-equilibrium game if and only if

$$\sigma_s^2 > \frac{1}{12w^2N^2} + (\frac{(N^2-4)}{3})(\mu_s - \beta)^2. \tag{9}$$

2. **P** strictly prefers to retain authority in an N-equilibrium whenever

$$\sigma_s^2 > \frac{(2N^2+2N-3)}{w^2N^2(N+1)^2}\ \sigma_\theta^2. \tag{10}$$

Part 2 of Proposition 8 offers a sufficient condition for dominance of authority by relating the variance in type uncertainty with the variance in the residual uncertainty. The actual magnitudes are interesting. For instance, if $N = 2$ then the residual uncertainty need only be 25% more variable than type-uncertainty; with $N = 4$, residual uncertainty needs to be only 10% more variable. In fact, the rate at which the ratio $\sigma_s^2/\sigma_\theta^2$ must fall is of the order of $O(N^{-2})$. In other words, as the communication equilibrium becomes more informative, the incentive to retain authority becomes more attractive at a fairly rapid rate. Of course, one needs to bear in mind that in order to ensure the existence of an N-equilibrium for N large enough, μ_s must be sufficiently close to the BCV.

□ **Non-monotonicity of delegation.** Part 1 of Proposition 8 offers a comparison that is based only on parameters of the residual uncertainty (μ_s, σ_s^2). Using this, the mean-variance pairs for which authority dominates delegation (and vice-versa) are depicted in Figure 2. The curve C describes the (μ_s, σ_s^2) pairs such that (9) holds as an equality under the further assumption that the players coordinate on the most informative equilibrium N_{b^*}.[18] Whenever (μ_s, σ_s^2) lies below the curve **C** (in the shaded area), delegation yields a higher payoff to **P** than authority whereas the *opposite* holds above C. Given Proposition 7, the interest is in what happens for smaller values of (μ_s, σ_s^2). When $\sigma_s^2 = 0$, which is precisely the case considered by Dessein (2002), whenever the distance of μ_s to BCV is above a certain threshold value, delegation is superior. As is evident from the picture, this is no longer true in the presence of some residual uncertainty. Indeed, for *any* positive σ_s^2, there is a threshold value such that if the distance of μ_s to the BCV is below the threshold value, the *opposite conclusion* holds: Retaining authority is better for **P**.

The non-monotonicity of the superiority of delegation/authority with respect to $|\mu_s - \beta|$ for

[18]See Chen, Kartik, and Sobel (2008) for a justification for selecting the most informative equilibrium.

a fixed $\sigma_s^2 > 0$ is not a-priori obvious. For both the loss from delegation and the loss from authority are (weakly) decreasing in $|\mu_s - \beta|$. In the context of the linear-quadratic example, the loss from delegation is $w^2(\Delta^2 + \sigma_s^2)$, and therefore the loss increases at the rate w^2 with respect to $\Delta^2 = (\mu_s - \beta)^2$. On the other hand, in an N-equilibrium, the rate of increase in payoff with respect to Δ^2 is $w^2 \left(N^2 - 1\right)/3$. For smaller values of Δ that permit an equilibrium with $N \geq 3$ elements, a *decrease* in Δ^2 is associated with a more than proportional decrease in the loss from keeping authority relative to delegation. Because for a $|\Delta|$ close enough to zero, (a) the loss from delegation is bounded away from zero due to σ_s^2 being positive, and (b) the loss from authority converges to zero as there is enough transmission of information (i.e. $N_{b^*}^*$ is sufficiently large), authority dominates for a $|\Delta|$ below a certain threshold. This non-monotonicity holds more generally, beyond quadratic preferences:

Proposition 9 Let ℓ_p and $\epsilon > 0$ be given. There exists an N_ϵ such that for any SITU game (in which ***P***'s loss function is ℓ_p) that satisfies Condition A, if $\sigma_s^2 > \epsilon$ and μ_s is sufficiently close to the BCV β, so that an N-equilibrium with $N \geq N_\epsilon$ exists, ***P*** prefers to retain authority in such an equilibrium.[19]

With the above set of results, we are now in a position to comment on the internal organization of a firm, a la Dessein (2002). Recall that in Dessein, when preferences are close, it is in fact optimal for the better informed player to have authority. As we are assuming that s is publicly observable following the communication stage, at all stages of the SITU game and under delegation, **A** being exclusively informed of θ is the better informed of the two players. An important implication of the above results is that when residual uncertainty is present, it is no longer necessarily optimal for the better informed party to exercise authority.

□ **Proximate preferences and delegation.** So far we have held the preference parameters fixed and investigated the implications of changes in residual uncertainty parameters. In this section, we fix the uncertainty parameters μ_s and $\sigma_s^2 > 0$, and examine the implications of changing preference parameters b, w_a and w_p. In particular, our focus is on delegation decision when the preferences of the two players are "close" to each other. Recall the key observation of Dessein (2002) that in a CS-game, if the preferences are close, then it is optimal for **P** to delegate authority. Because the preferences are identical when $b = 0$ and $w_a = w_p$, preferences

[19]Proof of Proposition 9 relies on a technical result from Agastya, Bag, and Chakraborty (2013), which establishes that almost full revelation can occur in the most informative equilibrium of a CS-game of fixed bias when players' preferences are sufficiently close.

in the SITU game are close when (b, w) is sufficiently close to $(0, 0)$. But we have the following result:

Proposition 10 Consider a SITU game in which $\ell_p(\xi) = \xi^2$, Condition A holds and θ is uniformly distributed over $[0, 1]$. For any $\epsilon > 0$, there are (infinitely many) values of (b, w) in the ϵ-neighborhood of $(0, 0)$ such that

1. **P**'s payoff in the most informative equilibrium in the SITU game is higher than under delegation.
2. **P**'s payoff in the most informative equilibrium in the SITU game is lower than under delegation.

In other words (when $\sigma_s^2 > 0$), no matter how close the preferences of the two players are, it is virtually impossible to say whether delegation is better than retaining authority, in sharp contrast to the result of Dessein (2002) that delegation is superior (when $\sigma_s^2 = 0$).

■ **Timing of Communication and Authority**

□ **Soft information and delegation.** So far we have assumed that, upon delegation, **A** can observe s prior to making a choice. This is a reasonable assumption where either s is publicly observable or represents hard information that **P** can commit to making available to **A**. In other cases, s may in fact be soft information privately available to **P** making him an expert on some aspects of the decision just as **A** is on certain other aspects. In such a case, delegation of authority to **A** could be accompanied with communication from **P** regarding s, i.e., we have a SITU game with the roles reversed. In fact, Harris and Raviv (2005) consider exactly this mechanism for the uniformly distributed uncertainty, quadratic loss functions and $w_a = w_p = 1$. Our approach enables us to generalize this analysis to environments where Condition A holds and θ is symmetrically distributed. Below we illustrate the payoff comparison for quadratic loss functions.

Given a partition $\mathbf{a} = (a_1, \ldots, a_N)$ of Θ, define

$$\sigma_\theta^2(\mathbf{a}) = \sum_{i=1}^{N} \int (x_i - \theta)^2 f(\theta)\, d\theta$$

where $x_i = E[\theta \mid [a_{i-1}, a_i]]$. The players' respective payoffs in the SITU game in the most informative equilibria are

$$\pi_p = -\sigma_\theta^2(\mathbf{a}) \quad \text{and} \quad \pi_a = -\sigma_\theta^2(\mathbf{a}) - \mu_b^2 - w^2\sigma_s^2,$$

where $\mathbf{a}$ is the equilibrium partition.

Assuming that s is continuously distributed on a compact interval, say $[s_\ell, s_h]$, and taking partition $\mathbf{a}$ of this interval, define $\sigma_s^2(\mathbf{a})$ similar to $\sigma_\theta^2(\mathbf{a})$. Now, if **P** delegates to **A** and communicates regarding s, we have a new SITU game where the roles of **P** and **A** are reversed as well as the roles of s and θ. Accordingly, if we denote by $\mathbf{a}'$ the most informative partition in this new SITU game, the corresponding payoffs are

$$\pi_p' = -\sigma_s^2(\mathbf{a}') - {\mu_b'}^2 - w^2\sigma_\theta^2 \quad \text{and} \quad \pi_a' = -\sigma_s^2(\mathbf{a}'),$$

where $\mu_b' = -b - w\mu_\theta$ is the new effective bias. **P** prefers authority to delegation if

$$\sigma_s^2(\mathbf{a}') + {\mu_b'}^2 + w^2\sigma_\theta^2 \quad > \quad \sigma_\theta^2(\mathbf{a}),$$

and the opposite is true if the (strict) reverse inequality holds. One may now proceed to obtain analogues of Propositions 8 and 9.

□ **Ex-post delegation vs. authority.** So far in our discussion of control vs. delegation, we have assumed that **P** elicits information from **A** *before* s is publicly observed. A natural question is whether **P**, prior to the resolution of any uncertainty, has an incentive to commit to postpone the decision to 'either elicit information about θ or delegate' until *after* s is publicly observed. Proposition 9 offers a ready answer: if the mean bias is close to zero, **P** has no such incentives. For, if communication occurs after s is publicly observed, then it is as if **P** and **A** play the CS-game with a constant bias. On the other hand, if the residual uncertainty is such that $|ws| \approx 0$ for all s, we know from Dessein (2002) that **P** is better off by delegating authority giving him a loss of $\ell_p(|ws|)$. Therefore, the expected loss from delaying either communication or delegation until after the realization of s leads to an expected loss for **P** that is bounded away from zero provided $\sigma_s^2 > 0$ and the support of s contains a set of positive measure close to 0. However, if the mean bias is sufficiently close to zero, then eliciting information on θ prior to public revelation of s allows the information loss to be arbitrarily close to zero (see Proposition A in the Appendix and Proposition 9). In this case, authority dominates delegation.

□ **Interim delegation.** We considered the question of delegation vs. control in a hierarchy with a sequential resolution of multiple sources of uncertainty. Given the timing of delegation considered so far, the resolution of residual uncertainty following delegation exposes **P** to risks as **A**'s ex-post optimal decisions might significantly differ from that of **P**. To avoid this risk, we now consider the case where **P** observes s privately and then decides whether to delegate or elicit information. We call this *interim delegation*. There are two differences from what we considered in the rest of the article. First, **P** is *privately* informed of s as opposed to s being public knowledge. Second, given **P**'s prior knowledge of s, the decision to delegate or not is likely to convey to **A** some information about s.

In principle, the entire analysis of Section 2 can be used to study the interim delegation. Given **P**'s equilibrium behavior, at any part of the game tree where he retains authority, there is a SITU game with **A**'s posterior (which must be consistent with **P**'s behavior in equilibrium) on s describing the nature of uncertainty. Our earlier analysis in this article can be applied to characterize equilibrium behavior on that sub-tree. One may then work recursively to solve the entire game. This analysis is sufficiently involved, even in the linear-quadratic case, for it to be pursued here.

5 Conclusion

■ By introducing residual uncertainty in a canonical version of Crawford and Sobel (1982)'s model involving a constant bias, the article revisits two important issues that have been previously analyzed extensively – cheap-talk style information transmission and delegation vs. authority. The role of residual uncertainty for both these issues has been discussed. Our analysis reveals conditions under which the risk of residual uncertainty has no bearing on the extent of information transmission. Equally, it also overturns a commonly held view that authority must lie with the better informed party especially when the preferences are close.

The formal analysis relies fairly heavily on the fact that the residual uncertainty is distributed independently of the type uncertainty. It would be of interest to see how these results generalize if one allowed for correlation between the two types. Following Che and Kartik (2009), it may be possible to study correlated uncertainty in a tractable manner if one makes the assumption that the probability distribution of the variables is bivariate normal. Finally, throughout in the analysis of delegation we assumed that **A** also learns about the realization of residual uncertainty upon receiving authority. It would be of interest to explore the tradeoff between delegation and authority under other timing considerations, briefly discussed under 'Timing of Communication and Authority' in Section 4. Each of these requires a separate and elaborate treatment that we hope will be pursued in the future.

A Appendix

Proof of Lemma 1. Recall that a real-valued random variable X is said to be symmetrically distributed if $Y = E[X] - X$ and $-Y$ have the same probability distribution. Moreover, for any integrable function f, $E[f(Y)] = E[f(-Y)]$. It follows from this that $E[\ell_a(|\xi+Y|)] = E[\ell_a(|\xi-Y|)] = E[\ell_a(|-\xi+Y|)]$. By setting $X = ws$, we have $\varphi(\xi + E[X]) = E[\ell_p(|\xi+Y|)] = E[\ell_p(|-\xi+Y|)] = \varphi(-\xi + E[X])$. In other words, φ is symmetric about $E[X] = w\mu_s$. The fact that φ inherits the strict convexity of ℓ_a tells us that the point of symmetry of φ must also be its unique minimum, i.e., $b_s = w\mu_s$. *Q.E.D.*

Certain preliminaries are in order before moving to the next Lemma. When the strategy profile (σ_a, σ_p) is played, **P** plays $Y(\theta, s)$ in state (θ, s). Therefore, having fixed an arbitrary strategy profile (σ_a, σ_p), the expected loss of **A** of type θ if she deviates and mimics the behavior of type θ' is:

$$L_a(\theta', \theta) \quad = \quad \int \ell_a(|Y(\theta', s) - \theta - w_a s - b|)\, dG(s). \tag{A.1}$$

Definition 1 (Equilibrium) An equilibrium consists of a strategy profile (σ_a, σ_p) such that

$$L_a(\theta', \theta) \quad \geq \quad L_a(\theta, \theta) \qquad \forall \theta, \theta' \in \Theta, \tag{A.2}$$

and for every $m \in R(\sigma_a)$, where $R(\sigma_a) \subseteq \mathcal{M}$ denotes the range of σ_a,

$$\sigma_p(m, s) \quad \in \operatorname{argmin}_\xi \int_{\theta' \in \sigma_a^{-1}(m)} \ell_p(|\xi - \theta' - w_p s|) f(\theta')\, d\theta', \tag{A.3}$$

whenever $\sigma_a^{-1}(m)$ is of non-zero probability.

Condition (A.2) is the usual incentive compatibility requirement on **A**'s behavior. Condition (A.3) is the requirement that at every m that is reported along the equilibrium path, **P**'s choice is a best response given his updated Bayesian posterior.[20]

We begin with the preliminary observation that any $Y(\theta, s)$ is additively separable in the two variables. This is intuitive as **P** chooses an action only *after* observing s.

Lemma 2 For every EOF Y of the SITU game, there is a unique function $\psi : \Theta \longrightarrow \mathbb{R}$ such that $Y(\theta, s) = \psi(\theta) + w_p s$.

Proof. Choose any equilibrium strategy profile (σ_a, σ_p). At θ, **P** hears the report $m = \sigma_a(\theta)$. The support of his posterior is $\sigma_a^{-1}(m)$. His expected loss from selecting an action ξ' after observing s is proportional to

$$\int_{\theta' \in \sigma_a^{-1}(m)} \ell_p(|\xi' - \theta' - w_p s|) f(\theta')\, d\theta'. \tag{A.4}$$

The best-response property requires choosing an action that minimizes the above expression. Now define ψ as follows:

$$\psi(\theta) \quad := \quad \arg\min_{\xi'} \int_{\theta' \in \sigma_a^{-1}(\sigma_a(\theta))} \ell_p(|\xi' - \theta'|) f(\theta')\, d\theta'. \tag{A.5}$$

Comparing the minimand expression in (A.5) with **P**'s expected loss in (A.4) gives us $\sigma_p(m, s) = \psi(\theta) + w_p s$. *Q.E.D.*

Proof of Proposition 1. Suppose a fully revealing equilibrium exists. The EOF is then $Y(\theta, s) = \theta + w_p s$. Substituting in (A.1) we have

$$\begin{aligned} L_a(\theta', \theta) &= \int \ell_a(|\theta' + w_p s - \theta - w_a s - b|)\, dG(s) \\ &= \int \ell_a(|\theta' - \theta - b - ws|)\, dG(s) \\ &= \varphi(\theta' - \theta - b), \end{aligned}$$

Note that for the equilibrium condition (A.2) to hold, $L_a(\theta', \theta)$ must have a minimum in its first

[20] The analysis here is presented in terms of behavioral (pure) strategies whereas CS work with distributional strategies. This difference is inessential here. Furthermore, the definition of an equilibrium must specify players' beliefs at all information sets, including out of the equilibrium path, as well as (A.2) and (A.3). Insofar as our concern is only in the characterization of the EOF, this is without loss of generality because, given a strategy profile (σ_a, σ_p) such that (A.2) and (A.3) hold, pick $\hat{\theta}$ arbitrarily and let $\hat{m} = \sigma_a(\hat{\theta})$. For any $m \in \mathcal{M} \setminus R(\sigma_a)$, which represents an unreached node in the candidate equilibrium (σ_a, σ_p), prescribe the beliefs of **P** at m to be the same as those at $\hat{m}$ and redefine $\sigma_p(m, s) = \sigma_p(\hat{m}, s)$. That is, **P** behaves at any unreached equilibrium message exactly as he does upon hearing $\hat{m}$. Because the original incentive compatibility conditions prevent any type (other than $\hat{\theta}$) from mimicking the behavior of $\hat{\theta}$, with the above prescribed beliefs, every type of **A** has an incentive to weakly report $\sigma_a(\theta)$ and makes (σ_a, σ_p) a Perfect Bayesian Equilibrium, in the sense of Fudenberg and Tirole (1990).

argument when $\theta' = \theta$ for all θ. On the other hand, from the above equation and definition of b_s, we also note that this minimum occurs when $\theta' - \theta - b = b_s$ or $\theta' = \theta + b^*$. To reconcile these two facts, we must have $b^* = 0$.

Conversely, assume $b^* = 0$, i.e. $b_s = -b$. Because $\mathcal{M}$ is sufficiently rich, there is no loss of generality in assuming that $\mathcal{M} \supseteq \Theta$. Now suppose that **A** reports truthfully, i.e. plays $\sigma_a(\theta) = \theta$ for all θ and **P** believes this. The equilibrium requirement (A.3) gives $\sigma_p(\theta, s) = \theta + w_p s$, which again gives $L_a(\theta', \theta) = \varphi(\theta' - \theta - b) = \varphi(\theta' - \theta + b_s)$. By reporting the truth, **A**'s loss is therefore $\varphi(b_s)$, which is the minimum value of φ (and hence of $L_a(\cdot, \theta)$). Mis-reporting her type cannot therefore improve **A**'s payoff. Hence, truthful reporting is an equilibrium strategy. *Q.E.D.*

The proofs of Proposition 2 and Lemma 3 below require the following preliminaries. Fixing **P**'s equilibrium behavior and the corresponding EOF $Y(\theta, s) = \psi(\theta) + w_p s$ (see Lemma 2), the payoff of **A** of type θ from mimicking to be type θ' and sending a signal $\xi' = \psi(\theta')$ is $-\varphi(\xi' - \theta - b)$.

For any $\xi_1 = \psi(\theta_1) < \psi(\theta_2) = \xi_2$, write

$$D(\theta, \xi_1, \xi_2) = \frac{\varphi(\xi_2 - \theta - b) - \varphi(\xi_1 - \theta - b)}{\xi_2 - \xi_1}. \tag{A.6}$$

Therefore, a type θ would prefer to mimic being type θ_2 instead of type θ_1 provided $(\xi_2 - \xi_1)D(\theta, \xi_1, \xi_2) \leq 0$ and conversely otherwise. $D(\theta, \xi, \xi')$ is the slope of φ between the points $\xi - \theta$ and $\xi' - \theta$. Because φ is strictly convex, this slope must be *decreasing* in θ.

Lemma 3 Consider an equilibrium where $\psi(\theta_1) = \xi_1$ and $\psi(\theta_2) = \xi_2$ are such that $\xi_1 \neq \xi_2$. Then $|\xi_1 - \xi_2| \geq |b^*|$. Consequently, Ψ can only take finitely many values.

Proof. Assume, with no loss in generality, that $\xi_1 < \xi_2$. Given **P**'s behavior described by Lemma 2, the payoff of a type θ from reporting ξ_i is $-\varphi(\xi_i - \theta)$. By incentive compatibility of equilibrium behavior of θ_1, we must have $D(\theta_1, \xi_1, \xi_2)(\xi_2 - \xi_1) \geq 0$, and similarly incentive compatibility of θ_2 requires $(\xi_2 - \xi_1)D(\theta_2, \xi_1, \xi_2) \leq 0$. By continuity of $D(\cdot, \xi_1, \xi_2)$, there must exist some $\theta^* \in [\theta_1, \theta_2]$ such that $D(\theta^*, \xi_1, \xi_2) = 0$. By monotonicity, those types to the right of θ^* would strictly prefer to report ξ_2 and those to its left strictly prefer to report ξ_1. Therefore, when **P** hears ξ_1 or ξ_2, he knows the true type is respectively bounded above or below by θ^*.

Looking at the definition of ψ in (A.5), we can readily conclude that

$$\xi_1 \leq \theta^* \leq \xi_2. \tag{A.7}$$

Furthermore, because φ is convex with a minimum at b_s, for $D(\theta^*, \xi_1, \xi_2)$ to be zero, we must have

$$\xi_1 - \theta^* - b < b_s < \xi_2 - \theta^* - b. \tag{A.8}$$

Combining (A.7) and (A.8), we obtain

$$\xi_1 < \theta^* < \theta^* + b^* < \xi_2 \qquad \text{for } b^* > 0,$$

$$\xi_1 < \theta^* + b^* < \theta^* < \xi_2 \qquad \text{for } b^* < 0,$$

implying $\xi_2 - \xi_1 \geq | b^* | > 0$. Because Θ is a compact set, it follows that Ψ can only take on finitely values in an equilibrium. *Q.E.D.*

Proof of Proposition 2. Pick an EOF Y. It follows from Lemma 3 and Lemma 2 that in any EOF, there are finitely many values, say $\xi_1 < \xi_2 < \cdots < \xi_N$, such that $Y(\theta, s) = \xi_i + w_p s$ for $1 \leq i \leq N$. Each ξ_i is a possible report that will be sent by some θ in the equilibrium. We now determine which types send a given report ξ_i. To this end, with no loss in generality assume that $N \geq 2$ and first define a_i by the equation

$$\varphi(\xi_i - a_i - b) = \varphi(\xi_{i+1} - a_i - b) \qquad \text{for } i = 1, \ldots, N-1. \tag{A.9}$$

Next, recall that the loss of a type θ from reporting ξ_i, given the behavior of $\mathbf{P}$, is $\varphi(\xi_i - \theta)$ and hence $(\xi_{i+1} - \xi_i)D(\theta, \xi_i, \xi_{i+1})$ is the difference in type θ's payoff from reporting ξ_{i+1} instead of ξ_i. Because $D(\cdot, \xi_i, \xi_{i+1})$ is decreasing, all the types in $[\theta_\ell, a_i)$ would strictly prefer reporting ξ_i instead of ξ_{i+1} whereas the opposite is true for types in $(a_i, \theta_h]$ and type a_i, by its definition above, is indifferent between either report. Consequently (a_{i-1}, a_i) is the set of types that strictly prefer to report ξ_i to any of the other reports. Type a_i is indifferent between reporting ξ_i and ξ_{i+1}, and strictly prefers those over any other report. Therefore, (A.5) reduces to

$$\xi_i = \arg\min_\xi \int_{a_{i-1}}^{a_i} \ell_p(\xi - \theta) f(\theta) \, d\theta \quad \forall i = 1, \ldots, N. \tag{A.10}$$

Noting that $\xi_i = x(a_{i-1}, a_i)$ completes the proof of Part 1 of the Proposition. For the proof of Part 2 of the Proposition, one need only follow the arguments given in the proof of Theorem 1 in Crawford and Sobel (1982).[21] *Q.E.D.*

Proof of Proposition 3. By Lemma 1, φ is symmetric about b_s. Therefore, (4) reduces to

$$b_s - (x_i - a_i - b) \quad = \quad (x_{i+1} - a_i - b) - b_s, \tag{A.11}$$

which in turn reduces to (6), given that $b_s = w\mu_s$. In fact, (6) is precisely the set of conditions listed as Eq. (9) and arbitrage conditions (A) in Crawford and Sobel (1982). They are obtained by requiring a cutoff type a_i to be indifferent between pooling with types in intervals $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$ and effectively characterize an equilibrium. In fact, (4) determine an equilibrium of a CS-game of constant bias b^* with *any* loss function of **A** that is symmetric around zero, not just ℓ_a, and the loss function of **P** being ℓ_p. This completes the proof of Part 1.

To see Part 2, we begin by noting that the isomorphism between the equilibrium partitions of such a CS-game of constant bias b^* and the SITU game, the payoff of **P** in the corresponding N-equilibrium is equal. Moreover, a CS-game of constant bias satisfies assumption (M) given in Crawford and Sobel (1982) that leads to their Theorem 2. As a result, all their comparative static results apply. In particular, **P**'s payoff is higher in the N-equilibrium than in the $N-1$-equilibrium.

To complete the proof, what remains to be shown is that **A**'s payoff in the SITU game also increases in N. But, a careful examination of the steps leading to Theorem 5 of CS reveals that the proof of that result can be carried forward *ad verbatim* after averaging out the residual uncertainty in **A**'s payoff to conclude that **A** also prefers the equilibrium with a greater number of steps. *Q.E.D.*

Proof of Proposition 5. Let B denote a random variable, initially distributed according to a probability distribution function H. Now consider a change in its distribution to H' so that it second order stochastically dominates H and the *mean remains unchanged.* An unchanged mean implies

$$\int_\alpha^\beta H(x)dx \quad = \quad \int_\alpha^\beta H'(x)dx. \tag{A.12}$$

[21] Repeat the proof arguments given on para 2, page 1438 onwards replacing their condition (A) with (A.9) and their Eq. (10) with (A.10) above.

Given that ℓ_a is convex, a direct application of the proof of Theorem 3′ of Hadar and Russell (1969) together with the above equality immediately gives us

$$E_H[\ell_a(|\, x - B\,|)] \quad > \quad E_{H'}[\ell_a(|\, x - B\,|)]. \tag{A.13}$$

Let $\mathfrak{a}$ be the partition in the N-equilibrium of the SITU game and let $x_i := x\,(a_{i-1}, a_i)$. Letting $B = ws + b$ and H denote its probability distribution, ex-ante payoff of **A** in this equilibrium is

$$\pi_N^a\,(\mathfrak{a}, H) \quad = \quad -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} E_H\,[\ell_a\,(|x_i - \theta - B|)]\, dF(\theta). \tag{A.14}$$

If the distribution H is changed to H' in the manner described above, the effective bias remains at $w\mu_s + b$. From Proposition 5, $\mathfrak{a}$ continues to be the equilibrium partition in the N-equilibrium following the change. Consequently, the **A**'s payoff is now $\pi_N^a\,(\mathfrak{a}, H') > \pi_N^a\,(\mathfrak{a}, H)$, the inequality following from (A.13). **P**'s payoff remains unchanged as the effective bias has not changed. *Q.E.D.*

Proof of Proposition 6. We have already discussed Part 1 following the statement. We turn to proving Part 2.

First, note that when θ is uniform, $x_i = (a_{i-1} + a_i)\,/2$ and (6) reduces to

$$a_{i+1} \quad = \quad 2a_i - a_{i-1} + 4\,(b + w\mu_s)\,. \tag{A.15}$$

Coming to the SITU* game, this after all satisfies the assumptions of CS. Hence every equilibrium must involve a finite partition of Θ. The arbitrage condition facing a marginal type a_i is

$$\varphi_a\left(x_i' - a_i - b\right) \quad = \quad \varphi_a\left(x_i' - a_i - b\right).$$

where

$$x_i' \quad = \quad \text{argmin}_\xi\; E_\theta\,[\varphi_p\,(\xi - \theta)\mid [a_{i-1}, a_i]]\,.$$

Again, applying Lemma 1 to φ_a, the unique solution to $\varphi_a\,(\xi) = \varphi\,(\xi')$ is $|\mu_s - \xi| = |\mu_s - \xi'|$.

Hence, above arbitrage condition reduces to

$$a_i = \frac{x'_i + x'_{i+1}}{2} + b + w_a\mu_s, \tag{A.16}$$

just as in Proposition 3.

To calculate x'_i, write $t = \theta + w_p s$ where the distribution of θ is now uniform on $[a_{i-1}, a_i]$. Write

$$\hat{\varphi}(\xi) = E[\ell_p(|\xi - t|)]$$

and note that $x'_i = \text{argmin}_\xi\ \hat{\varphi}(\xi)$. Noting that t is symmetrically distributed. Applying Lemma 1, we have $x'_i = E[t] = (a_{i-1} + a_i)/2 + b + w_p\mu_t$. Using this, (A.16) reduces to (A.15). So the equilibrium partitions must be identical in both the games.

Pareto-superiority of an equilibrium of the SITU game with a corresponding equilibrium of the SITU* game is a direct consequence of the facts that the outcome in the latter is a mean-preserving spread of the former and ℓ_i is convex. *Q.E.D.*

Proof of Proposition 8. By Proposition 3, the payoff from retaining authority and playing the SITU game is the same as **P**'s payoff in the CS-game with quadratic loss functions and bias μ_b. In an N-equilibrium of the latter, the expected loss of **P** is simply the residual variance of θ that **P** expects after hearing **A**'s report. That is, if $\xi^*_N : \Theta \longrightarrow \mathbb{R}$ denotes the equilibrium outcome function of the CS-game, then **P**'s loss from retaining authority, as shown in Section 4 of CS with the bias $b = \mu_b$, is

$$E[\ell_p(\xi_N(\theta))] = \frac{1}{12N^2} + \mu_b^2\frac{(N^2-1)}{3}.$$

On the other hand, if **P** delegates, **A** chooses the action $\theta + w_a s$ in state (θ, s) leaving **P** with an ex-post payoff of $\ell_p(b_s)$. Given that ℓ_p is assumed to be quadratic, the loss from delegation is $\mu_b^2 + w\sigma_s^2$. Retaining authority is therefore a superior choice whenever

$$\begin{aligned} \mu_b^2 + w^2\sigma_s^2 &> \frac{1}{12N^2} + \mu_b^2\frac{(N^2-1)}{3} \\ \Leftrightarrow \qquad w^2\sigma_s^2 &> \frac{1}{12N^2} + \mu_b^2\left(\frac{(N^2-4)}{3}\right). \end{aligned} \tag{A.17}$$

Dividing both sides of (A.17) by w^2 establishes Part 1. For Part 2, recall that CS show that the existence of an equilibrium which divides Θ into N sub-intervals requires that

$$N \leq -\frac{1}{2} + \frac{1}{2}(1 + \frac{2}{\mu_b})^{1/2} \quad \text{or equivalently,} \quad \mu_b \leq \frac{2}{(2N+1)^2 - 1}.$$

Therefore, the RHS of (A.17) is bounded above by

$$\frac{1}{12N^2} + (\frac{2}{(2N+1)^2 - 1})^2 \frac{(N^2-4)}{3} \quad = \quad \frac{(2N^2+2N-3)}{N^2(N+1)^2}\sigma_\theta^2.$$

Hence, (10) implies (A.17) and Part 2 follows. *Q.E.D.*

Proof of Proposition 9. Let $CS(b)$ denote a CS-game of fixed bias b and let $\pi_N(b)$ denote **P**'s payoff in that game. Recalling our discussion following Proposition 3, in a SITU game where Condition A holds, $\pi_N(\mu_b)$ is **P**'s payoff. Thus, we will routinely appeal to the following comparative statics developed in CS:

$$\pi_{N-1}(\mu_b) < \pi_N(\mu_b) \quad \text{whenever} \quad N \leq N^*_{\mu_b}; \qquad \textbf{(CS1)}$$

$$N^*_{\mu_b} \leq N^*_{\mu'_b} \quad \text{whenever} \quad |\mu'_b| < |\mu_b|; \qquad \textbf{(CS2)}$$

$$\pi_N(\mu_b) < \pi_N(\mu'_b) \quad \text{whenever} \quad |\mu'_b| < |\mu_b|. \qquad \textbf{(CS3)}$$

We will also need a result from Agastya, Bag, and Chakraborty (2013), Proposition A below. Recall that the *mesh* of a partition $\mathbf{a} = (a_0, a_1, \ldots, a_N)$ is the length of its longest sub-interval, denoted by $\| \mathbf{a} \|$.

Proposition A. Fix ℓ_p, ℓ_a and F and (b_k) be such that $\lim_{k\to\infty} b_k = 0$ and consider the corresponding family of CS-games $\{CS(b_k)\}_k$. Then,

1. $\lim_{k\to\infty} N^*_{b_k} = \infty$.

2. Consider any infinite sequence of integers (N_k) such that $N_k \leq N^*_{\mu_{b_k}}$ for all k and $N_k \to \infty$. Then, for the corresponding equilibrium partitions $\mathbf{a}^k = (a_0^k, a_1^k, \ldots, a_{N_k}^k)$,

$$\lim_{k\to\infty} \| \mathbf{a}_k \| = 0,$$

$$\text{and} \qquad \lim_{k\to\infty} \pi_{N_k}(b_k) = -\ell_p(0).$$

To complete the proof of our proposition then, we apply (7) to see that the payoff from delegation is at most $-K\epsilon$. Let $b_\epsilon = \sup\left\{|b| : \pi_{N^*_b}(b) \geq -K\epsilon\right\}$. Note that b_ϵ is well-defined, due to Proposition A. Define $N_\epsilon = N^*_{b_\epsilon} + 1$. For the existence of an N-equilibrium, where

$N \geq N_\epsilon$, we must have $|\mu_b| < |b_\epsilon|$ and by construction then $\pi_N(\mu_b) > -K\epsilon$, i.e., authority is superior choice for **P**. *Q.E.D.*

Proof of Proposition 10. Let B_ε denote an $\varepsilon > 0$ neighborhood of $(0,0)$. For any $(b,w) \in B_\varepsilon$ such that $w = 0$ and $b > 0$, it is as if there is no residual uncertainty and the situation is effectively as in Dessein (2002): Authority dominates delegation for all such (b,w), for ε sufficiently small.

We will now show that the converse can happen in any such neighbourhood.

Choose any sequence (b_N) such that $\lim_{N\to\infty} b_N = 0$ and $\lim_{N\to\infty} Nb_N = K$ where $|K| > |\mu_s/\sigma_s|$. Set

$$w_N = -\frac{b_N}{\mu_s} + \frac{1}{2\mu_s N(N+1)}.$$

We claim that along the infinite sequence (b_N, w_N), **P** prefers authority to delegation for all but finitely many N. Because there are infinitely many such sequences, all with $\lim_{N\to\infty}(b_N, w_N) \longrightarrow (0,0)$, establishing this claim completes the proof.

As Condition A holds, along the above sequence, the effective bias is $b_N^* = b_N + w_N\mu_s = \frac{1}{2N(N+1)}$, which is the critical value for the existence of an N-equilibrium. Therefore, it suffices to verify (10), namely $w_N^2 \geq \lambda_N$, where

$$\lambda_N = \frac{(2N^2 + 2N - 3)}{4\sigma_s^2 N^2 (N+1)^2} \tag{A.18}$$

holds for all N large enough. Note that $\lim_{N\to\infty} N^2\lambda_N = \frac{1}{2\sigma_s^2}$, whereas $\lim_{N\to\infty} N^2 w_N^2 = K^2/\mu_s^2$. Because $|K| > |\mu_s/\sigma_s|$ by hypothesis, $w_N^2 \geq \lambda_N$ for all N large enough. *Q.E.D.*

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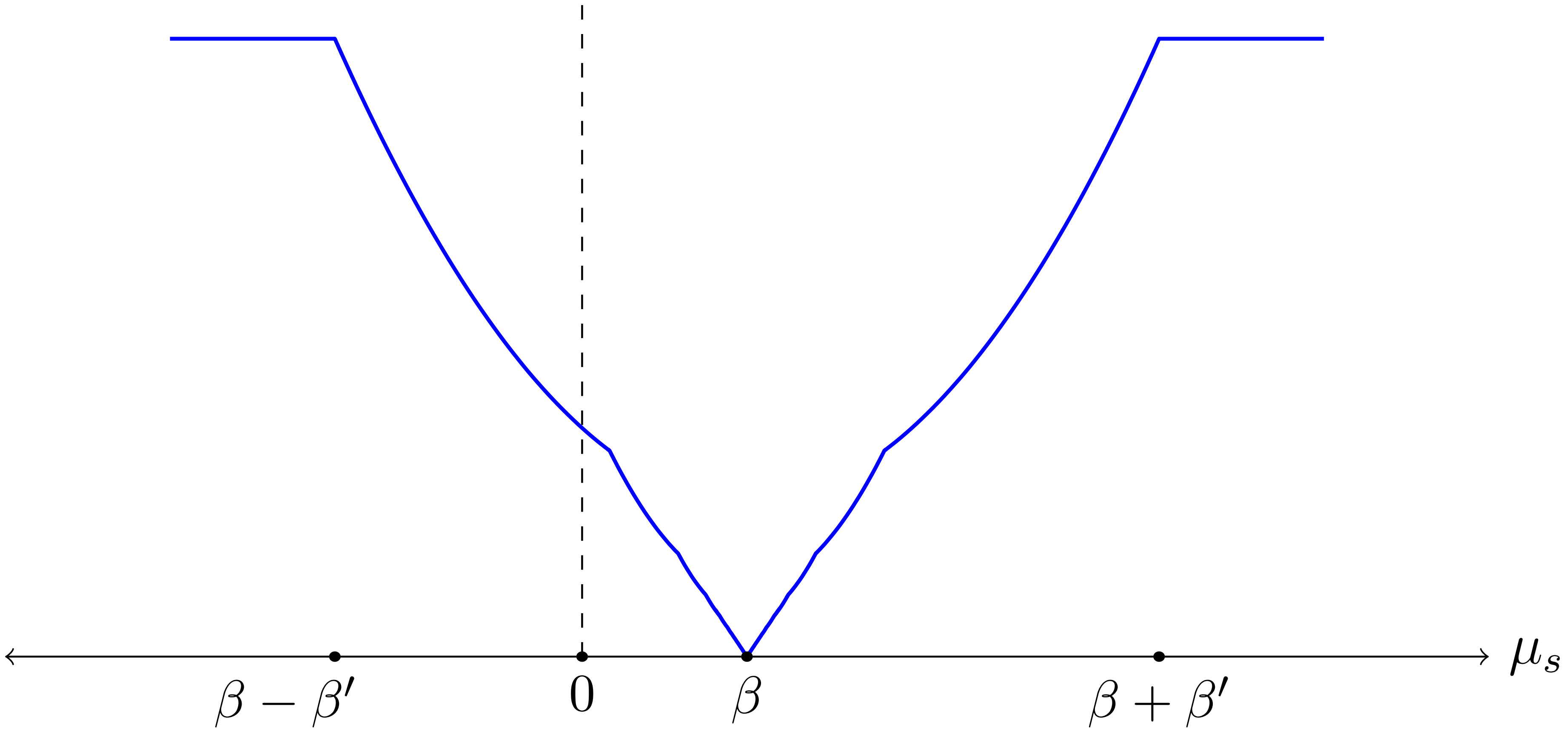

Figure 1: **P**'s loss as μ_s varies, plotted for quadratic loss function and uniformly distributed θ

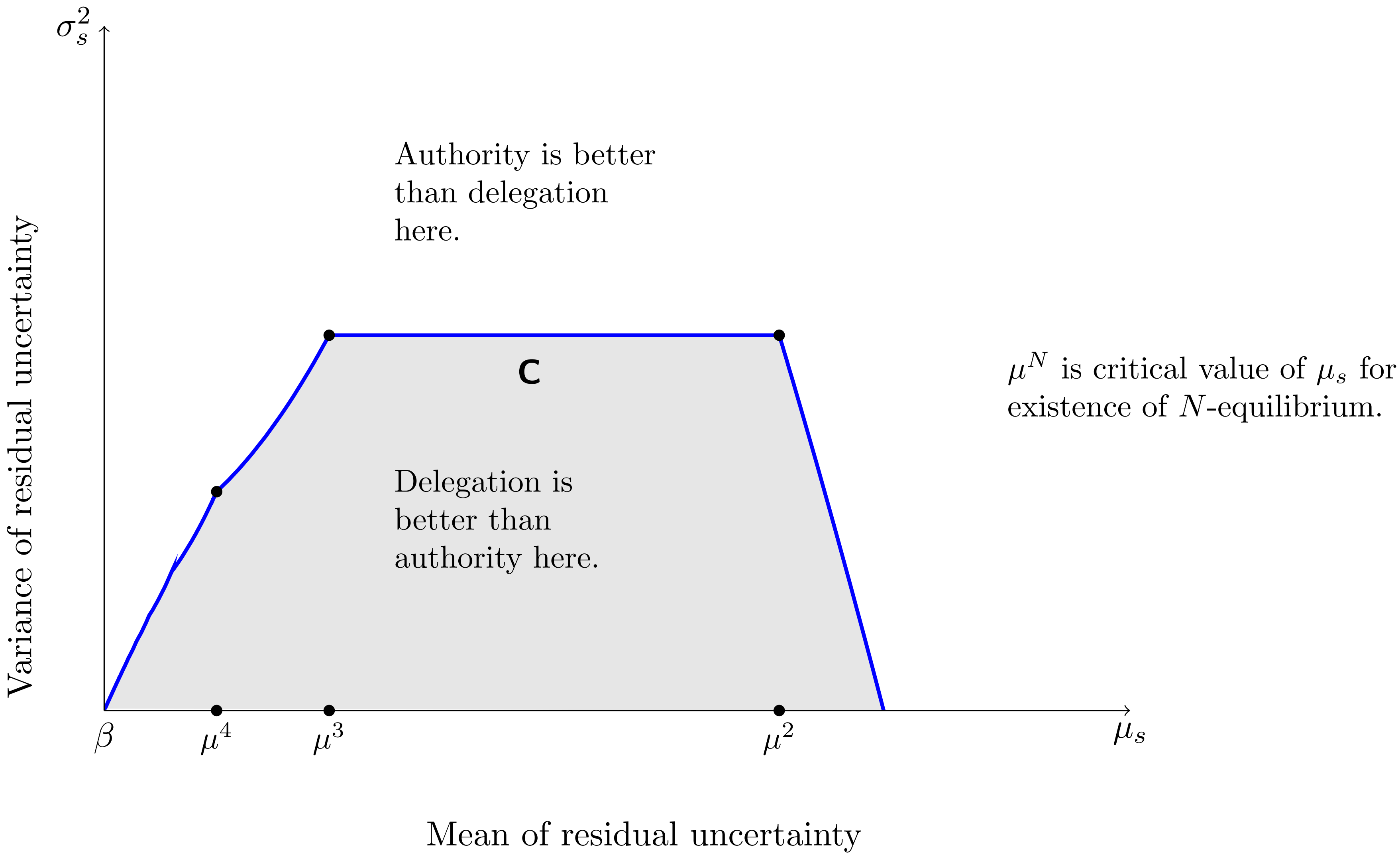

Figure 2: Delegation vs. Authority