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## Income tax evasion and audits under common and idiosyncratic shocks

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## ABSTRACT

Common shocks affect the profits of ex ante identical self-employed entrepreneurs. From aggregate tax return data or other industry expertise, the tax collection agency can better estimate the common shock. With three profit realizations, high, medium, or low, it is shown that this information asymmetry results in taxpayers engaging in at most one-step underreporting compared to a maximal two-step underreporting when the agency does not know the common shock. The evasion behavior also varies with profit levels: for high penalties, only high profit earners evade, whereas for moderate penalties, medium profit earners evade more often. In addition, the tax authorities audit low returns more intensively than they do medium returns when the common shock is favorable, and sometimes do not audit at all when the common shock is unfavorable. Finally, when idiosyncratic shocks depend on the abilities of the entrepreneurs, high-ability entrepreneurs will be more prone to underreporting.

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## 1. Introduction

Tax enforcement for self-employed individuals seems to be a major concern. Developing countries tend to have a large informal sector, making tax collection difficult.<sup>1</sup> Even in developed countries, a substantial part of the tax base consists of self-employment activities. Registered individual entrepreneurs in the professional service sector such as health, legal consultation, information technology, and financial services can easily hide incomes or profits. Successful businesses can engage in creative accounting. A report in the Wall Street Journal (March 26, 2017) observes that small-business owners tend

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<sup>1</sup> Burgess and Stern (1993) observe the hurdles of raising agricultural income tax in developing countries due to difficulties in measuring income (p. 793). Bhattacharyya (1999) (see Table 1, p. F357) estimate a substantially large hidden economy of the service and industrial sectors in India during 1960–1992. Besley and Persson (2014) point out the importance of a well-functioning legal system to make it “more attractive for firms to operate in the daylight of the formal economy” to widen the tax base (pp. 110–111). The authors view the preponderance of small-scale informal firms and the low-tax base as a salient characteristic of underdevelopment.

to be the largest group responsible for tax noncompliance.<sup>2,3</sup> Against this background, we study the strategic interactions between self-reporting entrepreneurs and the tax authority by introducing two types of shocks to incomes (or profits) – a common (market) shock and individual specific shocks, also known as idiosyncratic shocks.

Common shocks affect the profits of all entrepreneurs. From aggregate tax return data or other industry expertise, a tax collection agency is likely to have better information about the shock than the taxpayers. Not knowing the common shock creates more doubts in the taxpayers' minds about the probability of audits. If the economy experiences a positive (or favorable) shock, underreporting by a large margin is likely to draw more scrutiny by tax inspectors. Thus, the taxpayers must be more cautious about the extent of underreporting. If, in contrast, the tax agency does not have better information about the common shock, then the taxpayers can take more chances at evasion. One of the central messages of this study is that the tax agency's superior information makes tax evasion harder.

Another difference from many tax auditing models is our assumption of *non-commitment*: we generate the audit probabilities endogenously in equilibrium through auditor – taxpayer strategic interactions. Broadly, commitment to an auditing rule takes away a large part of the deliberations needed in tax return submissions and reduces the taxpayers' problem to a simple decision under uncertainty. Non-commitment, on the other hand, empowers the tax authority to utilize its superior information about the common shock to its advantage by exposing taxpayers to greater uncertainties.

In our model, individual taxpayers are *ex ante* identical in terms of the distribution of the idiosyncratic shock. Additionally, a small fraction of the taxpayers are intrinsically honest, while most are strategic. We assume three possible profit realizations – high, medium, and low. For any given profit realization, posteriors about the common shock for those realizing the specific profit will be identical, as will their reporting strategies.

We start with a baseline model in which the tax agency is uninformed about the common shock. The auditing is naive; it can be conditioned only on reported income or profit. We then enrich the model by allowing the tax agency to have superior information. The game between a taxpayer and the auditor thus pivots around the tax authority having to guess the idiosyncratic shock while the taxpayer has to guess the market shock. This makes the tax return/audit game much richer than those analyzed in the existing literature.

In general, the tax authority will be more confounded about the evasion of returns in the medium (low) range whenever the common shock realization is high (low). When the common shock is high, (i) high earners gamble knowingly, whereas medium earners gamble in hope;<sup>4</sup> and (ii) the tax authority will know that low submitters are surely evading and thus only need to incur small auditing costs to establish tax fraud, whereas for medium returns, auditing costs will be higher. When the common shock is low, again, the tax authority would learn it precisely due to the presence of a sufficient number of honest taxpayers. Their auditing should therefore focus exclusively on low submitters, as the absence of high profits leaves only the medium profit earners who can underreport.

In the equilibrium of the game with an informed tax agency, strategic tax return submissions involve *at most one-step underreporting*, or what we call controlled risk-taking: high profit earners will possibly mix between truthful submission and reporting medium profits, but never report low profits, and the ones with medium profits will possibly mix between reporting truthfully and reporting low profits. This result is very different from the baseline model in which *two-step underreporting* can occur in equilibrium. Such a characterization may seem like an artefact of two assumptions: (i) only three possible profits and (ii) high profit is a sure indication of a favorable common shock. In a more general continuum profits setting, this means that high profits can help eliminate, with a high probability, low common shocks. This, in turn, would make taxpayers avoid taking on undue risks by submitting a return that is too far below the realized profit – such submissions would make the auditor infer that the return is nontruthful. Thus, the tax authority's superior information of the common shock brings in discipline, eliminating exaggerated underreporting.

In addition, our results show that for moderate penalties for evasion (i.e., below a threshold rate), medium profit earners evade more, whereas for high penalties, high earners evade more. Under progressive taxation, there is more to gain from evasion for high profit earners, but there is also a higher penalty cost as the fine is proportional to the evaded tax. Empirically, it is perhaps a moot point which group of entrepreneurs, whether it is the more or less successful one, are bigger evaders. As for audits, when the common shock is favorable, low profit reports are audited more intensively than are medium reports.<sup>5</sup> This is because against the backdrop of some high returns, low returns naturally raise greater suspicion than medium returns.

We also derive an interesting comparative static. When the proportion of honest taxpayers increases, strategic taxpayers increase their rate of evasion. This is simply because the greater number of honest taxpayers provide a better cover for evaders, confounding the inspector in carrying out costly audits. On balance, audit intensity does not change.

<sup>2</sup> "IRS research has shown the largest amounts of noncompliance typically come from returns of taxpayers who own their own businesses, deal in large amounts of cash, receive payments that aren't subject to tax-withholding requirements and whose income isn't reported separately to the IRS by third parties." See <https://www.wsj.com/articles/chances-of-an-irs-audit-are-low-and-getting-lower-1490582326>.

<sup>3</sup> In field experiments on tax enforcement (Kleven et al., 2011), the tax evasion rate was significant for self-reported incomes. Much earlier, Pissarides and Weber (1989) observe that in 1982, Britain's "true self-employment income is 1.55 times as much as reported self-employment income." Other references on the importance of the informal sector in tax enforcement include studies by Ihriga and Moe (2004), Dabla-Norris et al. (2008), and Blackburn et al. (2012).

<sup>4</sup> High earners would know that the market shock must be favorable, whereas medium earners are uncertain.

<sup>5</sup> When the common shock is unfavorable, the auditor might not choose to audit at all.

In a more sophisticated model (see Section 4), individuals draw private types, *high* or *low*, according to a common distribution. Now with the idiosyncratic shock distribution depending on the entrepreneur's ability, the analysis shows richer strategic interactions between taxpayers and the tax authority. The ones drawing high (low) types will be more (less) likely to realize a high idiosyncratic draw. Thus, even with the same profit realization, two individuals will have different perceptions about the common shock if their types differ. This difference may lead them toward different tax submission strategies. For instance, an individual of high type who realizes medium profits is likely to think of it as a combination of innate high ability and a poor market shock. This will trigger underreporting. On the other hand, a low type with the same (medium) profit realization tends to attribute it to a favorable market shock. Thus, this individual would be reluctant to underreport. This not only generates heterogeneity in tax submissions by individuals with identical profits, which is more realistic, but also makes auditing a worthwhile guessing game.

**Literature review.** Villalba (2015)'s study is related to our study, but uses a much simpler model. His model assumes a collection of individuals whose incomes are perfectly correlated, drawn from  $\{0, 1\}$  according to a given distribution. The tax authority audits low declarations with a probability increasing in the average declaration. This induces taxpayers to coordinate their declarations with the possibility of multiple equilibria.

Our model substantially differs from Villalba's. By introducing idiosyncratic shocks, we can generate heterogeneity in profits. Our common shock mirrors Villalba's perfectly correlated income idea, except that our tax agency has the information about the common component. Given this difference, unlike in Villalba, individual filers are unable to precisely infer others' profits and the coordination among submitters will fail. The idiosyncratic shock also shifts the distribution of information more realistically – individuals are better informed about their individual shocks, whereas tax authorities should be better informed about the market shock; in Villalba, the tax authorities are completely uninformed whereas individuals are perfectly informed.

We also allow some taxpayers to be intrinsically honest. This seems to be a plausible description of the characteristics of tax-paying citizens. (Posner, 2000, page 1782) for instance,<sup>6</sup> observes that,

*A widespread view among tax scholars holds that law enforcement does not explain why people pay taxes. The penalty for ordinary tax convictions is small; the probability of detection is trivial; so the expected sanction is small. Yet large numbers of Americans pay their taxes. ... Some scholars therefore conclude that the explanation for the tendency to pay taxes must be that people are obeying a norm – presumably a norm of tax payment or a more general norm of law-abiding behavior.*

The fraction of strategic submitters will possibly underreport incomes, so de-trending the whole collection of tax returns and inverting the market shock is a difficult learning exercise. However, with a sufficiently high number of individuals, the population will contain some honest taxpayers whose high submission would almost perfectly reveal that the market shock has been favorable. Any absence of high returns would equally indicate that the market shock has been unfavorable. This setting places the tax authority in a unique position to learn about the market shock from aggregate data that will be unavailable to most submitters. Even without the presence of honest taxpayers, we have two reasons to believe that the tax authority holds better information about the common shock: (1) it can make better inferences from aggregate tax returns data and (2) the IRS makes 'Projections of Federal Tax Return Filings' (<https://www.irs.gov/statistics/projections-of-federal-tax-return-filings#p6961>), indicating it has more reliable information about underlying economic conditions.

Our study also shares some features with Erard and Feinstein (1994). Like us, these authors assume both honest and strategic taxpayers who face endogenously derived (i.e., without commitment) auditing. Their main theoretical finding resolves the earlier models' unrealistic prediction that taxpayers would underreport incomes by a constant amount such that the tax authority could infer the true incomes precisely. Additionally, they conclude that there is no reason to believe that low-income reports should necessarily be audited more intensively. Our model differs in two important respects. First, we introduce idiosyncratic and common shocks that generate differing informational positions, from which (strategic) taxpayers and the inspector formulate their strategies. The inspector decides on auditing based not only on reported incomes, but also on information about common shock gathered from the aggregate tax returns data. Erard and Feinstein generate the incomes from an exogenously given distribution function with no role for variations in common market conditions. Second, we do not consider a fixed budget for auditing. These differences lead to a very different tax audit game. With our restriction to three discrete profit levels, we derive explicitly the equilibrium audit and tax evasion probabilities that, in turn, enable us to derive some meaningful comparative statics.

Another work addressing a similar issue, but with very different modelling is that by Bigio and Zilberman (2011). The authors consider profit reporting by self-employed entrepreneurs as we do, but recognize that the entrepreneurs must be hiring workers for their enterprise. They analyze the tax collection agency's (the IRS's) problem. The IRS audits based only on the profit report: if the reported profit is below a threshold, then they conduct a random audit, resulting in only low-ability entrepreneurs reporting honestly. This result is similar to our result for the extended model (in Section 4), where high-ability entrepreneurs are more prone to underreporting. However, our logic is very different.

<sup>6</sup> See also Kopczuk (2001), who describes such behavior as "pathological honesty" (refer to Section 2 of the article). Earlier, Graetz et al. (1986) and Erard and Feinstein (1994) analyzed tax evasion in the presence of honest taxpayers and observed that the predictions of tax evasion models change significantly as a result.

Our observation of differing tax evasion behavior (in Section 4) relates both theoretically and empirically to similar issues examined in prior studies, such as those by [Kopczuk \(2001\)](#) and [Alstadsaeter et al. \(2019\)](#). This latter study deserves special mention in our context as the authors find that offshore tax evasion is highly concentrated among the wealthy.<sup>7</sup> Our assumption that entrepreneurs know their types is only a reduced-form simplification; entrepreneurs' history of successes or failures in the business, giving rise to high or low accumulated wealth that will build up their reputations and confidence in their own abilities. While we do not analyze a dynamic model, in our one-shot static model, high (low) ability can be linked to high (low) wealth, and thus the evasion behavior, as our theoretical analysis predicts, can be mapped to the empirical observation of [Alstadsaeter et al. \(2019\)](#). [Kopczuk's](#) paper provides a theoretical model.

[Border and Sobel \(1987\)](#), and later [Chander and Wilde \(1998\)](#), use direct revelation mechanisms to derive the optimal tax schedules and enforcement schemes, i.e., the audit strategy and a penalty function. [Border and Sobel's](#) mechanism aimed to guarantee "audit efficiency" whereas [Chander and Wilde's](#) mechanism sought to achieve a broader goal of "efficiency," i.e., maximization of tax revenues plus penalty less audit costs.<sup>8</sup> The Revelation Principle requires the mechanism designer, which is the tax authority in our application, to commit to the mechanism (e.g., [Poitevin, 2000](#); [Mookherjee, 2006](#)). These authors use truth-telling constraints to determine the feasible programs for their optimal taxation and enforcement objectives. In contrast, we do away with commitment, take the tax rates and punishments as given exogenously, and study the equilibrium audit and evasion strategies.

We also differ from earlier papers in the literature on tax enforcement (e.g., [Allingham and Sandmo, 1972](#); [Reinganum and Wilde, 1985](#); [Cremer et al., 1990](#)). A common feature in these studies is the assumption that the tax department can commit to an audit strategy, and often the commitment was in the form of a threshold (or cutoff for reported incomes) audit strategy that naturally deterred taxpayers from underreporting too much. Our tax authority's superior information and flexible audit rule achieves the same discipline to keep taxpayers from underreporting too much, as we discussed at the beginning of this Introduction.<sup>9,10</sup>

The rest of the paper is organized as follows. We present the tax auditing game in Section 2. We provide the main analysis in Sections 3 and 4. In Section 5, we conclude by discussing an alternative tax-enforcement mechanism. All proofs are in the Appendix. A supplementary file contains two additional results.

## 2. Model

There are  $n$  self-employed individuals (or entrepreneurs) whose profits in a one-period economy can be either *high*, *medium*, or *low*.<sup>11,12</sup> We refer to them as taxpayers, indexed by  $i$ , with an  $\alpha$  proportion being *honest* and  $1 - \alpha$  proportion being *strategic*,  $0 < \alpha < 1$ .<sup>13</sup> An honest taxpayer will always report truthfully, whereas a strategic taxpayer will evade if the (net) expected benefit of evasion is positive. For the purpose of evasion, taxpayers are risk neutral.

We will assume  $\alpha$  to be common knowledge, known to both the tax authority and the taxpayers. The common knowledge assumption can be justified based on (i) country-specific *tax morale* (see the discussion in [Luttmer and Singhal \(2014\)](#)) and (ii) tax authorities' estimates of the "residual" proportion of honest taxpayers based on past data.<sup>14</sup>

Taxpayer  $i$ 's profit  $\pi_i$  consists of two parts, an idiosyncratic shock  $x_i$  and a common shock  $\epsilon$ , and is given by  $\pi_i = x_i + \epsilon$ . We assume that  $x_i \in \{0, 1\}$  are i.i.d. draws, with  $\Pr(x_i = 0) = q$ ,  $0 < q < 1$ . In Section 4, we assume that the idiosyncratic shock depends on the entrepreneur's type (or ability). The common shock is also binary,  $\epsilon \in \{0, 1\}$  (where  $\epsilon = 1$  is *favorable*, and  $\epsilon = 0$  is *unfavorable*), with  $\Pr(\epsilon = 0) = p$ ,  $0 < p < 1$ , and is independent of idiosyncratic shocks. Thus, profits are

<sup>7</sup> [Landier and Plantin \(2017\)](#) observe that the rich will have better access to a tax-avoidance technology, the implication being that the rich are more likely to evade.

<sup>8</sup> Consider a mechanism consisting of a triplet of {tax function, audit rule, penalty function}. Roughly, audit efficiency means that there is no other mechanism such that at least the same expected gross revenue (from tax and penalty combined) can be collected for any given distribution of taxpayer wealth, as in the original mechanism without increasing the audit probability for some ranges of income reports. [Chander and Wilde's](#) notion of efficiency is broader in that they also allow a principal to seek to maximize a welfare function that includes both (audit) efficiency and equity, in addition to audit efficiency.

<sup>9</sup> [Melumad and Mookherjee \(1989\)](#) endogenize commitment to an audit policy through the government's delegation to a tax agency in the form of an incentive contract: reward the agency for meeting an audit budget target and give a proportion of the fines collected to the agency.

<sup>10</sup> In [Reinganum and Wilde \(1986\)](#), auditing is endogenous rather than part of a commitment mechanism. [Yitzhaki \(1987\)](#) assumes the probability of being caught to be an increasing function of the undeclared income. In [Cremer and Gahvari \(1994\)](#), a tax evader can affect the probability of being caught, if audited, by spending money to conceal evasion.

<sup>11</sup> In the following analysis, we use the words "profit" and "income" interchangeably. Self-employment does play an important role in our model. If the subjects are employed in companies or institutions for which their salaries are paid regularly and recorded, then it will be difficult for them to hide their incomes during tax declaration. On the other hand, we are not interested in incomes obtained from many different sources either, as the common shock feature may not be present. Therefore, we focus on the self-employed individuals who experience the industry-specific common shock and who also have opportunities to evade taxes.

<sup>12</sup> An analysis of a multi-period economy is more involved, and we intend to pursue this line of study in the future.

<sup>13</sup> For a similar setup, see [Erard and Feinstein \(1994\)](#).

<sup>14</sup> The residual (proportion of) honesty is honesty after accounting for strategic honesty due to enforcement such as the intensity of monitoring and penalties for evasion. [Erard and Feinstein \(1994\)](#) (see Section 4 of their article) report a study by [Alexander and Feinstein \(1987\)](#), based on 1982 IRS audit data, that, "as many as one-half of all taxpayers may in fact have honest intentions." [Erard and Feinstein](#) also discuss evidence of inherent honesty reported by economists and sociologists ([Graetz et al., 1986](#); [Alm et al., 1992](#); [Alexander and Feinstein, 1987](#); [Spicer and Lundstedt, 1976](#); [Smith and Stalans, 1991](#)) to motivate their theoretical analysis, similar to ours, of the impact of taxpayers' intrinsic honesty on tax compliance and enforcement.

positively correlated. Moreover, if the realization of the common shock is 0, then no entrepreneurs realize profit 2, and if the common shock is 1, then the profit is at least 1. Due to the presence of idiosyncratic shocks, there will be some heterogeneity in profits.

**Tax-Audit Game.** Taxpayers will submit profit reports for auditing by the tax inspectors, henceforth inspector(s), who are interested in recovering evaded taxes plus fines. We do not consider the moral hazard problem of auditing. On conducting an audit, the true profit will be uncovered with certainty. Additionally, we assume that the inspectors are honest, so they do not collude with taxpayers to suppress underreporting. Tax filings and inspections give rise to an extensive form game that runs as follows:

1. The tax department announces the fine structure, and the common shock and idiosyncratic shocks are drawn.
2. Taxpayers observe their individual profit realizations, form beliefs about idiosyncratic and common shocks, and submit tax returns simultaneously and independently.
3. Upon receiving all tax returns, the tax department updates its belief about the common shock and informs the inspectors.<sup>15</sup>
4. The inspectors then make auditing decisions on the tax returns within their jurisdiction, recover evaded taxes, and impose fines appropriate for the extent of underreporting.

While many inspectors are involved in auditing for the tax department, we conduct our analysis using a representative inspector.

We look for a *Perfect Bayesian equilibrium* (PBE). In particular, for information sets on the equilibrium path, the inspector's beliefs are determined by the Bayes' rule using equilibrium tax submission strategies, and the audit strategy must be sequentially rational given the inspector's beliefs. If any of the three tax returns,  $\{0, 1, 2\}$ , is off the equilibrium path, then the Bayes' rule should be applied wherever possible; otherwise, the inspector is free to assign any belief and determine the audit decision accordingly.

■ **Learning the common shock.** It is trivial that for at least one report of 2, the tax authority can infer that  $\epsilon$  is 1 because no one will over-report profit. Conditional on the common shock being 1, the probability of at least one submission of 2 by an honest taxpayer is

$$1 - q^{\alpha n}. \quad (1)$$

For a sufficiently large  $n$ , the probability (1) is close to 1, so it is almost certain that at least one taxpayer will report 2 when the common shock is 1. This also implies that a complete absence of a report of 2 informs the tax authority that  $\epsilon$  is almost certainly 0. In the benchmark model, we assume that the tax authority does not know the common shock. From Section 3.2 onwards, we assume that  $n$  is sufficiently large so the tax authority will act in full knowledge of  $\epsilon$ .

■ **Differential auditing costs.** The auditing cost will depend on the inspector's knowledge about the particular group of taxpayers being audited: if the inspector knows that the market shock is favorable and yet the submitted return is low, then the taxpayer clearly lied, so finding the relevant evidence of the lie is relatively less costly, which we denote by  $c_1 \geq 0$ . On the other hand, if no such inference is possible, then the audit cost is higher, denoted by  $c_2$ , where  $c_2 \geq c_1$ .<sup>16</sup> The *differential auditing cost* assumption is somewhat different from the treatment in earlier studies, such as that by Reinganum and Wilde (1986), who assume that despite knowing a taxpayer has evaded, the inspector would need to incur different costs to prove evasion with different probabilities of success. In our model, the inspector spends different costs based on inferential certainty or uncertainty about the evasion behavior. However, the evasion, if any, will be detected with certainty.

■ **Tax and penalty structures.** Each individual filer needs to pay a tax based on the reported profit at a *progressive* rate, starting with zero tax at profit of 0. If the reported profit is 1, then the corresponding tax is  $t_1$ , where  $0 < t_1 < 1$ ; if the reported profit is 2, then the corresponding tax is  $t_1 + t_2$ , where  $0 < t_2 < 1$  and  $t_2 \geq t_1$ . If the taxpayer is audited and found to have under-reported, then the taxpayer must pay back the evaded tax plus a penalty. The penalty is proportional to the amount of evaded tax at the rate of  $f$ : when underreporting by one unit (of profit), the penalty is  $ft_1$  if the taxpayer's true profit is 1 and the penalty is  $ft_2$  if the taxpayer's true profit is 2; if the underreporting is by two units, then the penalty is  $f(t_1 + t_2)$ .<sup>17</sup>

<sup>15</sup> We also report a benchmark case in which the tax department is naive and does not update its belief.

<sup>16</sup> For the generality of our results, we allow for  $c_2 = c_1$  and  $c_1 = 0$ . However, we consider the assumption  $c_2 > c_1 > 0$  to be more plausible for several reasons. Some documents on financial transactions may have ambiguity. For example, while the transaction may have occurred at a certain date, it could be for a service provided at a much earlier date. When the inspector "knows" that a particular taxpayer has evaded, the inspector can examine the financial transaction documents with more certainty and the investigation will possibly involve less scrutiny than if the inspector did not know about the truthfulness of the taxpayer's tax submissions. In the language of communication games (Lipman and Seppi, 1995), the very inferential knowledge of one's non-truthfulness expands the *provability* of the same accusation (of nonpayment) than if the same inference cannot be made with certainty. At the minimum, the inspector's more confident questioning of the authenticity of certain documents (e.g., the number of hours worked for a certain job, start and end date of completion, etc.) submitted by the taxpayer is likely to lead to a natural unraveling of the truth. The inspector can even verify from the other party involved in the financial transaction whether the amount of payment reported for the job is accurate or not. This is easily implemented, say, if the buyer in a transaction can receive a credit for the VAT (value-added tax) by submitting a sales receipt. When the inspector is not certain about the accuracy of a tax submission, the inspector might not know where to look for evidence corroborating the lie.

<sup>17</sup> The maxim that the penalty should reflect the severity of the crime dates back to Stigler (1970). Andreoni (1991) offers a "reasonable doubt test" explanation of increasing fines that does not necessarily imply progressive fines.

### 3. Analysis of tax-audit game

A typical taxpayer’s strategy is a function that maps  $\pi_i$  to  $\hat{\pi}_i$ , where  $\pi_i$  and  $\hat{\pi}_i$  are the true and declared profits, respectively.<sup>18</sup> Let  $\gamma(\hat{\pi}_i|\pi_i)$  denote the probability of reporting  $\hat{\pi}_i$  when the true profit is  $\pi_i$ . It is trivial that no one will report a profit higher than the true profit. Thus, those receiving zero profit will report only 0 and those realizing a profit of 1 will never report 2; that is,  $\gamma(0|0) = 1$  and  $\gamma(2|1) = 0$ .

We consider the audit to be very much in the textbook style, with the audit to be conducted whenever it is justified by the cost-benefit tradeoff. Underlying this approach is the important assumption that audits are run in a profit-maximizing style without concern for a fixed audit budget. We specify the auditing strategy later.

We impose the following assumption for the rest of the paper.

**Assumption 1.** The costs of auditing are never so excessive to rule out auditing as a viable strategy from the start:

$$(1 + f)t_1 > c_2. \tag{2}$$

Assumption 1 also implies

$$(1 + f)t_2 > c_2, \tag{3}$$

and  $(1 + f)t_1 > c_1, \tag{4}$

because  $t_2 \geq t_1$  and  $c_2 \geq c_1$ .

#### 3.1. Benchmark: Tax authority uninformed about the common shock

First, we start with a benchmark case in which the tax authority is uninformed about the common shock  $\epsilon$ . Instead of using complicated belief updating based on the collection of reports, the tax inspector applies a “naive” strategy; that is, the auditing rule depends only on the individual reported profit:  $\rho(\hat{\pi}_i)$ . In particular, even if the tax inspector sees a report of 2, the inspector does not use the information that the common shock is favorable and alter the auditing of any non-maximal submission. We now present the taxpayers’ and inspector’s problems separately.

■ **Taxpayers’ problem.** Given that the inspector’s audit strategy depends only on the reported profit, updating about the common shock is of no relevance to a taxpayer.

Taxpayer  $i$  with  $\pi_i = 0$  would always report 0.

If  $\pi_i = 2$ , then we can express  $i$ ’s expected utility for different reports as follows:

$$EU_i(\hat{\pi}_i = 2|\pi_i = 2) = 2 - t_1 - t_2,$$

$$\begin{aligned} EU_i(\hat{\pi}_i = 1|\pi_i = 2) &= \rho(1)(2 - t_1 - t_2 - ft_2) + [1 - \rho(1)](2 - t_1) \\ &= 2 - t_1 - \rho(1)(1 + f)t_2, \end{aligned}$$

$$\begin{aligned} EU_i(\hat{\pi}_i = 0|\pi_i = 2) &= \rho(0)(2 - t_1 - t_2 - f(t_1 + t_2)) + [1 - \rho(0)] \times 2 \\ &= 2 - \rho(0)(1 + f)(t_1 + t_2). \end{aligned}$$

If  $\pi_i = 1$ , then we can express the expected utility as:

$$EU_i(\hat{\pi}_i = 1|\pi_i = 1) = 1 - t_1,$$

$$EU_i(\hat{\pi}_i = 0|\pi_i = 1) = 1 - \rho(0)(1 + f)t_1.$$

For each profit realization, the taxpayer will choose the profit report that leads to the highest expected utility.

■ **Tax inspector’s problem.** Given any tax rate and fine structure, the authority’s objective is to maximize tax revenue net of audit cost.<sup>19</sup> To assess the extent of underreporting, the inspector will use the prior belief of the common shock  $\epsilon$ .

For any report of 1, it will be the case that

1. the common shock is 1, and
  - (a) the honest taxpayer with  $x_i = 0$  (probability is  $\alpha(1 - p)q$ ), or
  - (b) the strategic taxpayer with  $x_i = 1$  and underreports by 1 unit (probability is  $(1 - \alpha)(1 - p)(1 - q)\gamma(1|2)$ ), or
  - (c) the strategic taxpayer with  $x_i = 0$  and reports truthfully (probability is  $(1 - \alpha)(1 - p)q[1 - \gamma(0|1)]$ );
2. the common shock is 0, and
  - (a) the honest taxpayer with  $x_i = 1$  (probability is  $\alpha p(1 - q)$ ), or
  - (b) strategic taxpayer with  $x_i = 1$  and reports truthfully (probability is  $(1 - \alpha)p(1 - q)[1 - \gamma(0|1)]$ ).

<sup>18</sup> The strategy can be pure or/and mixed.

<sup>19</sup> The maximization of net revenue collections as an objective was previously studied by [Scotchmer \(1989\)](#) and [Cremer et al. \(1990\)](#), among others.

Thus, using Bayes' rule, the inspector's updated belief about the (accuracy of) the taxpayer's submission is

$$\begin{aligned} \Pr(\pi_i = 2|\hat{\pi}_i = 1) &= \frac{(1 - \alpha)(1 - p)(1 - q)\gamma(1|2)}{\alpha(1 - p)q + (1 - \alpha)(1 - p)(1 - q)\gamma(1|2) + (1 - \alpha)(1 - p)q[1 - \gamma(0|1)] + \alpha p(1 - q) + (1 - \alpha)p(1 - q)[1 - \gamma(0|1)]} \\ &= \frac{(1 - \alpha)(1 - p)(1 - q)\gamma(1|2)}{[(1 - p)q + p(1 - q)][\alpha + (1 - \alpha)(1 - \gamma(0|1))] + (1 - \alpha)(1 - p)(1 - q)\gamma(1|2)}. \end{aligned} \tag{5}$$

The inspector compares the expected (net) profit from auditing a particular taxpayer reporting 1,  $\Pr(\pi_i = 2|\hat{\pi}_i = 1) \times (t_1 + t_2 + ft_2) + (1 - \Pr(\pi_i = 2|\hat{\pi}_i = 1)) \times t_1 - c_2$ , with the profit from not auditing, which is  $t_1$ .<sup>20</sup> This is equivalent to comparing  $\Pr(\pi_i = 2|\hat{\pi}_i = 1) \times (1 + f)t_2 - c_2$  with 0.

For any report of 0, it will be the case that

1. the common shock is 1, and
  - (a) the strategic taxpayer with  $x_i = 0$  and underreports (probability is  $(1 - \alpha)(1 - p)q\gamma(0|1)$ ), or
  - (b) the strategic taxpayer with  $x_i = 1$  and underreports by 2 units (probability is  $(1 - \alpha)(1 - p)(1 - q)\gamma(0|2)$ );
2. the common shock is 0, and
  - (a) the honest taxpayer with  $x_i = 0$  (probability is  $\alpha pq$ ) or
  - (b) the strategic taxpayer with  $x_i = 0$  and reports truthfully (probability is  $(1 - \alpha)pq$ ), or
  - (c) strategic taxpayer with  $x_i = 1$  and underreports (probability is  $(1 - \alpha)p(1 - q)\gamma(0|1)$ ).

Again, using Bayes' rule, the inspector's updated beliefs about the taxpayer's submission are

$$\begin{aligned} \Pr(\pi_i = 1|\hat{\pi}_i = 0) &= \frac{(1 - \alpha)(1 - p)q\gamma(0|1) + (1 - \alpha)p(1 - q)\gamma(0|1)}{(1 - \alpha)(1 - p)q\gamma(0|1) + (1 - \alpha)(1 - p)(1 - q)\gamma(0|2) + \alpha pq + (1 - \alpha)pq + (1 - \alpha)p(1 - q)\gamma(0|1)} \\ &= \frac{(1 - \alpha)[(1 - p)q + p(1 - q)]\gamma(0|1)}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]\gamma(0|1) + (1 - \alpha)(1 - p)(1 - q)\gamma(0|2)}. \end{aligned} \tag{6}$$

$$\begin{aligned} \Pr(\pi_i = 2|\hat{\pi}_i = 0) &= \frac{(1 - \alpha)(1 - p)(1 - q)\gamma(0|2)}{(1 - \alpha)(1 - p)q\gamma(0|1) + (1 - \alpha)(1 - p)(1 - q)\gamma(0|2) + \alpha pq + (1 - \alpha)pq + (1 - \alpha)p(1 - q)\gamma(0|1)} \\ &= \frac{(1 - \alpha)(1 - p)(1 - q)\gamma(0|2)}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]\gamma(0|1) + (1 - \alpha)(1 - p)(1 - q)\gamma(0|2)}. \end{aligned} \tag{7}$$

Again, the inspector compares the expected gain from auditing a particular taxpayer reporting 0,  $\Pr(\pi_i = 1|\hat{\pi}_i = 0)(1 + f)t_1 + \Pr(\pi_i = 2|\hat{\pi}_i = 0)(1 + f)(t_1 + t_2) - c_2$ , with 0.

■ **Analysis of the problem.** We will present one equilibrium of the above model and discuss its properties.

**Proposition 1 (Two-step underreporting).** Suppose

$$\alpha < \frac{pq(1 - p)(1 - q)}{[(1 - p)q + p(1 - q)]^2}, \text{ and } \frac{(1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]} < c_2 < \frac{(1 - \alpha)(1 - p)(1 - q)(1 + f)t_2 + (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{1 - \alpha(1 - p)(1 - q)}.$$

Then, the following constitute a PBE of the tax returns/auditing game:

- the taxpayers submit returns according to the strategy

$$\begin{aligned} \gamma(0|2) &= \frac{pq c_2 - (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1 - c_2}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)(t_1 + t_2) - c_2]}, \\ \gamma(1|2) &= \frac{[(1 - p)q + p(1 - q)]\alpha c_2}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)t_2 - c_2]}, \\ \gamma(2|2) &= 1 - \gamma(0|2) - \gamma(1|2), \quad \gamma(0|1) = 1, \quad \gamma(1|1) = 0; \text{ and} \end{aligned}$$

- the inspector audits according to the strategy  $\rho(0) = \frac{1}{1+f}$  and  $\rho(1) = \frac{1}{1+f}$ .

<sup>20</sup> The inspector will need to incur a higher auditing cost,  $c_2$ , given the lack of knowledge about the common shock as well as the truthfulness of the submission.

In this equilibrium, the high profit earners may report truthfully or underreport up to two steps, while middle profit earners always underreport. Two-step underreporting can be easily explained: given that tax authorities do not have the information about the common shock, high profit earners can take calculated risks of more extensive evasion. Risk-taking is no lower among middle profit earners either – they discern that tax authorities attach a positive probability that the common shock is unfavorable and so zero profit is a possible scenario. With the positive cost of auditing, the inspector would audit only randomly, and so taking a chance on no-auditing is worthwhile. Note that the extensive underreporting is not an artefact of the discretization of profits. If profits were a continuum, then some taxpayers’ evasion can stretch up to whatever realizations seem feasible with a positive probability. As we shall soon see, when tax authorities are better informed about the common shock, very high profit earners would know that the common shock is favorable and the tax authorities know that the left tail of the profits must be nontruthful reports, thus engaging in less extensive underreporting.

**Corollary 1.** *The following comparative statics of the taxpayers’ and inspectors’ strategies follow from Proposition 1:*

- When  $f$  increases, both medium and high profit earners evade (weakly) less often, and inspectors audit those reporting 1 and 0 less intensively.
- When  $\alpha$  increases, both medium and high profit earners evade (weakly) more often.

When the fines increase, taxpayers are less likely to evade, so the probability of underreporting and audit intensity will drop. This reflects the familiar substitution between punishment and monitoring in deterring crimes (Becker, 1968). When the proportion of honest taxpayers increases, evaders increase underreporting due to a better cover for their lie. This last observation is interesting when viewed against the perspective in prior works that corruption begets more corruption due to its increasing social acceptability (Rasmusen, 1996). In the alternative view of people’s attitude towards corruption due to cultural norms, then an increase in the proportion of honest taxpayers should lower the rate/extent of evasion due to the increased stigma.

### 3.2. Tax authority informed about the common shock

In our model, the only source of correlation in taxpayers’ profits is the common shock. Now, given that the tax authority can precisely infer the common shock from the reported profits (for the reason discussed in “Learning the common shock” in Section 2), we can simply focus on the audit strategy based on the information about the common shock,  $\epsilon$ , and the individual reported profit but no other aggregate measure such as the average return.<sup>21</sup> Let the audit probability for a particular taxpayer  $i$  be denoted by  $\rho(\hat{\pi}_i|\epsilon)$  if the inspector learns that the common shock is  $\epsilon$  and  $i$ ’s reported profit is  $\hat{\pi}_i$ . Clearly, the inspector will never audit a report of 2; that is,  $\rho(2|1) = \rho(2|0) = 0$ .<sup>22</sup> Additionally,  $\rho(1|0) = 0$  because a report of 1 must be truthful given that the common shock is  $\epsilon = 0$ .

■ **Taxpayers’ problem.** Upon realizing the profit, a taxpayer will update his or her belief about both the common and idiosyncratic shocks.

For taxpayer  $i$ , if  $\pi_i = 0$ , then the taxpayer knows that  $\epsilon = 0$ , but this knowledge is immaterial as he or she would always report 0.

If  $\pi_i = 2$ , then the taxpayer knows that  $\epsilon = 1$  (and  $x_i$  is also 1), so we can express the expected utility for different reports as follows:

$$\begin{aligned} EU_i(\hat{\pi}_i = 2|\pi_i = 2) &= 2 - t_1 - t_2, \\ EU_i(\hat{\pi}_i = 1|\pi_i = 2) &= \rho(1|1)(2 - t_1 - t_2 - ft_2) + [1 - \rho(1|1)](2 - t_1) \\ &= 2 - t_1 - \rho(1|1)(1 + f)t_2, \\ EU_i(\hat{\pi}_i = 0|\pi_i = 2) &= \rho(0|1)(2 - t_1 - t_2 - f(t_1 + t_2)) + [1 - \rho(0|1)] \times 2 \\ &= 2 - \rho(0|1)(1 + f)(t_1 + t_2). \end{aligned}$$

If  $\pi_i = 1$ , then the taxpayer does not know the true common shock and the expected utility is

$$\begin{aligned} EU_i(\hat{\pi}_i = 1|\pi_i = 1) &= 1 - t_1, \\ EU_i(\hat{\pi}_i = 0|\pi_i = 1) &= p_1[\rho(0|0)(1 - t_1 - ft_1) + (1 - \rho(0|0)) \times 1] \\ &\quad + (1 - p_1)[\rho(0|1)(1 - t_1 - ft_1) + (1 - \rho(0|1)) \times 1], \end{aligned}$$

where  $p_1$  is the updated belief that the common shock is 0 given that the profit is 1; that is,

$$p_1 \equiv \Pr(\epsilon = 0|\pi_i = 1) = \frac{p(1 - q)}{p(1 - q) + (1 - p)q}. \tag{8}$$

<sup>21</sup> Villalba (2015) considers the audit probability to be declining in the average return due to a perfect correlation in profits and the tax authority not knowing the common shock. He did not consider the presence of honest taxpayers. One deficiency of focusing only on the common shock is that tax authorities should immediately know the true profit after auditing even one tax submission. So even under a budget constraint for auditing, non-truthful reporting can be easily broken if either the penalty is sufficiently large or an additional auditing budget is sought from the higher-up fiscal planning department in the knowledge that all taxpayers are lying. This unraveling does not happen under idiosyncratic shocks.

<sup>22</sup>  $\rho(2|0)$  is off the equilibrium path.



For each profit realization, the taxpayer will choose the profit report that leads to the highest expected utility.

■ **Tax inspector’s problem.** Again, the authority’s objective is to maximize tax revenue net of audit cost. We separate the analysis into two cases.

**Economy in a good state**

The group reporting 1 may consist of honest taxpayers, strategic taxpayers who report truthfully, and strategic taxpayers with profit 2 but who report 1. Given the reporting strategy, in particular  $\gamma(1|1)$  and  $\gamma(1|2)$ , we can calculate the inspector’s beliefs about the truthfulness of tax returns.

Conditional on the common shock being 1, for any particular taxpayer, the probability of being honest with the true profit of 1 is  $\alpha q$ ; the probability of being strategic with the true profit of 1 and reporting truthfully is  $(1 - \alpha)q\gamma(1|1)$ ; and the probability of being strategic with the true profit of 2 and reporting 1 is  $(1 - \alpha)(1 - q)\gamma(1|2)$ . Thus, using Bayes’ rule, the inspector’s updated belief about the (accuracy of) the taxpayer’s submission is

$$\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) = \frac{(1 - \alpha)(1 - q)\gamma(1|2)}{\alpha q + (1 - \alpha)q\gamma(1|1) + (1 - \alpha)(1 - q)\gamma(1|2)}. \tag{9}$$

The inspector compares the expected (net) profit from auditing a particular taxpayer reporting 1,  $\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) \times (t_1 + t_2 + ft_2) + (1 - \Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1)) \times t_1 - c_2$ , to the profit from not auditing, which is  $t_1$ .<sup>23</sup> This is equivalent to comparing  $\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) \times (1 + f)t_2 - c_2$  to 0.

We address the analysis for the group reporting 0 in Proposition 2.

**Economy in a bad state**

Given the reporting strategy  $\gamma(0|1)$ , for any individual submitting a return of 0 profit, the inspector’s updated belief that the taxpayer is honest is  $\alpha q$ ; the probability that the taxpayer is strategic and realized 0 profit is  $(1 - \alpha)q$ ; and the probability that the taxpayer is strategic with a profit of 1 but reporting 0 is  $(1 - \alpha)(1 - q)\gamma(0|1)$ . Again, using Bayes’ rule, the inspector’s updated belief about the truthfulness of a submission is

$$\Pr(\pi_i = 1|\hat{\pi}_i = 0, \epsilon = 0) = \frac{(1 - \alpha)(1 - q)\gamma(0|1)}{q + (1 - \alpha)(1 - q)\gamma(0|1)}. \tag{10}$$

The inspector compares the expected (net) profit from auditing a particular taxpayer reporting 0,  $\Pr(\pi_i = 1|\hat{\pi}_i = 0, \epsilon = 0) \times (1 + f)t_1 - c_2$ , with the profit from not auditing, which is 0.

■ **Equilibrium Analysis.** Hereafter, we present the equilibrium results describing who might evade more, when the inspector likely to be more vigilant, and which group the inspector should concentrate on.

**Proposition 2.**

1. The inspector with the information that  $\epsilon = 1$  will audit any return of zero profit with probability 1; that is,  $\rho(0|1) = 1$ . In addition, any strategic taxpayer with a profit of 2 never reports 0 in equilibrium; that is,  $\gamma(0|2) = 0$ .
2. For any strategic taxpayer, always reporting truthfully when the profit is 2 cannot occur in equilibrium; that is,  $\gamma(2|2) \neq 1$ .
3. Auditing a report of 0 with probability 1 when the inspector knows that the common shock is 0 cannot occur in equilibrium; that is,  $\rho(0|0) \neq 1$ .

The taxpayers with profit  $\pi = 2$  know that the inspector will learn that the common shock has been favorable, so reporting  $\hat{\pi} = 0$  will inevitably lead to auditing and detection. They are also aware that they can hide behind at least some truthful submissions by reporting  $\hat{\pi} = 1$ . On the other hand, when the common shock is unfavorable, some truthful low submissions are expected, so mixed strategy audits for low reports is likely to be more sensible for the inspector. In the following propositions, we will identify these evasion and audit behaviors more precisely.

One point we would like to highlight here is that in our benchmark case, where the tax authority does not know the common shock or disregards the information about the common shock and uses a “naive” auditing strategy, we do observe that high profit earners’ severe underreporting can occur in equilibrium. However, with precise knowledge about the occurrence of a favorable common shock, the tax inspector can infer the “two-step” underreporting as nontruthful, so high profit earners would engage in only “one-step” underreporting, if at all.

Our next result predicts randomized audit and evasion in equilibrium with useful comparative statics.

**Proposition 3 (Mixed strategies).** Suppose the penalty rate is moderate, i.e.,  $f < \frac{p_1}{1-p_1}$ , and the auditing costs are also moderate, i.e.,  $c_2 < \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q}$ .

1. The following strategies constitute a PBE of the tax-returns/inspection game.<sup>24</sup>

<sup>23</sup> The inspector will need to incur a higher auditing cost,  $c_2$ , given that the pool submitting  $\hat{\pi}_i = 1$  contains some truthful submissions.

<sup>24</sup> We can derive the inspector’s belief from (9) and (10) by substituting the taxpayers’ equilibrium strategy. We omit this derivation from the description of the equilibrium presented here and later.

*Taxpayers' return submission strategies:*<sup>25</sup>

- (a) If the realized profit is  $\pi = 2$ , then the taxpayer will submit a return of  $\hat{\pi} = 1$  with a probability of  $\gamma(1|2) = \frac{c_2 q [(1-q)(1+f)t_1 - c_2]}{(1-\alpha)(1-q)^2((1+f)t_1 - c_2)((1+f)t_2 - c_2)}$ , and a return of  $\hat{\pi} = 2$  with a probability of  $1 - \gamma(1|2)$ .
- (b) If the realized profit is  $\pi = 1$ , then the taxpayer will submit a return of  $\hat{\pi} = 0$  with a probability of  $\gamma(0|1) = \frac{c_2 q}{(1-\alpha)(1-q)((1+f)t_1 - c_2)}$ , and a return of  $\hat{\pi} = 1$  with a probability of  $1 - \gamma(0|1)$ .

*Inspector's audit strategy:*<sup>26</sup>

- (a) If the common shock is  $\epsilon = 1$ , then the inspector audits  $\hat{\pi} = 1$  with a probability of  $\rho(1|1) = \frac{1}{1+f}$ .
- (b) If the common shock is  $\epsilon = 0$ , then the inspector audits  $\hat{\pi} = 0$  with a probability of  $\rho(0|0) = \frac{p_1(1+f) - f}{p_1(1+f)}$ .

2. Taxpayers with medium profits ( $\pi = 1$ ) underreport more often than those with high profits ( $\pi = 2$ ):  $\gamma(0|1) > \gamma(1|2)$ .

**Proposition 3** is an intuitive prediction with the inspector and taxpayers playing cat-and-mouse games that manifest in mixed strategies. Because the tax authority learns the common shock (almost) surely and there is no aggregate budget constraint, auditing reduces to the inspector dealing with tax returns individually based on the expected cost-benefit calculation.

The uncertainty about the inspector's auditing is more substantial for entrepreneurs with medium profits because they cannot pin it down uniquely to idiosyncratic or market shock. They must therefore take a chance on reporting zero profit and sometimes become easy for the inspector to catch.

The result that medium profit earners evade more than high earners is quite subtle. When the penalty rate is moderate, medium profit earners are more willing to take the risk of under-reporting. Thus, it will be more difficult for the high profit earners to hide behind those reporting profit 1, because there will be fewer people with a true profit of 1 reporting truthfully. In addition, the probability of auditing a report of 1 (i.e.,  $\frac{1}{1+f}$ ) will be relatively high when the penalty rate is low, which makes high profit earners more reluctant to evade. On the other hand, a high  $f$  would lead medium profit earners to not risk evasion, thus making it easier for high earners to hide. We will revisit the last point when explaining the next proposition.

**Corollary 2.** *The following comparative statics of taxpayers' and inspectors' strategies follow from Proposition 3:*

- When  $f$  increases, both medium and high profit earners evade less often, and inspectors audit those reporting 1 (or 0) less intensively given a common shock of 1 (or 0).
- When  $\alpha$  increases, both medium and high profit earners evade more often.

The above results are similar to the one in the benchmark model. Though the knowledge about the common shock will affect the magnitude of underreporting, qualitatively there is no effect on the changes in the intensity of underreporting and auditing due to changes in fines or the proportion of honest taxpayers.

Below, we present a complete characterization of the equilibria in terms of penalties and auditing costs.<sup>27</sup>

**Proposition 4 (Reporting concentration in the middle).** *Suppose the penalty rate is high:  $f \geq \frac{p_1}{1-p_1}$ . The following constitute PBEs of the tax returns/auditing game, with two principal features: reporting concentration in the middle range and the complete absence of auditing under an unfavorable common shock.*

1. (i) In a range of high auditing costs  $c_2 > \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+q}$ ,

- taxpayers submit returns according to the strategy  $\gamma(0|1) = 0$  and  $\gamma(1|2) = 1$ , and
- the inspector audits according to the strategy  $\rho(1|1) = 0$  and  $\rho(0|0) = 0$ .

(ii) For  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+q}$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 0$  and  $\gamma(1|2) = 1$ , and
- the inspector audits according to  $0 \leq \rho(1|1) \leq \frac{1}{1+f}$  and  $\rho(0|0) = 0$ .

(iii) In the moderate auditing costs range  $c_2 < \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+q}$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 0$ ,  $\gamma(1|2) = \frac{qc_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , and  $\gamma(2|2) = 1 - \gamma(1|2)$ , and
- the inspector audits according to  $\rho(1|1) = \frac{1}{1+f}$  and  $\rho(0|0) = 0$ .

2. Only high profit earners will evade.

When medium profit earners are considering tax evasion, they know that if the common shock is 1, they will be audited and detected, and if the common shock is 0, there is still some chance that their underreporting will not be audited. Given

<sup>25</sup> We omit the taxpayer's equilibrium action  $\gamma(0|0) = 0$  here, and the same applies to the other equilibria presented later.

<sup>26</sup> We omit the inspector's equilibrium actions  $\rho(2|1) = \rho(2|0) = \rho(1|0) = 0$  and  $\rho(0|1) = 1$  here, and the same applies to other equilibria presented later.

<sup>27</sup> The equilibria that occur only when  $f = \frac{p_1}{1-p_1}$  are listed in the Supplementary material.

the high penalty for evasion, they do not want to take the risk of underreporting. Consequently, the inspector will not audit low reports when the common shock is unfavorable.

On the other hand, the high profit earners now anticipate a large cluster of the population with reported profits of 1, making it easier for them to hide. Therefore, they always underreport when the auditing cost is high, and underreport less often when the auditing cost is low.

**Proposition 5 (Evasion at all levels).** Suppose the penalty rate is moderate:  $f \leq \frac{p_1}{1-p_1}$ . The following constitute PBEs of the tax returns/auditing game.

1. (i) In a sufficiently high auditing cost range  $c_2 > \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+\alpha q}$ ,

- taxpayers submit returns according to the strategy  $\gamma(0|1) = 1$  and  $\gamma(1|2) = 1$ , and
- the inspector audits according to the strategy  $\rho(1|1) = 0$  and  $\rho(0|0) = 0$ .

(ii) For  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+\alpha q}$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 1$  and  $\gamma(1|2) = 1$ , and
- the inspector audits according to  $0 \leq \rho(1|1) \leq \frac{1}{1+f}$  and  $\rho(0|0) = 0$ .

(iii) In the intermediate range  $\frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q} < c_2 < \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+\alpha q}$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 1$ ,  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , and  $\gamma(2|2) = 1 - \gamma(1|2)$ , and
- the inspector audits according to  $\rho(1|1) = \frac{1}{1+f}$  and  $\rho(0|0) = 0$ .

(iv) For  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q}$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 1$ ,  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , and  $\gamma(2|2) = 1 - \gamma(1|2)$ , and
- the inspector audits according to  $\rho(1|1) = \frac{1}{1+f}$  and  $0 \leq \rho(0|0) \leq \frac{p_1(1+f)-f}{p_1(1+f)}$ .

(v) For  $c_2 < \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q}$ , the equilibrium is as described in Proposition 3.

2. Medium profit earners underreport (weakly) more often than high profit earners do.

When the penalty rate  $f$  is not high, medium profit earners are more willing to gamble on underreporting. With high auditing cost, the inspector will not audit low profit submissions when the common shock is 0, making evasion by medium earners a certainty. With moderate auditing cost, the inspector will audit with a positive probability, lowering the intensity of evasion.

For a not high  $f$ , high profit earners thus expect a low concentration of medium profit submissions, so underreporting will attract greater scrutiny. That is, the inspector would guess that the medium submissions are more likely from high profit earners, thus providing a better incentive to audit. Thus, high profit earners will underreport for sure only when the auditing cost is prohibitively high, but will underreport less often otherwise.

■ **Overall summary.** The first takeaway message is about auditing. Low returns are audited more intensively, if at all, relative to the returns of medium profits, but not all audits are necessarily carried out because the credible threat of auditing may discipline low submissions.<sup>28</sup> Reinganum and Wilde (1985) report a similar result under the commitment to an audit cutoff rule in which the audit should be triggered if the reported profit is “too low” and no audit is triggered if the reported profit is “sufficiently high.” The key reason behind our audit partition result is the tax authority’s exclusive knowledge of the common shock, which obviates the need for a commitment to an auditing rule; just the threat of a potential audit provides sufficient discipline.<sup>29</sup>

Our second message is that underreporting is always at most by one step. Two-step underreporting in the benchmark case of uninformed/naive tax audits (Proposition 1) is now deterred because the inspector would know such submissions to be nontruthful and can thus easily expose them. Even though we derive the result under discrete profit possibilities, we expect a similar type of result to prevail with a continuum of profits: so long as the tax authority can learn about the favorable common shock from honest submissions, it will become evident to the high income earners that very low submissions will be uncovered, deterring them from grossly underreporting their earnings.

The third point is the absence of a uniform pattern of evasion: sometimes, medium profit earners evade more than those realizing high profits (Propositions 3 and 5), on other occasions, only high profit earners would evade (Proposition 4). The difference in the two outcomes depends on whether the penalty rate for evasion ( $f$ ) is high or moderate. With high profit, the taxpayer holds superior information compared to medium profit earners (knowing precisely both the common

<sup>28</sup> For instance, in Proposition 4, taxpayers never report 0 when the true profit is 1 and the inspector chooses  $\rho(0|1) = 1$ .

<sup>29</sup> There is no commonly accepted wisdom, however, on whether high or low income reports should be audited more intensively. See, for instance, Erard and Feinstein (1994), who raise a similar point (see the discussion in their simulation results).

shock and their idiosyncratic shocks) and can thus risk evasion knowing the probability of being audited. Medium profit earners are unsure of the common shock and may thus err too much in either direction – evading when they should not given moderate penalties (when the common shock is  $\epsilon = 1$  and  $f \leq \frac{p_1}{1-p_1}$ , as in Proposition 5) and not evading when they could do so under high penalties ( $\epsilon = 0$  and  $f \geq \frac{p_1}{1-p_1}$ , as in Proposition 4). This result yields a policy implication: given that evasion is inevitable, should the tax authority set a high or moderate penalty rate  $f$ ? If the government cares about redistribution, then it might prefer to set moderate penalties and tolerate more evasion by medium profit earners than high earners. Although we do not study optimal taxation here, we can still raise the issue of redistribution in a positive analysis of tax evasion. In an optimal taxation analysis, Kopczuk (2001) studies a similar issue and observes that for redistribution purposes, the optimal taxes may permit tax avoidance even when prevention is not costly.

The final point is the inspector's decision to sometimes abandon auditing when the common shock is unfavorable.

The results – more intensive auditing of low submissions, moderate underreporting, and evasion by only the high profit earners under high penalties – appeared in one form or another in the earlier optimal tax auditing literature with commitment<sup>30</sup> discussed briefly in the Introduction. Our modelling differs in two respects, non-commitment and the tax authority's superior knowledge of the common shock, which delivers similar stylized predictions. Thus, it opens the possibility that better sources of information (on the tax authority's side) can act as an imperfect commitment device.

■ **No honest taxpayers.** If we assume that all taxpayers are strategic ( $\alpha = 0$ ) but the tax authority has full knowledge about the common shock (through other means), then our results do not change, or change very little, by substituting  $\alpha = 0$  into all the relevant expressions, except for the results in parts (iii) and (iv) of Proposition 5. The changes in the last two cases are as follows:

(iii) In the intermediate range  $(1 - q)(1 + f)t_1 < c_2 < (1 + f)t_2$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 1$ ,  $\gamma(1|2) = 0$ , and  $\gamma(2|2) = 1$ , and
- the inspector audits according to  $\frac{1}{1+f} < \rho(1|1) \leq 1$  and  $\rho(0|0) = 0$ .

(iv) For  $c_2 = (1 - q)(1 + f)t_1$ ,

- taxpayers submit returns according to  $\gamma(0|1) = 1$ ,  $\gamma(1|2) = 0$ , and  $\gamma(2|2) = 1$ , and
- the inspector audits according to  $\frac{1}{1+f} < \rho(1|1) \leq 1$  and  $0 \leq \rho(0|0) \leq \frac{p_1(1+f)-f}{p_1(1+f)}$ .

Note that in these two cases, taxpayers with profit 1 will report 0 and taxpayers with profit 2 will report 2, so authorities will observe no reports of 1. Thus, the inspector's auditing rule  $\rho(1|1)$  is off the equilibrium path, and is determined according to sequential rationality.

■ **A more informed IRS.** Even in the absence of honest taxpayers, it is plausible to take the position that tax agencies, for example the IRS, will have superior knowledge about the common shock. Besides the aggregate trend in tax returns that reveals economic conditions, the IRS sometimes makes detailed projections on employment and job losses. For instance, fortune.com reports, “The IRS forecasts there will be about 229.4 million employee-classified jobs in 2021—about 37.2 million fewer than it had estimated last year, before the virus hit, according to updated data released Thursday. The statistics are an estimate how many of the W-2 tax forms that are used to track employee wages and withholding the agency will receive.” (source: <https://fortune.com/2020/08/21/the-u-s-economy-is-shedding-over-1-million-jobs-per-week-they-wont-come-back-for-years-the-irs-says/>). In fact, year-by-year “Projections of Federal Tax Return Filings” by the IRS are available from <https://www.irs.gov/statistics/projections-of-federal-tax-return-filings#p6961> which suggests that the tax agency must have more reliable information about the underlying economic conditions than any individual taxpayer does.

In this study, allowing for a (small) fraction of honest taxpayers simplifies the analysis and provides another rationale for the tax authority's superior knowledge of the common shock. However, this informational advantage can be the product of not only some special economic expertise at their disposal, such as researchers and data analysts, but also due to sophisticated updating from the collection of tax submissions. In a related work (Bag and Wang, 2020), we analyze a more complicated bounded learning model of tax audits in which the tax authority gleans the information about the common shock with individual agents submitting tax returns based on their knowledge of some of their neighborhood peers' profit realizations.

#### 4. Evasion and skills

Now, suppose entrepreneurs are differentiated according to their abilities to generate good idiosyncratic outcomes, i.e.,  $\Pr(x_{it} = 0) = q_t$ , where  $t \in \{L, H\}$  is entrepreneur  $i$ 's type with  $q_H < q_L$ .<sup>31</sup> Suppose a  $\beta$  proportion of entrepreneurs are high

<sup>30</sup> See Border and Sobel (1987), for example.

<sup>31</sup> We assume that the ability is related to each entrepreneur's professionalism or experience, such as the number of years in the business, good networks, related skills improvement training, and so on.

types ( $H$ ) and  $1 - \beta$  proportion are low types ( $L$ ). Thus, each taxpayer's strategy will depend on his or her type  $t$  and can be written as  $\gamma_t(\hat{\pi}_i|\pi_i)$ .

The strategy of any taxpayer with a profit of 2 should be type-independent since the taxpayer knows the true values of the common shock and the idiosyncratic shock. On the other hand, if profit is 1, beliefs about the shocks depend on the taxpayer's type, influencing the strategy. In particular,

$$p_t \equiv \Pr(\epsilon = 0|\pi_{it} = 1) = \frac{p(1 - q_t)}{p(1 - q_t) + (1 - p)q_t}. \tag{11}$$

It is easy to see that  $p_H > p_L$ .

We derive the taxpayer's expected utility in the same way as before, by replacing  $p_1$  with  $p_H$  or  $p_L$  according to the type when the profit is 1. For the inspector, when  $\epsilon = 1$ , the updated belief about the (accuracy of) taxpayer's submission is

$$\begin{aligned} \Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) &= \frac{(1 - \alpha)[\beta(1 - q_H)\gamma(1|2) + (1 - \beta)(1 - q_L)\gamma(1|2)]}{\alpha[\beta q_H + (1 - \beta)q_L] + (1 - \alpha)[\beta q_H\gamma_H(1|1) + (1 - \beta)q_L\gamma_L(1|1) + \beta(1 - q_H)\gamma(1|2) + (1 - \beta)(1 - q_L)\gamma(1|2)]}. \end{aligned} \tag{12}$$

When  $\epsilon = 0$ , the inspector's updated belief is

$$\Pr(\pi_i = 1|\hat{\pi}_i = 0, \epsilon = 0) = \frac{(1 - \alpha)[\beta(1 - q_H)\gamma_H(0|1) + (1 - \beta)(1 - q_L)\gamma_L(0|1)]}{\beta q_H + (1 - \beta)q_L + (1 - \alpha)[\beta(1 - q_H)\gamma_H(0|1) + (1 - \beta)(1 - q_L)\gamma_L(0|1)]}. \tag{13}$$

With true profit 1, high-type entrepreneurs consider their profit to be more likely due to their high skill whereas the low types think it is more due to a favorable common shock. Thus, high types are more likely to evade, thinking perhaps that the realization of a market shock is  $\epsilon = 0$ . The low types are likely to report truthfully as they put higher odds on a market shock of  $\epsilon = 1$ , so underreporting is riskier. We have the following result.<sup>32</sup>

**Proposition 6 (Evasiveness & skill).** *Suppose*

$$\begin{aligned} c_2 > \max \left\{ \frac{(1 - \alpha)\beta(1 - q_H)(1 + f)t_1}{\beta q_H + (1 - \beta)q_L + (1 - \alpha)\beta(1 - q_H)}, \frac{(1 - \alpha)[\beta(1 - q_H) + (1 - \beta)(1 - q_L)](1 + f)t_2}{\alpha[\beta q_H + (1 - \beta)q_L] + (1 - \alpha)(1 - \beta q_H)} \right\}, \\ \text{and } \frac{p_L}{1 - p_L} < f < \frac{p_H}{1 - p_H}. \end{aligned}$$

The following constitutes a PBE of the tax returns/auditing game:

- taxpayers submit returns according to the strategy  $\gamma_H(0|1) = 1$ ,  $\gamma_L(0|1) = 0$ , and  $\gamma_H(1|2) = \gamma_L(1|2) = 1$ , and
- the inspector audits according to the strategy  $\rho(0|1) = 1$ ,  $\rho(1|1) = 0$ , and  $\rho(0|0) = 0$ .

**Remark.** Our equilibrium construction involves auditing only when the common shock is favorable and the profit report is low. While medium reports make up a large concentration, with both truthful/honest and nontruthful declarations, the audit cost makes it unprofitable to investigate such submissions. We want to emphasize that this auditing is not special or unique for our main result of interest - differential reporting by the high- and low-type entrepreneurs. In the supplementary file, we report another construction in which the inspector chooses a mixed strategy inspection of medium reports when the common shock is favorable and high profit earners submit medium reports with a (positive) probability less than one. However, the existence of that equilibrium requires  $t_2$  to be sufficiently larger than  $t_1$ , a condition that is not necessary for Proposition 6. ||

Proposition 6 highlights how evasion behavior may vary with taxpayers' characteristics, a theme expounded by various authors we will discuss, in particular, Kopczuk (2001) and Alstadsaeter et al. (2019). In an optimal tax model, Kopczuk studies individuals differentiated along two independent dimensions, skills and tax evasiveness, the latter being a taste characteristic. The author suggests why the tax system might be designed to allow targeted tax avoidance to achieve the goal of redistribution, even if the administrative costs of monitoring could be negligible. In our case, evasion is endogenously determined, which prompts two individuals with the same income level but differentiated skills to behave very differently, even though both would prefer to evade if they could get away with it. We do not determine the optimal tax rates but simply assume exogenously given progressive taxation.

Alstadsaeter et al. analyze, based on the leaked micro-data of off-shore financial institutions (the "Swiss leaks" of 2007 and the "Panama Papers" of 2016), the distribution of tax evasion in Norway, Sweden, and Denmark. They find that tax evasion rises sharply with wealth. Assuming wealth is positively correlated with skills, our result that high-skilled taxpayers evade more than low-skilled taxpayers do resembles these empirical observations. However, the similarity is only in the aggregate behavior, given that the main difference in evasion in our case occurs for the middle-income (earning a profit of 1) group.<sup>33</sup>

<sup>32</sup> Besides the equilibrium studied in Proposition 6, other types of equilibria can exist. We do not intend to offer a comprehensive characterization. Instead, we report the specific equilibrium as we thought that the result would be intuitively appealing and should be of interest for its empirical relevance.

<sup>33</sup> All individuals in the upper-income group earning a profit of 2, whether ex ante of high or low ability, will evade.

Another motivation for the extension is to suggest a generalization of the tax-enforcement problem to a multi-period setting. With period-by-period sector-specific common shocks and random individual earnings, how to audit individual returns is an interesting question. With individual types being invariant, past submissions and audit outcomes should reveal valuable information for tax authorities to determine individualized future audit strategies. Taxpayers with high (or low) submissions in the past are also expected to make similar submissions in the future. This should create a ratchet effect.

■ **Significance of Proposition 6.** What do we take away from this result? Stated simply, we observe that in a favorable market shock, the nontruthful low submitters are entirely of high-ability types. However, how does this specific information help the inspector? In an alternative world in which idiosyncratic shocks are type-independent, seeing low submissions under a favorable market shock will prompt the inspector to react the same way as in Proposition 6; that is, it would audit them for sure at low auditing cost  $c_1$  and recover evaded taxes and impose fines. Thus, there is no special gain to the inspector from the type-related information gathered from low submitters. This *irrelevance of informational knowledge*, however, is unlikely to be true if one considers a multi-period extension. Then, any type-related information gathered in period 1 can be utilized for future auditing by making the auditing history dependent. If idiosyncratic shocks are type-independent, then the first period behavior does not empower the inspector for future monitoring. Empirically, it is well known that a tainted history (due to the detection of past tax evasion) is likely to put the individual under greater scrutiny. This acts as a disciplining device, restraining future evasion. Our type-enriched model in this section and Proposition 6 therefore have greater significance for a multi-period extension of the current analysis, which we will take up in future research.

### 5. Concluding remarks

Standard tax-audit models assume one-sided information asymmetry. By introducing a common shock that, together with idiosyncratic shocks, determine the profits of self-employed entrepreneurs turns the tax-audit game into a two-sided asymmetric information problem. That the audit will be conditional upon the realization of the common shock creates an additional layer of uncertainty for the taxpayers, as they will not know with probability that they might be audited.

We assume no fixed budget for auditing. Instead, the tax authority or the agency acting on its behalf will audit whenever the expected benefits of auditing (recovered tax plus fines) exceed the cost of auditing.<sup>34</sup> Thus, auditing is internalized with the sole objective of profit maximization, or at the minimum outsourced, so the constraint of a fixed auditing budget does not compromise tax collections.

Doing away with the explicit budget is not an insignificant point, especially in view of how declining enforcement budgets have been harming tax collections.<sup>35</sup> Thus, dealing with the problem of the enforcement budget should be an important policy consideration as it impacts the efficiency of tax collections. In a modern world with increasing reliance on delegation and expertise for specific tasks, tax auditing can be outsourced to an external agency. Even better, the government’s tax department can internalize such commissions by conducting audits in the manner of a profit-seeking firm.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### Appendix

**Proof of Proposition 1.** Given the taxpayers’ strategies  $\gamma(1|2) = \frac{[(1-p)q+p(1-q)]\alpha c_2}{(1-\alpha)(1-p)(1-q)[(1+f)t_2-c_2]}$  and  $\gamma(0|1) = 1$ , the posterior (5) becomes  $Pr(2|1) = \frac{c_2}{(1+f)t_2}$ . Since  $Pr(2|1)(1+f)t_2 - c_2 = 0$ ,  $\rho(1) = \frac{1}{1+f}$  is (one of) the best responses. Given

$$\gamma(0|2) = \frac{pq c_2 - (1-\alpha)[(1-p)q + p(1-q)][(1+f)t_1 - c_2]}{(1-\alpha)(1-p)(1-q)[(1+f)(t_1 + t_2) - c_2]}$$

<sup>34</sup> In the absence of moral hazard in auditing, the agency’s interests will be aligned with that of the tax authority.

<sup>35</sup> The Chicago Tribune (March 5, 2017) has the following report: “The IRS blames budget cuts as money for the agency shrunk from \$12.2 billion in 2010 to \$11.2 billion last year. Over that period, the agency has lost more than 17,000 employees, including nearly 7000 enforcement agents. A little more than 80,000 people work at the IRS. IRS Commissioner John Koskinen said budget cuts are costing the federal government between \$4 billion and \$8 billion a year in uncollected taxes. ‘We are the only agency if you give us more people and money, we give you more money back,’ Koskinen said in an interview.” Source: <http://www.chicagotribune.com/news/nationworld/ct-irs-audits-budget-cuts-20170305-story.html>

and  $\gamma(0|1) = 1$ , the posterior (6) becomes

$$Pr(1|0) = \frac{(1 - \alpha)[(1 - p)q + p(1 - q)][(1 + f)(t_1 + t_2) - c_2]}{pq(1 + f)(t_1 + t_2) + (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_2},$$

and the posterior (7) becomes

$$Pr(1|0) = \frac{pq c_2 - (1 - \alpha)[(1 - p)q + p(1 - q)][(1 + f)t_1 - c_2]}{pq(1 + f)(t_1 + t_2) + (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_2}.$$

Now verify that  $Pr(1|0)(1 + f)t_1 + Pr(2|0)(1 + f)(t_1 + t_2) - c = 0$ , so  $\rho(1) = \frac{1}{1+f}$  is a best response.

On the other hand, since  $\rho(0) = \frac{1}{1+f}$  and  $\rho(1) = \frac{1}{1+f}$ , we have  $2 - t_1 - t_2 = 2 - t_1 - \rho(1)(1 + f)t_2 = 2 - \rho(0)(1 + f)(t_1 + t_2)$ . Thus, having  $0 < \gamma(2|2) < 1$ ,  $0 < \gamma(1|2) < 1$ , and  $0 < \gamma(0|2) < 1$  is a best response. Also, we have  $1 - t_1 = 1 - \rho(0)(1 + f)t_1$ , so  $\gamma(0|1) = 1$  is a best response.

To make sure that  $0 < \gamma(2|2) < 1$ ,  $0 < \gamma(1|2) < 1$ , and  $0 < \gamma(0|2) < 1$ , it is sufficient to check the conditions  $\gamma(1|2) > 0$ ,  $\gamma(0|2) > 0$ , and  $\gamma(1|2) + \gamma(0|2) < 1$ , since  $\gamma(1|2) > 0$  implies that  $\gamma(2|2) = 1 - \gamma(1|2) - \gamma(0|2) < 1$ ,  $\gamma(1|2) + \gamma(0|2) < 1$  implies  $\gamma(1|2) < 1$ ,  $\gamma(0|2) < 1$ , and also  $\gamma(2|2) > 0$ .

Since  $c_2 < (1 + f)t_2$  by our assumption,  $\gamma(1|2) > 0$  is satisfied. Given that  $c_2 > \frac{(1-\alpha)[(1-p)q+p(1-q)](1+f)t_1}{pq+(1-\alpha)[(1-p)q+p(1-q)]}$ , we can see that  $\gamma(0|2) > 0$  is satisfied.

Now we are going to verify that  $\gamma(1|2) + \gamma(0|2) < 1$ .

$$\begin{aligned} & \gamma(1|2) + \gamma(0|2) \\ & < \frac{[(1 - p)q + p(1 - q)]\alpha c_2}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)t_2 - c_2]} + \frac{pq c_2 - (1 - \alpha)[(1 - p)q + p(1 - q)][(1 + f)t_1 - c_2]}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)t_2 - c_2]} \\ & = \frac{pq c_2 + [(1 - p)q + p(1 - q)]c_2 - (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)t_2 - c_2]}. \end{aligned}$$

Given that

$$c_2 < \frac{(1 - \alpha)(1 - p)(1 - q)(1 + f)t_2 + (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{1 - \alpha(1 - p)(1 - q)},$$

we have

$$\frac{pq c_2 + [(1 - p)q + p(1 - q)]c_2 - (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{(1 - \alpha)(1 - p)(1 - q)[(1 + f)t_2 - c_2]} < 1.$$

Thus,  $\gamma(1|2) + \gamma(0|2) < 1$ .

In addition, to make sure that there exists values for  $c_2$ , we need to have the following condition:

$$\frac{(1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]} < \frac{(1 - \alpha)(1 - p)(1 - q)(1 + f)t_2 + (1 - \alpha)[(1 - p)q + p(1 - q)](1 + f)t_1}{1 - \alpha(1 - p)(1 - q)},$$

which is equivalent to

$$\frac{[(1 - p)q + p(1 - q)]t_1}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]} < \frac{(1 - p)(1 - q)t_2 + [(1 - p)q + p(1 - q)]t_1}{1 - \alpha(1 - p)(1 - q)}. \tag{14}$$

If we can show that

$$\frac{[(1 - p)q + p(1 - q)]}{pq + (1 - \alpha)[(1 - p)q + p(1 - q)]} < \frac{(1 - p)(1 - q) + [(1 - p)q + p(1 - q)]}{1 - \alpha(1 - p)(1 - q)}, \tag{15}$$

then condition (14) is satisfied. It can be shown that condition (15) is equivalent to

$$\alpha < \frac{pq(1 - p)(1 - q)}{[(1 - p)q + p(1 - q)]^2},$$

as stated in the proposition.  $\square$

**Proof of Corollary 1.** The results are trivial from the expressions.  $\square$

**Proof of Proposition 2.**

1. When the inspector knows that  $\epsilon = 1$ , the inspector infers that any profit report of 0 must be non-truthful. Given condition (4), the inspector will audit for sure.

When the profit is 2, the taxpayer knows that  $\epsilon = 1$ , and the inspector having the same information will audit any report of 0 with certainty. Hence the taxpayer will not report 0.

2. Suppose any taxpayer with the true profit of 2 always reports 2. Then the inspector knows that when the common shock is 1, those with reported profit of 1 must all be truthful and thus the inspector will not audit them. However, given that the inspector will not audit any report of 1 when the common shock is 1, the taxpayer with a profit of 2 should choose to report 1. A contradiction arrives.

3. Suppose the inspector audits 0 with probability 1 when the inspector knows that the common shock is 0. This means a report of 0 will always be audited regardless of the common shock, given Proposition 2. Hence the taxpayer with a profit of 1 will never choose to underreport. However knowing this, the inspector should not audit 0 when the common shock is 0, a contradiction.  $\square$

**Proof of Proposition 3.**

1. Since we are looking for a mixed strategy equilibrium, a taxpayer with a profit of 2 is indifferent between reporting a profit of 2 and a profit of 1, and a taxpayer with a profit of 1 is indifferent between reporting a profit of 1 and a profit of 0:

$$EU_i(\hat{\pi}_i = 2|\pi_i = 2) = EU_i(\hat{\pi}_i = 1|\pi_i = 2),$$

and  $EU_i(\hat{\pi}_i = 1|\pi_i = 1) = EU_i(\hat{\pi}_i = 0|\pi_i = 1),$

or using the payoff expressions derived earlier,

$$2 - t_1 - t_2 = 2 - t_1 - \rho(1|1)(1 + f)t_2, \tag{16}$$

$$\text{and } 1 - t_1 = [p_1\rho(0|0) + 1 - p_1](1 - t_1 - ft_1) + p_1(1 - \rho(0|0)). \tag{17}$$

Solving (16) and (17) will yield the inspector’s equilibrium audits as stated in the proposition. It can be easily seen that  $0 < \rho(1|1) < 1$  and  $\rho(0|0) < 1$ . Given  $f < \frac{p_1}{1-p_1}$ , it also follows that  $\rho(0|0) > 0$ .

For the inspector to use mixed strategies, the inspector must be indifferent between auditing a submission of 1 and not auditing when the common shock is 1, and be indifferent between auditing 0 and not auditing when the common shock is 0, as follows:

$$\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) \times (1 + f)t_2 - c_2 = 0,$$

and  $\Pr(\pi_i = 1|\hat{\pi}_i = 0, \epsilon = 0) \times (1 + f)t_1 - c_2 = 0,$

or using the expressions derived earlier (see (9) and (10)),

$$\frac{(1 - \alpha)(1 - q)\gamma(1|2)}{\alpha q + (1 - \alpha)q\gamma(1|1) + (1 - \alpha)(1 - q)\gamma(1|2)} \times (1 + f)t_2 - c_2 = 0, \tag{18}$$

$$\text{and } \frac{(1 - \alpha)(1 - q)\gamma(0|1)}{q + (1 - \alpha)(1 - q)\gamma(0|1)} \times (1 + f)t_1 - c_2 = 0. \tag{19}$$

Solving (18) and (19) together with the condition that  $\gamma(0|1) + \gamma(1|1) = 1$ , we can derive the taxpayers’ reporting strategies  $\gamma(0|1)$  and  $\gamma(1|2)$  as stated in the proposition. It is clear that  $\gamma(0|1) > 0$ , and condition  $c_2 < \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q}$  implies that  $\gamma(0|1) < 1$ .  $c_2 < \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q)+q}$  also implies that  $c_2 < (1 - q)(1 + f)t_1$ . Thus,  $\gamma(1|2) > 0$ .  $\gamma(1|2)$  can also be expressed as  $\gamma(1|2) = \gamma(0|1) \times \frac{(1-q)(1+f)t_1 - c_2}{(1-q)(1+f)t_2 - (1-q)c_2}$ . Since  $\gamma(0|1) < 1$  and  $\frac{(1-q)(1+f)t_1 - c_2}{(1-q)(1+f)t_2 - (1-q)c_2} < 1$  as  $t_1 < t_2$ , we have  $\gamma(1|2) < 1$ .

2. That  $\gamma(1|2) < \gamma(0|1)$  follows given that  $\gamma(0|1) > 0$  and  $\frac{(1-q)(1+f)t_1 - c_2}{(1-q)(1+f)t_2 - (1-q)c_2} < 1$ .  $\square$

**Proof of Corollary 2.** The results are also trivial from the expressions.  $\square$

**Proof of Proposition 4.**

1. (i) Given the taxpayers’ strategies  $\gamma(1|1) = \gamma(1|2) = 1$ , the posterior (9) becomes  $\Pr(2|1) = \frac{(1-\alpha)(1-q)}{q+(1-\alpha)(1-q)}$ .<sup>36</sup>  $\Pr(2|1)(1 + f)t_2 - c_2 < 0$  given that  $c_2 > \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+q}$ , so we arrive at  $\rho(1|1) = 0$ . The posterior (10) becomes  $\Pr(1|0) = 0$ , thus  $\rho(0|0) = 0$  as  $\Pr(1|0)(1 + f)t_1 - c_2 < 0$ .

On the other hand, given the inspector’s strategy  $\rho(1|1) = 0$  and  $\rho(0|0) = 0$ , for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = 2 - t_1 - t_2$  and  $EU_i(1|2) = 2 - t_1$ .<sup>37</sup> Thus we should have  $\gamma(1|2) = 1$ . For a taxpayer with the true profit of 1, the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1 = 1 - t_1 + p_1t_1 - (1 - p_1)ft_1$ . Given that  $f \geq \frac{p_1}{1-p_1}$ , we have  $EU_i(1|1) \geq EU_i(0|1)$ , and thus  $\gamma(1|1) = 1$  is a best response.

(ii) Given the taxpayers’ strategies  $\gamma(1|1) = \gamma(1|2) = 1$ , the posterior (9) becomes  $\Pr(2|1) = \frac{(1-\alpha)(1-q)}{q+(1-\alpha)(1-q)}$ . Since  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q)+q}$ , the inspector is indifferent between auditing and not auditing. Thus,  $0 \leq \rho(1|1) \leq \frac{1}{1+f}$  is a best response. The posterior (10) becomes  $\Pr(1|0) = 0$ , thus  $\rho(0|0) = 0$  is a best response as  $\Pr(1|0)(1 + f)t_1 - c_2 < 0$ .

On the other hand, for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = 2 - t_1 - t_2$  and  $EU_i(1|2) = 2 - t_1 - \rho(1|1)(1 + f)t_2$ . Since  $\rho(1|1) \leq \frac{1}{1+f}$ , we have  $EU_i(2|2) \leq EU_i(1|2)$ , thus  $\gamma(1|2) = 1$  is a best response.

<sup>36</sup> We write  $\Pr(2|1)$  as a short form of  $\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1)$ , and the same applies to similar other expressions.

<sup>37</sup> We write  $EU_i(2|2)$  as a short form of  $EU_i(\hat{\pi}_i = 2|\pi_i = 2)$ , and the same applies to similar other expressions.



For a taxpayer with the true profit of 1, given that  $\rho(0|0) = 0$ , the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1 = 1 - t_1 + p_1t_1 - (1 - p_1)ft_1$ . Given that  $f \geq \frac{p_1}{1-p_1}$ , we have  $EU_i(1|1) \geq EU_i(0|1)$ , and thus  $\gamma(1|1) = 1$  is a best response.

(iii) Given the taxpayers' strategies  $\gamma(1|1) = 1$  and  $\gamma(1|2) = \frac{qc_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , the posterior (9) becomes  $\Pr(2|1) = \frac{c_2}{(1+f)t_2}$ , so  $\Pr(2|1)(1+f)t_2 - c_2 = 0$ . Thus,  $\rho(1|1) = \frac{1}{1+f}$  is a best response. The posterior (10) becomes  $\Pr(1|0) = 0$ , thus  $\rho(0|0) = 0$  is a best response as  $\Pr(1|0)(1+f)t_1 - c_2 < 0$ .

On the other hand, given the inspector's strategy  $\rho(1|1) = \frac{1}{1+f}$ , for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = EU_i(1|2) = 2 - t_1 - t_2$ . Thus  $\gamma(1|2) = \frac{qc_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$  is a best response. For a taxpayer with the true profit of 1, given that  $\rho(0|0) = 0$ , the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1 = 1 - t_1 + p_1t_1 - (1 - p_1)ft_1$ . Since  $f \geq \frac{p_1}{1-p_1}$ , we have  $EU_i(1|1) \geq EU_i(0|1)$ , and thus  $\gamma(1|1) = 1$  is a best response.

2. The observation follows from the first part.  $\square$

**Proof of Proposition 5.**

1. (i) Given the taxpayers' strategies  $\gamma(0|1) = \gamma(1|2) = 1$ , the posterior (9) becomes  $\Pr(2|1) = \frac{(1-\alpha)(1-q)}{\alpha q + (1-\alpha)(1-q)}$ .  $\Pr(2|1)(1+f)t_2 - c_2 < 0$  given that  $c_2 > \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q) + \alpha q}$ , so  $\rho(1|1) = 0$  is the best response. The posterior (10) becomes  $\Pr(1|0) = \frac{(1-\alpha)(1-q)}{q + (1-\alpha)(1-q)}$ , thus  $\rho(0|0) = 0$  is the best response as  $\Pr(1|0)(1+f)t_1 - c_2 < \Pr(2|1)(1+f)t_2 - c_2 < 0$ .

On the other hand, given the inspector's strategy  $\rho(1|1) = 0$  and  $\rho(0|0) = 0$ , for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = 2 - t_1 - t_2$  and  $EU_i(1|2) = 2 - t_1$ , thus we should have  $\gamma(1|2) = 1$  as the best response. For a taxpayer with the true profit of 1, the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1 = 1 - t_1 + p_1t_1 - (1 - p_1)ft_1$ . Given that  $f \leq \frac{p_1}{1-p_1}$ , we have  $EU_i(1|1) \leq EU_i(0|1)$ , and thus  $\gamma(0|1) = 1$  is a best response.

(ii) Given the taxpayers' strategies  $\gamma(0|1) = \gamma(1|2) = 1$ , the posterior (9) becomes  $\Pr(2|1) = \frac{(1-\alpha)(1-q)}{\alpha q + (1-\alpha)(1-q)}$ . Since  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_2}{(1-\alpha)(1-q) + \alpha q}$ , the inspector is indifferent between auditing and not auditing. Thus,  $0 \leq \rho(1|1) \leq \frac{1}{1+f}$  is a best response. The posterior (10) becomes  $\Pr(1|0) = \frac{(1-\alpha)(1-q)}{q + (1-\alpha)(1-q)}$ , thus  $\rho(0|0) = 0$  is the best response as  $\Pr(1|0)(1+f)t_1 - c_2 < \Pr(2|1)(1+f)t_2 - c_2 < 0$ .

On the other hand, for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = 2 - t_1 - t_2$  and  $EU_i(1|2) = 2 - t_1 - \rho(1|1)(1+f)t_2$ . Since  $\rho(1|1) \leq \frac{1}{1+f}$ , we have  $EU_i(2|2) \leq EU_i(1|2)$ , thus  $\gamma(1|2) = 1$  is a best response. For a taxpayer with the true profit of 1, given that  $\rho(0|0) = 0$ , the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1 = 1 - t_1 + p_1t_1 - (1 - p_1)ft_1$ . Given that  $f \leq \frac{p_1}{1-p_1}$ , we have  $EU_i(1|1) \leq EU_i(0|1)$ , and thus  $\gamma(0|1) = 1$  is a best response.

(iii) Given the taxpayers' strategies  $\gamma(0|1) = 1$  and  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , the posterior (9) becomes  $\Pr(2|1) = \frac{c_2}{(1+f)t_2}$ , so  $\Pr(2|1)((1+f)t_2 - c_2) = 0$ . Thus,  $\rho(1|1) = \frac{1}{1+f}$  is a best response. The posterior (10) becomes  $\Pr(1|0) = \frac{(1-\alpha)(1-q)}{q + (1-\alpha)(1-q)}$ , so  $\Pr(1|0)(1+f)t_1 - c_2 < 0$  as  $c_2 > \frac{(1-\alpha)(1-q)((1+f)t_1)}{(1-\alpha)(1-q) + q}$ . Thus,  $\rho(0|0) = 0$  is the best response.

On the other hand, given the inspector's strategy  $\rho(1|1) = \frac{1}{1+f}$  and  $\rho(0|0) = 0$ , for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = EU_i(1|2) = 2 - t_1 - t_2$ . Thus  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$  is a best response. Since  $c_2 < \frac{(1-\alpha)(1-q)((1+f)t_2)}{(1-\alpha)(1-q) + \alpha q}$ , we have  $\gamma(1|2) < 1$ . For a taxpayer with the true profit of 1, the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = (1 - p_1)(1 - t_1 - ft_1) + p_1$ .  $EU_i(0|1) \geq EU_i(1|1)$  is equivalent to  $(1 - p_1)(1 - t_1 - ft_1) + p_1 \geq 1 - t_1$ , which is true when  $f \leq \frac{p_1}{1-p_1}$ . Thus  $\gamma(0|1) = 1$  is a best response.

(iv) Given the taxpayers' strategies  $\gamma(0|1) = 1$  and  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$ , the posterior (9) becomes  $\Pr(2|1) = \frac{c_2}{(1+f)t_2}$ , so  $\Pr(2|1)(1+f)t_2 - c_2 = 0$ . Thus,  $\rho(1|1) = \frac{1}{1+f}$  is a best response. The posterior (10) becomes  $\Pr(1|0) = \frac{(1-\alpha)(1-q)}{q + (1-\alpha)(1-q)}$ , so  $\Pr(1|0)(1+f)t_1 - c_2 = 0$  as  $c_2 = \frac{(1-\alpha)(1-q)(1+f)t_1}{(1-\alpha)(1-q) + q}$ . Thus,  $0 \leq \rho(0|0) \leq \frac{p_1(1+f)-f}{p_1(1+f)}$  is a best response.

On the other hand, given the inspector's strategy  $\rho(1|1) = \frac{1}{1+f}$ , for a taxpayer with the true profit of 2, the expected utilities are  $EU_i(2|2) = EU_i(1|2) = 2 - t_1 - t_2$ . Thus  $\gamma(1|2) = \frac{q\alpha c_2}{(1-\alpha)(1-q)((1+f)t_2 - c_2)}$  is a best response. For the taxpayer with true profit of 1, the expected utilities are  $EU_i(1|1) = 1 - t_1$  and  $EU_i(0|1) = [p_1\rho(0|0) + 1 - p_1](1 - t_1 - ft_1) + p_1(1 - \rho(0|0))$ .  $EU_i(0|1) \geq EU_i(1|1)$  is equivalent to  $[p_1\rho(0|0) + 1 - p_1](1 - t_1 - ft_1) + p_1(1 - \rho(0|0)) \geq 1 - t_1$ . After rearranging the expression, we obtain  $\rho(0|0) \leq \frac{p_1(1+f)-f}{p_1(1+f)}$  as stated in the proposition.<sup>38</sup> Thus  $\gamma(0|1) = 1$  is a best response.

2. The observation follows from part 1.  $\square$

**Proof of Proposition 6.** Fix the taxpayers' strategies as stated in the proposition. When  $\epsilon = 1$ , the inspector's updated belief is

$$\Pr(\pi_i = 2|\hat{\pi}_i = 1, \epsilon = 1) = \frac{(1 - \alpha)[\beta(1 - q_H) + (1 - \beta)(1 - q_L)]}{\alpha[\beta q_H + (1 - \beta)q_L] + (1 - \alpha)(1 - \beta q_H)} \tag{20}$$

<sup>38</sup> Here, since  $f < \frac{p_1}{1-p_1}$ ,  $p_1(1+f) - f > 0$ .

Given that  $\frac{(1-\alpha)[\beta(1-q_H)+(1-\beta)(1-q_L)](1+f)t_2}{\alpha[\beta q_H+(1-\beta)q_L]+(1-\alpha)(1-\beta q_H)} - c_2 < 0$ , the inspector should not audit those reporting 1, i.e.,  $\rho(1|1) = 0$ .

Similarly, when  $\epsilon = 0$ , the inspector's updated belief is

$$\Pr(\pi_i = 1|\hat{\pi}_i = 0, \epsilon = 0) = \frac{(1-\alpha)\beta(1-q_H)}{\beta q_H + (1-\beta)q_L + (1-\alpha)\beta(1-q_H)}. \quad (21)$$

Given that  $\frac{(1-\alpha)\beta(1-q_H)(1+f)t_1}{\beta q_H+(1-\beta)q_L+(1-\alpha)\beta(1-q_H)} - c_2 < 0$ , the inspector should not audit those reporting 0, i.e.,  $\rho(0|0) = 0$ .

Now, take the inspector's strategy as given. For a taxpayer with the true profit of 2, if the taxpayer reports 2, the expected utility is

$$EU_i(\hat{\pi}_i = 2|\pi_i = 2) = 2 - t_1 - t_2;$$

if the taxpayer reports 1, the expected utility is

$$EU_i(\hat{\pi}_i = 1|\pi_i = 2) = 2 - t_1 > 2 - t_1 - t_2.$$

Thus, the taxpayer should report 1, i.e.,  $\gamma_H(1|2) = \gamma_L(1|2) = 1$ .

For a taxpayer with the true profit of 1, if the taxpayer reports 1, the expected utility is

$$EU_i(\hat{\pi}_i = 1|\pi_i = 1) = 1 - t_1.$$

If the taxpayer is of high type and reports 0, the expected utility is

$$EU_{iH}(\hat{\pi}_i = 0|\pi_i = 1) = (1 - p_H)(1 - t_1 - ft_1) + p_H.$$

Given that  $f < \frac{p_H}{1-p_H}$ , the taxpayer should report 0, i.e.,  $\gamma_H(0|1) = 1$ .

If the taxpayer is of low type and reports 0, the expected utility is

$$EU_{iL}(\hat{\pi}_i = 0|\pi_i = 1) = (1 - p_L)(1 - t_1 - ft_1) + p_L.$$

Given that  $f > \frac{p_L}{1-p_L}$ , the taxpayer should report 1, i.e.,  $\gamma_L(1|1) = 1$ .  $\square$

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