

Group Size Paradox and Public Goods

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Abstract

In an economy with voluntarily provided public goods and private product varieties, and a general class of *CES* preferences, it is shown that aggregate public good contribution follows an inverted-U pattern with respect to group size when private and public goods are substitutable in preferences. With complementarity, however, aggregate provision grows monotonically with group size.

JEL Classification: H41. **Key Words:** Public good, group size, monopolistic competition, general equilibrium, partial equilibrium.

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1 Introduction

How group size affects voluntary provision of public goods is an old issue. Olson (1965) had argued that the free-rider problem would worsen in large groups. Chamberlin (1974), McGuire (1974), Andreoni (1988) etc. partly countered this view by showing that as the number of agents grew large total public good contribution would approach a finite upper bound. Pecorino (2009a) added to this debate by arguing that more population meant less public goods because individuals switch to greater varieties of private consumption goods that become available with the increased market size.¹

In this paper, consumers buy a composite private good made up of different varieties of private goods and contribute voluntarily to a public good. In such public good economy, an increase in group size (or, population) will endogenously support a larger variety of private goods, lowering the shadow price of the composite private good. As composite good becomes cheaper, the demand for public good, and consequently its aggregate provision level, will depend on the elasticity of substitution between the composite good and the public good. There is also a traditional income effect resulting from increased group size: the consumers' budgets will be relaxed as any public good level produced in the economy will serve a larger population. So how the combined effects of a larger group size impact on the level of public good will depend very much on the elasticity of substitution between the composite good and the public good, as well as the elasticity of substitution between the various private good varieties.

We show that, when the elasticity of substitution between the composite good and the public good is less than or equal to unity, the conventional wisdom on group-size effect (i.e., larger groups lead to higher public goods) prevails. However, if this elasticity exceeds unity, then often as the group size increases initially the public good level will increase and then decrease. Thus, the relationship between public good and group size exhibits an inverted U-shape.

The model is specified in section 2, equilibrium analysis appears in section 3, comparative statics in section 4, and concluding summary in section 5.

2 The model

There are L individuals who each inelastically supplies one unit of labor, earns a competitive wage w and spends it on the composite good and contribution towards the public good. Denoting g_j to be the dollar contribution towards the public good by consumer j , $G = \sum_{j=1}^L g_j$ is the total voluntary contribution by L consumers. We normalize the price of the public good at unity so that G is the total amount of public good consumed.

¹Pecorino (2009b) analyzes the effect of group size on public good in a much simpler economy without production but allowing for rivalry in public good's consumption.

The consumers have identical preferences. Representative consumer j solves:²

$$\max_{X_j, g_j} U_j = [\eta X_j^r + (1 - \eta)G^r]^{\frac{1}{r}}, \quad 0 \neq r \leq 1, \quad 0 < \eta < 1 \quad (1)$$

$$\text{subject to} \quad p_X X_j + g_j = w, \quad (2)$$

$$\text{where} \quad X_j = \left(\sum_{i=1}^n c_{ij}^\theta \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \quad (3)$$

$$\text{and} \quad p_X X_j = \sum_{i=1}^n p_i c_{ij}. \quad (4)$$

The composite good X_j for consumer j as defined in (3) is also a *CES* function, of j 's consumption of n private goods (c_{ij}) ($i = 1, 2, \dots, n$). The price of the composite good is denoted by p_X . The price of the private variety i is given by p_i . Finally, we define $\epsilon = \frac{1}{1-r} \geq 0$ as the elasticity of substitution between the composite good and the public good. We also define $\sigma = \frac{1}{1-\theta} > 1$ as the elasticity of substitution between any two private varieties. For the rest of our analysis, we impose the following assumption.

Assumption 1 *Suppose $\sigma \geq \epsilon$. That is, within the group the private goods are more substitutable than they are as a group vis-à-vis the public good.*

Treating the differentiated private products and the public good to be inherently different (such as different food items vs. community policing), it is natural to assume that the private goods (say, different varieties of food) are more substitutable than they would be as a whole vis-à-vis the public good (i.e., the community policing).

One can have the following solutions from the consumer's problem:³

$$G = \frac{w}{\frac{1}{\epsilon} + \left(\frac{1-\eta}{\eta} p_X \right)^{-\epsilon} p_X}, \quad (5)$$

$$X_j = \frac{w \left(\frac{1-\eta}{\eta} p_X \right)^{-\epsilon}}{\frac{1}{\epsilon} + \left(\frac{1-\eta}{\eta} p_X \right)^{-\epsilon} p_X}, \quad (6)$$

$$c_{ij} = \frac{p_i^{-\sigma} (p_X X_j)}{\sum_{k=1}^n p_k^{1-\sigma}}, \quad (7)$$

$$p_X = \left(\sum_{i=1}^n p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (8)$$

This completes the description of the demand side of the model. We now turn to the production side.

²When $r \downarrow 0$, the utility function (1) approaches the standard Cobb-Douglas form $U_j = X_j^\eta G^{1-\eta}$, with unitary elasticity of substitution between the composite good and the public good.

³Derivations are available in an online appendix at http://web.iitd.ac.in/~debasis/appendix_groupsizeparadox.pdf

3 Monopolistic competition: general equilibrium

There are n varieties of private goods, each produced under monopolistic competition as in Krugman (1980) and Pecorino (2009a). The production technology of good i is:

$$l_i = \alpha + \beta y_i, \quad i = 1, 2, \dots, n. \quad (9)$$

Here α represents a fixed cost and β is the marginal cost. Total labor employment in the production of n private goods is

$$L_Y = \sum_{i=1}^n (\alpha + \beta y_i).$$

For the public good production, we assume a one-to-one transformation from labor to public goods. This along with perfectly competitive production structure implies that the price of the public good becomes its marginal cost of production which is the wage rate (denoted by w). By normalizing the price of the public good, we get $w = 1$. The public good is financed entirely from voluntary contributions. So the total labor employed in public good production is

$$L_G = G.$$

Labor market clearing requires

$$L = L_Y + L_G. \quad (10)$$

With L consumers in the economy and individual demand for the i th variety given by (7), the aggregate demand for the i th variety is $\sum_{j=1}^L c_{ij} = c_i L$, suppressing j from c_{ij} . Firm i maximizes profit:

$$\pi_i = p_i y_i - \alpha w - \beta y_i w, \quad (11)$$

$$\text{where } y_i = c_i L. \quad (12)$$

We assume, similar to Krugman (1980) and Pecorino (2009a), that the monopolist treats the price-index (8) as given while maximizing profit.⁴ Then the profit-maximizing price can be solved as

$$p_i = \frac{\beta}{\theta}, \quad i = 1, 2, \dots, n. \quad (13)$$

Free entry and exit in monopolistically competitive industries guarantee zero profit. So setting $\pi_i = 0$ in (11) and noting that $w = 1$, we get

$$y_i = \frac{\theta \alpha}{(1 - \theta) \beta}, \quad i = 1, 2, \dots, n. \quad (14)$$

Using the solved prices and output, obtain $L_Y = \frac{n\alpha}{1-\theta}$. Using (10), the number of differentiated

⁴Note that the price-index p_X involves the prices p_i 's.

varieties is solved as follows:

$$n = \frac{(L - G)(1 - \theta)}{\alpha}. \quad (15)$$

We can also solve for the composite good price-index in (8), using (13), as follows:

$$p_X = n^{\frac{1}{1-\sigma}} \frac{\beta}{\theta}. \quad (16)$$

This completes the general equilibrium structure of the model. To solve for G , rewrite eq. (5), using $w = 1$ and eqs. (15) and (16), as follows:

$$(L - G)^{\frac{\sigma-\epsilon}{\sigma-1}} = LG \left(\frac{\eta}{1-\eta} \right)^\epsilon \left(\frac{\theta}{\beta} \right)^{\epsilon-1} \left(\frac{1-\theta}{\alpha} \right)^{\frac{\epsilon-1}{\sigma-1}}. \quad (17)$$

The left-hand side of (17) is a decreasing function of G as long as $\sigma > \epsilon$ (and constant for $\sigma = \epsilon$). The right-hand side is always strictly increasing in G . Then there must exist a unique solution of G under Assumption 1. Let us denote the solution as G^* . We now analyze some comparative static properties of G^* for the case when $\sigma > \epsilon$.

4 Comparative statics

From (17) one can show that

$$\frac{dG^*}{dL} = \frac{(\Gamma - 1)L + G^*}{\Gamma G^* + (L - G^*)} \frac{G^*}{L}, \quad (18)$$

where $\Gamma \equiv \frac{\sigma-\epsilon}{\sigma-1}$. The denominator of the above expression is always positive since $L > G^* > 0$, $\sigma > \epsilon$ and $\sigma > 1$ (so that, $\Gamma > 0$). So, the sign of $\frac{dG^*}{dL}$ in (18) depends on the sign of the numerator. We now consider two cases.

■ **Case 1:** $\epsilon \leq 1$ (or, $\Gamma \geq 1$). In this case, the public and private goods are complements to each other in the preferences. Then, it is easy to see that $\frac{dG^*}{dL} > 0$ from eq. (18). In other words, with complementarity, any increase in population should always raise the aggregate provision of the public good. This result agrees with the standard predictions of the effect of population growth on public goods in partial equilibrium models, see, for instance, the earlier literature (Chamberlin, 1974; McGuire, 1974; Andreoni, 1988, etc.).

Intuitively, there are three different effects driving this comparative statics when L increases. The first is the *traditional income effect* that comes from an increase in L : the consumers' budgets will be relaxed as any public good level produced in the economy will serve a larger population. This leads to an increase in aggregate provision. The second effect is the pure *substitution effect* that originates from the resulting change in the price of the private composite goods (denoted by p_X). As L increases, the number of private varieties goes up. This leads to a fall in the price of the composite private good. Due to this substitution effect, consumers are driven more toward the private good. This leads to a decline in the demand for the public good and consequently its lesser provision. The third is the *income effect* from a decrease in p_X . Consumers perceive themselves

to be richer and contribute a bit more toward the public good. The second and third effects, combined, can be termed as the *total price effect*. The general CES utility function we use in (1) with its varied range of substitution possibilities will determine the relative sizes of these effects.

With $\epsilon < 1$, X and G are gross complements. So, any decrease in the price of the private good should raise the demand for the public good. As consumers demand a bit more of the public good, its total provision goes up. Moreover, there is also the traditional income effect associated with larger population, reinforcing the price effect. So, the aggregate provision is strictly increasing in population size.

The preference structure becomes Cobb-Douglas with $\epsilon = 1$. Here the income and substitution effects associated with the price change exactly cancel out each other, leaving only the traditional income effect as the residual. Thus, we see that the public good level is increasing in L when $\epsilon = 1$. However, this traditional income effect gets weaker in larger economies, with the public good level approaching an upper bound $\frac{1-\eta}{\eta}$ asymptotically as L grows to infinity when $\epsilon = 1$. These results can be formally written in the following proposition:

Proposition 1 *Suppose in the preferences (1), $\tau \leq 0$. Equivalently, suppose $\epsilon \leq 1$ where ϵ is the elasticity of substitution between the public good and the composite good. Then the voluntary provision of public goods is strictly increasing in population size.*

Interestingly, in the case of strong complementarity, it is even possible for the aggregate provision to grow unbounded as the size of the economy grows large. This happens if $\tau > 2$ (which is possible if ϵ is close to zero). Then average contribution grows with population (i.e., $\frac{dg}{dL} > 0$). This result contrasts with some of the earlier findings where average contribution goes to zero in replicated economies (Theorem 1, Andreoni 1988; Theorem 3, Furusawa and Konishi 2011).⁵

■ **Case 2: $\epsilon > 1$ (or, $\tau < 1$).** Here private and public goods are gross substitutes. One can show that in this case,

$$\frac{dG^*}{dL} > (=; <) 0 \quad \text{iff} \quad G^* > (=; <) \left(\frac{\epsilon - 1}{\sigma - 1}\right)L. \quad (19)$$

Then there is a critical level of public good, $\left(\frac{\epsilon-1}{\sigma-1}\right)L$, such that whenever equilibrium provision is above this critical level, we get $\frac{dG^*}{dL} > 0$. On the other hand, we get $\frac{dG^*}{dL} < 0$, whenever the equilibrium provision falls short of this critical value.

Intuitively, with $\epsilon > 1$, X and G are gross substitutes and the *total price effect* due to population growth tends to lower the equilibrium G. But the *traditional income effect* tends to increase G. Depending on the relative strengths of these opposing effects, G^* may be increasing or decreasing in L. Since income effect tends to be weaker in larger groups, G^* should fall with large enough L. We summarize these observations as follows:

Proposition 2 *Given the preference specification in (1), monopolistic competition in the differentiated product varieties and the assumption that $\sigma > \epsilon$, voluntary provision of public goods, G^* ,*

⁵Andreoni (1988) shows this using a general preference structure. Furusawa and Konishi (2011) assume quasi-linear preferences, and use a special type of replication.

is

- (i) increasing in population size L if $G^* > (\frac{\epsilon-1}{\sigma-1})L$,
- (ii) decreasing in population size L if $G^* < (\frac{\epsilon-1}{\sigma-1})L$, and
- (iii) stationary as population size L varies if $G^* = (\frac{\epsilon-1}{\sigma-1})L$.

However, the equilibrium product varieties, \mathfrak{n}^* , always increases with the population size L .

We next show the possibility of an inverted-U relationship between G^* and L in case 2. When $\epsilon > 1$ (or, $\Gamma < 1$), one can show that G^* has a unique maximum with respect to L . This is proved in the following expression, derived using (18):

$$\frac{d^2G^*}{dL^2} \Big|_{\frac{dG^*}{dL}=0} = \frac{\Gamma - 1}{\Gamma G^* + L - G^*} \frac{G^*}{L} < 0 \quad (\text{since, } \Gamma < 1).$$

Thus, G^* has a maximum at a point where $\frac{dG^*}{dL} = 0$ or, where $G^* = \frac{\epsilon-1}{\sigma-1}L$. Plugging this value back in (17), one can solve for the critical value of population (denoted by \hat{L}) where G^* attains its maximum as follows:

$$\hat{L} = \Gamma^{\frac{\Gamma}{2-\Gamma}} A^{\frac{1}{2-\Gamma}} \left(\frac{\sigma-1}{\epsilon-1} \right)^{\frac{1}{2-\Gamma}}, \quad (20)$$

where Γ is defined earlier and $A \equiv \left(\frac{\eta}{1-\eta} \right)^\epsilon \left(\frac{\theta}{\beta} \right)^{\epsilon-1} \left(\frac{1-\theta}{\alpha} \right)^{\frac{\epsilon-1}{\sigma-1}}$.

Proposition 3 *For the general CES preferences specified in (1), suppose $\sigma > \epsilon > 1$. Also assume that there is monopolistic competition in the differentiated goods market. Then, total voluntary contribution to public goods will initially rise with the population size L up to $L = \hat{L}$ as determined in (20), and then starts falling. This gives rise to an inverted-U shape public good provision.*

We now present an example. Consider parameter values $\alpha = 100, \beta = 20, \eta = 2/3, \sigma = 5, \epsilon = 3$. For these values, $G^*(L)$ against L is obtained in Fig. 1 by plotting the graph of eq. (17). The critical population $\hat{L} = 182$ and the corresponding public good $G^*(\hat{L}) = 91.38$.

■ **Technological change.** Any technological improvement in the production of private goods can be thought of as reductions in α and β . One can easily show by using eq. (17) that the following results hold.

Proposition 4 *Reductions in the fixed cost, α , and the marginal cost, β , both result in reduced (increased) voluntary contributions to public goods, if the elasticity of substitution between the public good and the private composite good, ϵ , is greater (less) than unity. Only when the utility is of Cobb-Douglas form (i.e., $\epsilon = 1$), the size of the public good is neutral with respect to any technological change.*

Intuitively, a reduction in α increases the available varieties of private goods. This lowers the price of the composite private good. When $\epsilon > 1$, the substitution effect is stronger than the income

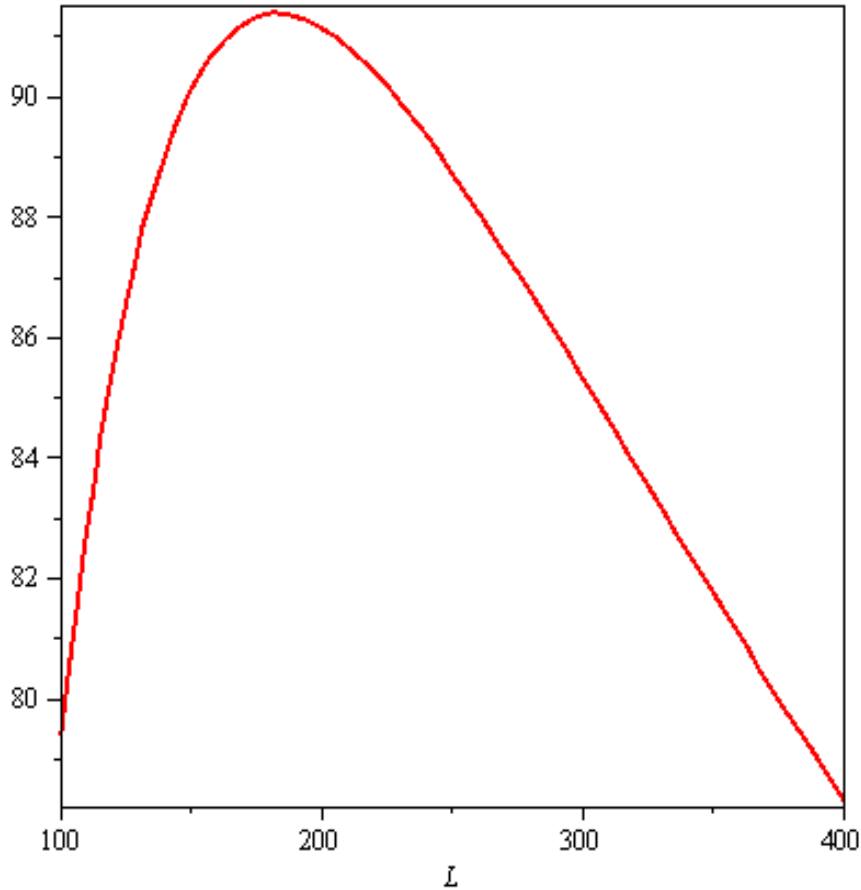


Figure 1: A typical inverted-U provision

effect, and the public good provision falls and the opposite happens when $\epsilon < 1$. In the case of Cobb-Douglas preferences ($\epsilon = 1$), these two opposite effects cancel out each other, leaving G neutral to any change in α . Effect of a decrease in β follows similarly.

5 Conclusion

Olson's (1965) conjecture about the effectiveness (or rather ineffectiveness) of large groups in voluntary provision of collective goods has prompted an interesting debate. Based on a generalization of Pecorino (2009a), this paper provides a complete characterization of the relationship between group size and public goods.

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