Dominance of contributions monitoring in teams

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Abstract

In team problems it has been previously argued that there is no loss to the principal from monitoring team output compared to monitoring of individual contributions, a result known as monitoring equivalence. Optimal output monitoring, however, sometimes required up front payment from the agents to the principal. By introducing limited liability (LL) on the part of agents that rules out positive monetary transfers to the principal, it is shown that the principal strictly benefits by monitoring individual contributions. Positive rent of the lowest type under output monitoring with LL implies there will be a dominating contributions monitoring contract that further transfers some of this rent to the principal. Thus, unlimited agent liability is necessary for the equivalence result.

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Key words: Principal, team, joint project, adverse selection, moral hazard, limited liability, monitoring, punishing contract

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1 Introduction

McAfee and McMillan (1991) studied a team monitoring problem where the principal can incentivize either by joint output without observing the team members' individual contributions, or by giving rewards based on individual contributions. The team members' (or agents') abilities are private information. The authors show that under appropriate conditions a compensation scheme linear in the team's aggregate output is optimal. That is, the disaggregated information on individual contributions is of no extra value to the principal: the two types of information are *equivalent*. This is a very intriguing result – with the disaggregated information the principal is expected to monitor more directly the individual agents in a team environment and incentivize them better.

We seek to gain a better understanding of the monitoring equivalence puzzle and contribute to the debate of *contributions vs. output monitoring*. To this end, we will use a parallel, discretized team production technology. The purpose in studying a different technology is to take McAfee and McMillan's broad economic message and subject it to further scrutiny. But given the difference in technology and other modelling assumptions, our analysis is not going to be an exact parallel of that of McAfee and McMillan. Instead, any difference in results and insights should be viewed within the context of our model. Still we hope to offer an enhanced understanding of the core debate.

To set off, we ask what aspect of McAfee and McMillan's optimal output monitoring mechanism could have been critical to their equivalence result, and how plausible that might be. In our discrete technology model, we identify one feature that plays a critical role in determining whether output monitoring could possibly be as powerful as McAfee and McMillan's result projects it to be: whether the principal can really ask the agents to pay upfront strictly positive amount of money which they stand to lose if the team output turns out to be "unsatisfactory". That is, effectively, the principal will be asking the agents to pay a fee to be able to participate in the team activity. This feature, to be described as *unlimited agent liabilities*, will be a contrast to what our study proposes as the more plausible assumption, the one that imposes on the principal the requirement of *limited liability* by the team members: the agents will not make any payment to the principal at any stage of contracting, in any eventuality. Any positive transfer can only be one-sided – from the principal to the agents and not the other way around. In most team based work arrangements, it might not be plausible for the main initiator of a project, the principal, to ask its members to contribute to the project and yet to post at the start a bond that they might forfeit if team performances do not go according to plans.[1]

Che and Yoo (2001) have pointed out, "Limited liability of the agents may arise from workers' having the freedom to quit but it may also arise from institutional constraints such as laws banning firms' exacting payments from workers, workers' liquidity constraints, or their extreme risk aversion for bearing loss." While imposing this limited liability restriction, we still retain McAfee and McMillan's assumption that the agents are risk neutral.[2] With this modification,

[1] Exceptions could be law firms or a group of medical doctors in private practices where junior partners may have to pledge compensation at the start in case the firm (or the group) does not perform well. Simon Grant suggested this example.

[2] Vander Veen (1995) observed that McAfee–McMillan's equivalence result should break down if the agents

we will show that the principal should strictly prefer monitoring individual contributions over output monitoring, which shows that unlimited agent liabilities, an assumption implicit in McAfee and McMillan, is *necessary* for the equivalence result.

Our argument for the dominance of contributions monitoring will proceed as follows. We first propose a feasible contributions monitoring contract which could generate the same expected payoff for the agents and the principal as the output monitoring contract. This contributions monitoring contract is of the take-it-or-leave-it form, with the principal setting a target individual contribution for each declared type of an agent (and a reported type profile of the other agents) and giving him an agreed performance reward only if the set target has been met. This disciplines agents' misbehavior in type reporting as their contribution choices following non-truthful reporting are restricted. We then show that under the optimal output monitoring contract the principal needs to leave positive rent to all types of at least one agent. Since the optimal output monitoring contract can be replicated by contributions monitoring contract with equivalent expected payoff for the principal as well as all types of all agents, and we prove that the replicated contract can be improved by extracting the rent from the lowest type agent, this suggests the superiority of contributions monitoring contract.

A more direct intuition can be given as follows. When limited liability condition is imposed, the principal cannot punish the agents sufficiently for failing to meet their target contributions, which makes inducing any intended contributions more costly. This problem is exacerbated when the agents' contributions cannot be directly observed so that the principal has to rely on output monitoring. In this case, if the output level turns out to be below expectation, the principal is not able to identify who are the main delinquents. In other words, limited liability and moral hazard, together, make it harder for the principal to induce contributions. In the contributions monitoring contract on the other hand, once the principal specifies a type contingent target performance for each agent, any deviation in individual contributions is always detected by the principal. Thus, less information rents are needed under the contributions monitoring contract.

As mentioned earlier, our setting differs in an important way from that of McAfee and McMillan (2001). They considered a continuum of agent types whereas we consider discrete/finite number of types.[3] This distinction will be an important factor in highlighting how limited liability plays a central role in delivering an intuitive result, that the principal's ability to monitor individual agents' contribution efforts helps mitigate the moral hazard problem and lifts contribution monitoring above output monitoring. We lose McAfee–McMillan's surprising result (that disaggregated information is of no extra value) but gain an understanding of what factor, among a multiple number of factors,[4] could have played a role behind the equivalence result. Our work should thus be viewed as adding value to the output vs. contributions monitoring debate, helping generate insights albeit in a different discretized environment with its own technical hurdles that are different from the ones in McAfee–McMillan's continuum types model. We must repeat, and emphasize, that our analysis is not a 'ceteris paribus' analysis of McAfee

are risk averse.

[3]There are other differences as well that will become clear in the model section.

[4]See the discussion of Holmström's (1982) conjectures below.

and McMillan but nonetheless applies to a natural variant given that the contract theory literature has devoted significant efforts in developing both continuous and discrete technology formulations.

Earlier in one of the first generation analysis of team moral hazard problems, Holmström (1982) had noted that “if there is uncertainty in production and if agents are risk averse or have limited endowments, monitoring becomes an important instrument in remedying moral hazard.” In McAfee and McMillan's work, agents are risk neutral and can be asked to make upfront payment. Modifying McAfee and McMillan's model, we verify Holmström's claim about the relevance of limited endowments by showing that the *type of monitoring* matters.

Ollier and Thomas (2013) studied a principal-agent problem with adverse selection and moral hazard. They consider incentives based on the agent's binary output, success or failure, and an ex post participation constraint that prevents the agent's payoff (net of the effort cost) from becoming negative even at the worse output, failure. Our limited liability condition is simply a positivity restriction on payments to the agents, rather than the agents' payoffs net of effort cost.[5] Different aspects of monitoring in principal-agent and team settings have also appeared in Varian (1990), Khalil and Lawarrée (1995), Raith (2008), Rahman (2012), and Gershkov and Winter (2015).

The rest of the paper proceeds as follows. The formal model with two types of monitoring contracts is analyzed in Sect. 2. The results comparing the monitoring contracts are presented in Sect. 3, with some further discussions in Sect. 4. The proofs appear in the Appendix. Detailed proof of Lemma 1 and some additional materials are included in a Supplementary file.

2 Model

A principal wants a team of n agents to undertake a project. Both the principal and the agents are risk neutral. Each agent i is endowed with ability level z_i, $i = 1, ..., n$. There are m different ability levels ranging from the lowest to the highest, i.e., from θ_1 to θ_m, and $m \geq 2$. It is common knowledge that z_i's are identically and independently distributed with the density function $\Pr(z_i = \theta_j) = q_j > 0$, $j = 1, ..., m$. Only agent i knows his true ability z_i. Each agent i chooses an effort level privately. Let $y_i \in \mathcal{R}_+$ represent agent i's individual contribution which is a combined outcome of i's ability and effort. The agent's cost of contribution, $c(y_i, z_i)$, is differentiable in y_i. Assume that $c(0, z_i) = 0$, $c_y \geq 0$ and $c_{yy} > 0\ \forall z_i$, and for any value of $y_i \neq 0$, $c(y_i, \theta_1) > c(y_i, \theta_2) > ... > c(y_i, \theta_m)$. Also, $\forall z_i$, $c(y_i, z_i) \to \infty$ as $y_i \to \infty$. Let $\mathbf{z} = (z_1, ..., z_n)$, $\mathbf{z}_{-i} = (z_1, ..., z_{i-1}, z_{i+1}, ..., z_n)$, $\mathbf{y} = (y_1, ..., y_n)$, and $\mathbf{y}_{-i} = (y_1, ..., y_{i-1}, y_{i+1}, ..., y_n)$.

The output level x, which is observed by the principal, depends on each one of the team members' contributions and some noise. Assume that the possible output level $x \in X$, where X is a non-empty finite set. Denote the conditional probability of the output x given $\mathbf{y}$ by $f(x|\mathbf{y})$. We impose the following assumption:

Assumption 1 *The team production technology, $f(x|\mathbf{y})$, is uniformly bounded below by some number $a > 0$, i.e., $f(x|\mathbf{y}) \geq a > 0$, $\forall x, \mathbf{y}$. Also, $f(x|\mathbf{y})$ is continuous in $\mathbf{y}$ and concave in y_i.*

[5] Thus, our limited liability restriction is weaker: ex post participation constraint implies limited liability but necessarily the other way around.

This is an important assumption. Positive probability of each possible output level for any contributions profile makes the inference of individual contributions a more meaningful exercise in the spirit of team problems. An agent may make a zero contribution and yet output realized can be at the highest level. So, unless an agent of any given type is given an extreme contract involving zero or negative payments for all output realizations, there will always be some output realization at which the agent will receive a positive payment under output monitoring. Further, when limited liability condition is imposed (which we will introduce soon), an agent's expected payment from the principal even under complete shirking (i.e., $y_i = 0$) will be positive. This last point will play a critical role in establishing Proposition 3 and consequently, our main result, Proposition 4.

The principal's utility is $U(x, \mathbf{z})$ which depends not only on the realized output, but also on the agents' abilities. We also normalize the value of each agent's outside option to be 0.

For any function $\Psi(x, \mathbf{z})$ or $\Psi(\mathbf{y}, \mathbf{z})$ or $\Psi(x, \mathbf{y}, \mathbf{z})$, let

$$E_{-i}\Psi(\cdot, \mathbf{z}) = \sum_{z_1=\theta_1}^{\theta_m} \dots \sum_{z_{i-1}=\theta_1}^{\theta_m} \sum_{z_{i+1}=\theta_1}^{\theta_m} \dots \sum_{z_n=\theta_1}^{\theta_m} \Psi(\cdot, \mathbf{z})\Big[\prod_{\ell \neq i} \Pr(z_\ell)\Big];$$

$$E_{\mathbf{z}}\Psi(\cdot, \mathbf{z}) = \sum_{z_1=\theta_1}^{\theta_m} \dots \sum_{z_n=\theta_1}^{\theta_m} \Psi(\cdot, \mathbf{z})\Big[\prod_{\ell=1}^{n} \Pr(z_\ell)\Big];$$

$$E_x\Psi(x, \cdot) = \sum_x \Psi(x, \mathbf{z}) \cdot f(x|\mathbf{y}).$$

By the Revelation Principle (Myerson, 1982), we will focus on truth-telling mechanisms. Let $\hat{z}_i$ denote agent i's report to the principal about his ability, and $\hat{\mathbf{z}}$ and $\hat{\mathbf{z}}_{-i}$ are the reported type profiles with similar meaning as before. Moreover, we will restrict to deterministic mechanisms similar to McAfee and McMillan (1991). This need not be without loss of generality as stochastic mechanisms may well improve the principal's payoff.[6]

2.1 Output monitoring contract

In the output monitoring contract, the principal can verify aggregate team output. He thus commits to a payment rule $p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i})$ specifying transfers to agent i based on realized output, i's report of his own ability (or type), and the other agents' declared types. Also, the principal makes contribution recommendations to each agent i, denoted by $y_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$. We assume that the principal has the technology to credibly disclose the actual type communications *after* the agents have made their contribution decisions.

The principal's profit and the agents' payoffs are respectively

$$\phi(x, \mathbf{z}, \hat{\mathbf{z}}) = U(x, \mathbf{z}) - \sum_{i=1}^{n} p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}),$$

$$\pi_i(x, z_i, \hat{z}_i, y_i, \hat{\mathbf{z}}_{-i}) = p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}) - c(y_i, z_i), \quad i = 1, \dots, n.$$

[6]Strausz (2006) had argued that in most principal-agent applications, focusing on deterministic mechanisms can be justified so long as the optimal (deterministic) mechanism satisfies a "no-bunching" condition. To our knowledge no such result is available in the principal-multi-agent setting, which is our focus.

Thus, the principal solves the following program:

$$[\mathcal{P}_{out}] \qquad \max_{\{p_i(\cdot), y_i(\cdot)\}} E_{\mathbf{z}} \sum_x \big[U(x, \mathbf{z}) - \sum_{i=1}^{n} p_i(x, z_i, \mathbf{z}_{-i})\big] f(x|\mathbf{y}(\mathbf{z}))$$

subject to the feasibility constraints:

$$E_{-i}\big[\sum_x p_i(x, z_i, \mathbf{z}_{-i}) f(x|\mathbf{y}(z_i, \mathbf{z}_{-i})) - c(y_i(z_i, \mathbf{z}_{-i}), z_i)\big]$$
$$\geq E_{-i}\big[\sum_x p_i(x, \hat{z}_i, \mathbf{z}_{-i}) f(x|\delta_i(y_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), y_{-i}(\hat{z}_i, \mathbf{z}_{-i})) - c(\delta_i(y_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), z_i)\big],$$
$$\forall i, z_i,\ \hat{z}_i,\ \delta_i(y_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i) \in \mathcal{R}_+ \qquad \text{(IC(out))}$$
$$E_{-i}\big[\sum_x p_i(x, z_i, \mathbf{z}_{-i}) f(x|\mathbf{y}(z_i, \mathbf{z}_{-i})) - c(y_i(z_i, \mathbf{z}_{-i}), z_i)\big] \geq 0, \quad \forall i, z_i \qquad \text{(PC(out))}$$
$$p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}) \geq 0, \quad \forall i, x, \hat{z}_i, \hat{\mathbf{z}}_{-i}. \qquad \text{(LL(out))}$$

where $\delta_i(y_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i)$ is any arbitrary choice of contribution by agent i with type z_i and reported type $\hat{z}_i$ when he receives recommendation $y_i(\hat{z}_i, \mathbf{z}_{-i})$.

Different from McAfee and McMillan's (1991) analysis we impose the limited liability constraints for the agents as in LL(out), i.e., regardless of the output level or type declarations the agents cannot be asked to make positive transfers to the principal. In reality, the agents might be financially constrained to make upfront payment infeasible.

Denote $\mathbf{p}(.) = (p_1(.), ..., p_n(.))$. The following result gives the optimal monitoring question a proper benchmark.

Lemma 1 *An optimal output monitoring contract solving the program* $[\mathcal{P}_{out}]$ *always exists.*

2.2 Contributions monitoring contract

Now, suppose the principal could costlessly monitor each individual's contribution, so that he could pay agent i according to his contribution y_i as well as the reported profile of abilities $(\hat{z}_i, \hat{\mathbf{z}}_{-i})$. Denote the payment by $\bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i})$ and the recommended contribution by $\bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$.[7]

The principal's profit and the agents' payoffs are respectively

$$\bar{\phi}(x, \mathbf{y}, \mathbf{z}, \hat{\mathbf{z}}) = U(x, \mathbf{z}) - \sum_{i=1}^{n} \bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}),$$
$$\bar{\pi}_i(z_i, \hat{z}_i, y_i, \hat{\mathbf{z}}_{-i}) = \bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) - c(y_i, z_i), \quad i = 1, ..., n.$$

[7]Note that the payment to agent i is restricted to depend only on i's contribution. Admittedly this weakens the principal's hand but given that ultimately we are going to show dominance of contributions monitoring, allowing a more general payment function that depends on other agents' contributions as well would retain the dominance result if not strengthen it further.

Thus, the principal solves the following program:

$$[\mathcal{P}\text{con}] \qquad \max_{\{\bar{p}_i(\cdot),\bar{y}_i(\cdot)\}} E_{\mathbf{z}} \sum_x \Big[U(x,\mathbf{z}) - \sum_{i=1}^{n} \bar{p}_i(\bar{y}_i(z_i,\mathbf{z}_{-i}), z_i, \mathbf{z}_{-i})\Big] f(x|\bar{\mathbf{y}}(z_i,\mathbf{z}_{-i}))$$

subject to the feasibility constraints:

$$E_{-i}\bar{\pi}_i(z_i, z_i, \bar{y}_i(z_i,\mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \bar{\delta}_i(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}), \quad \forall i, z_i, \hat{z}_i, \bar{\delta}_i(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}), z_i, \hat{z}_i) \tag{IC(con)}$$

$$E_{-i}\bar{\pi}_i(z_i, z_i, \bar{y}_i(z_i,\mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq 0, \quad \forall i, z_i \tag{PC(con)}$$

$$\bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) \geq 0, \quad \forall i, y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}, \tag{LL(con)}$$

where $\bar{\delta}_i(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}), z_i, \hat{z}_i)$ is any arbitrary choice of contribution by agent i.

2.3 Timing

Stage 1. Principal first announces the contract $\{p_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i}), y_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})\}$ (or $\{\bar{p}_i(y_i,\hat{z}_i,\hat{\mathbf{z}}_{-i}), \bar{y}_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})\}$) to the agents.

Stage 2. Each agent decides whether to participate or not. If all of them accept the contracts, then each agent reports his type privately to the principal.

Stage 3. After receiving all the reported types, the principal makes contribution recommendations according to $y_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})$ (or $\bar{y}_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})$) to each agent i privately.

Stage 4. Each agent then exerts effort independently and privately. Principal could observe all the agents' contributions if contributions monitoring contract is chosen in Stage 1.

Stage 5. Output is realized. Principal pays each agent $p_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i})$ (or $\bar{p}_i(y_i,\hat{z}_i,\hat{\mathbf{z}}_{-i})$) according to the contract chosen in Stage 1. ||

3 Contributions vs. output monitoring

3.1 Numerical example for payoff equivalence and dominance results

Before we start with the formal analysis, let us first provide a numerical example to show that without limited liability, the principal's payoffs are the same under contributions and output monitoring, but principal is better off under contributions monitoring with limited liability. The example is solved using the MATLAB program, after deriving analytically the relevant feasibility conditions that are included in the Supplementary file.

Suppose there are two agents, and each agent can be high type or low type. The probability for any agent to be of high type is 0.7, i.e, $\Pr(z_i = \theta_H) = 0.7$ and $\Pr(z_i = \theta_L) = 0.3$, where $\theta_H = 0.7$ and $\theta_L = 0.2$. The cost function is $c(y_i, z_i) = (1 - z_i)y_i^2$. The output x is also binary with $x^L = 1$ and $x^H = 2$, and $f(x^H|(y_1, y_2)) = \sqrt{y_1 y_2} + \epsilon$ if $y_1 y_2 \leq (1-\epsilon)^2$ and equals 1 otherwise, where $\epsilon = 10^{-8}$.

	Contributions Monitoring		Output Monitoring
$y_i(\theta_H, \theta_H)$	0.833	$\bar{y}_i(\theta_H, \theta_H)$	0.833
$y_i(\theta_H, \theta_L)$	0.521	$\bar{y}_i(\theta_H, \theta_L)$	0.521
$y_i(\theta_L, \theta_H)$	0.203	$\bar{y}_i(\theta_L, \theta_H)$	0.203
$y_i(\theta_L, \theta_L)$	0.127	$\bar{y}_i(\theta_L, \theta_L)$	0.127
$\bar{p}_i(\bar{y}_i(\theta_H, \theta_H), \theta_H, \theta_H)$	0.213	$p_i(x_L, \theta_H, \theta_H)$	-1.514
		$p_i(x_L, \theta_H, \theta_L)$	-0.102
$\bar{p}_i(\bar{y}_i(\theta_H, \theta_L), \theta_H, \theta_L)$	0.127	$p_i(x_L, \theta_L, \theta_H)$	0.035
		$p_i(x_L, \theta_L, \theta_L)$	-0.126
$\bar{p}_i(\bar{y}_i(\theta_L, \theta_H), \theta_L, \theta_H)$	0.084	$p_i(x_H, \theta_H, \theta_H)$	0.558
		$p_i(x_H, \theta_H, \theta_L)$	0.602
$\bar{p}_i(\bar{y}_i(\theta_L, \theta_L), \theta_L, \theta_L)$	-0.106	$p_i(x_H, \theta_L, \theta_H)$	0.187
		$p_i(x_H, \theta_L, \theta_L)$	0.027
Principal's payoff	1.278	Principal's payoff	1.278

Table 1: Optimal contract under contributions monitoring and output monitoring without limited liability.

Table 1 summarizes the results under contributions monitoring and output monitoring without limited liability.[8] The principal's optimal payoff is 1.278 under both contributions and output monitoring, and the induced contributions are also the same, i.e., $y_i(z_i, z_{-i}) = \bar{y}_i(z_i, z_{-i})$, $\forall i, z_i, z_{-i}$. The payments on the equilibrium path under contributions monitoring are presented in Table 1,[9] while other off-equilibrium payments can be set at arbitrarily large negative numbers. The payments under output monitoring are also listed above.[10] If we compare these two payment rules under contributions monitoring and output monitoring, it can be seen that

$$\begin{aligned}\bar{p}_i(\bar{y}_i(z_i, z_{-i}), z_i, z_{-i}) \quad = \quad & \Pr(x = x_L | (y_i(z_i, z_{-i}), y_{-i}(z_{-i}, z_i)))\, p_i(x_L, z_i, z_{-i}) \\ & + \Pr(x = x_H | (y_i(z_i, z_{-i}), y_{-i}(z_{-i}, z_i)))\, p_i(x_H, z_i, z_{-i}), \end{aligned} \tag{1}$$

i.e., if agent i has truthfully reported his type and followed the recommendation, his (expected) payments are the same under the two monitoring mechanisms given that the other agent has truthfully reported his type and followed the recommendation.

Table 2 summarizes the results under contributions monitoring and output monitoring with limited liability. With limited liability, the principal's optimal payoff and the induced contributions are the same as those in the absence of limited liability under contributions monitoring. The respective on-equilibrium-path payments are listed in the table, and the rest of the pay-

[8] Note that there are multiple equilibria for each type of contract, all leading to the same contributions and principal's payoffs. In particular, we do find optimal contracts such that payments under contributions monitoring are all positive. As can be clearly seen later, the optimal contract under contributions monitoring in Table 2 is also optimal without imposing the limited liability constraint.

[9] There are multiple solutions for the optimal payment, and only one of them is presented here. Note that for some solutions some of the payments on the equilibrium path can even be negative, but the optimal objective values are still the same.

[10] Similar to contributions monitoring, there are multiple solutions for the optimal payment under output monitoring without limited liability. Multiplicity of optimal payments will also be observed under contributions and output monitoring when limited liability applies, but only one set of optimal contributions is obtained for each case.

ments are 0. However, the principal's optimal payoff under output monitoring is 1.140 which is smaller than his optimal payoff under contributions monitoring, 1.278.

	Contributions Monitoring		Output Monitoring
$y_i(\theta_H, \theta_H)$	0.833	$\bar{y}_i(\theta_H, \theta_H)$	0.424
$y_i(\theta_H, \theta_L)$	0.521	$\bar{y}_i(\theta_H, \theta_L)$	0.261
$y_i(\theta_L, \theta_H)$	0.203	$\bar{y}_i(\theta_L, \theta_H)$	0.099
$y_i(\theta_L, \theta_L)$	0.127	$\bar{y}_i(\theta_L, \theta_L)$	0.061
$\bar{p}_i(\bar{y}_i(\theta_H, \theta_H), \theta_H, \theta_H)$	0.220	$p_i(x_L, \theta_H, \theta_H)$	0
		$p_i(x_L, \theta_H, \theta_L)$	0
$\bar{p}_i(\bar{y}_i(\theta_H, \theta_L), \theta_H, \theta_L)$	0.110	$p_i(x_L, \theta_L, \theta_H)$	0.039
		$p_i(x_L, \theta_L, \theta_L)$	0.015
$\bar{p}_i(\bar{y}_i(\theta_L, \theta_H), \theta_L, \theta_H)$	0.033	$p_i(x_H, \theta_H, \theta_H)$	0.240
		$p_i(x_H, \theta_H, \theta_L)$	0.239
$\bar{p}_i(\bar{y}_i(\theta_L, \theta_L), \theta_L, \theta_L)$	0.013	$p_i(x_H, \theta_L, \theta_H)$	0.112
		$p_i(x_H, \theta_L, \theta_L)$	0.088
Principal's payoff	1.278	Principal's payoff	1.140

Table 2: Optimal contract under contributions monitoring and output monitoring with limited liability.

Without limited liability, low type's Individual Rationality (IR) constraint and high type's Incentive Compatibility (IC) constraint are binding whereas low type's IC and high type's IR are non-binding under both contributions and output monitoring. This is standard in the contract theory literature. It is worth mentioning that with the introduction of limited liability, both low type and high type's IRs are non-binding and both their ICs are binding under output monitoring, while there is no change under contributions monitoring. The principal is thus unable to extract all the surplus from the low-type agent under output monitoring but is able to do so under contributions monitoring, hence the strict dominance of the latter. We are now going to establish this result formally.

3.2 Payoff equivalent contributions monitoring

Under contributions monitoring contract, for each reported type profile, the principal needs to specify the payment for every possible contribution. In order to simplify the optimal payment structure, we first introduce a special type of contract such that the principal only needs to focus on the payments on the equilibrium path while setting other off-equilibrium payments to 0.

Definition 1 (Punishing contract) *A punishing contract $\mathcal{M}_{con}$ consists of a target level of contribution $y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})$ and a payment function $\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i})$ of the following form:*

$$\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) = \begin{cases} r(\hat{z}_i, \hat{\mathbf{z}}_{-i}), & \text{if } \; y_i = y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i}) \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $r(\hat{z}_i, \hat{\mathbf{z}}_{-i}) > 0$.

That is, the principal specifies a contribution level so that for contributions differing from the specified level agent i is penalized.[11]

[11] A more natural mechanism would be to give a non-negative reward so long as i's contribution is at least $\bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$ and zero reward otherwise. All our analysis will hold for this alternative mechanism.

Proposition 1 *The equilibrium outcome of any feasible contributions monitoring contract can be replicated by a punishing contract as in Definition 1.*

From here onwards we need to consider only the punishing contract in the class of contributions monitoring contract.

Proposition 2 (Payoff equivalence) *Given any feasible output monitoring mechanism* $\{p'_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y'_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ *specified in the program* $[\mathcal{P}_{out}]$*, the contributions monitoring contract with target contributions* $y^c_i(\hat{z}_i, \hat{\mathbf{z}}_{-i}) = y'_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$ *and payment function*

$$\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) = \begin{cases} \sum_x p'_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}) f(x|\mathbf{y}'(\hat{z}_i, \hat{\mathbf{z}}_{-i})), & \text{if } y_i = y'_i(\hat{z}_i, \hat{\mathbf{z}}_{-i}) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

for each i*, is feasible, induces the same contributions and generates the same interim and ex-ante expected profit and payoffs for the principal and the agents (for each of their types), as in the given output monitoring contract.*

Payoff equivalence works as follows. Given any feasible output based contract, the principal could calculate the interim expected payoff for each type of each agent. Thus, in the situation where each agent's individual contribution can be perfectly monitored, the principal could induce the same profile of contributions so as to maintain the same expected output, by promising each agent the same interim payoff (as that under output monitoring) if the agent meets the target. Thus, the principal as well as the agents' ex-ante expected equilibrium payoff will be the same as before. On the other hand, if the agent deviates to report as another type under output monitoring, he has the freedom to choose his deviation contribution, whereas under contributions monitoring the deviation contribution level is specified by the principal. If the agent does not meet the target, his contribution yields no reward. Such deviation contribution level chosen by the principal may not necessarily coincide with the agent's interest. Therefore, we can see that indeed agents have (weakly) less incentives to deviate under contributions monitoring.

3.3 Strict dominance of contributions monitoring

To show strict dominance of contributions monitoring, we first present a useful property for the output monitoring contract.

Proposition 3 *For any feasible output monitoring contract* $\{p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ *specified in the program* $[\mathcal{P}_{out}]$*, for any agent* i*, either every type of him earns* 0 *information rent, or every type of him earns strictly positive information rent.*

Thus, we know that for the optimal output monitoring contract, it must be true that either all types of all agents get zero information rent or for at least one agent, every type of him earns strictly positive information rent. Due to limited liability, the former implies that $p_i(x, z_i, \mathbf{z}_{-i}) = 0$ and $y_i(z_i, \mathbf{z}_{-i}) = 0$ $\forall i, x, z_i, \mathbf{z}_{-i}$, which we call a *null contract*. This is because if $p_i(x, z_i, \mathbf{z}_{-i}) > 0$ for some $(x, z_i, \mathbf{z}_{-i})$, the agent can always put in zero contribution in all contingencies and earn a positive rent, whereas if $y_i(z_i, \mathbf{z}_{-i}) > 0$ for some $\mathbf{z}$ (and $p_i(x, z_i, \mathbf{z}_{-i}) = 0$ for all $(x, \mathbf{z})$) then agent i's payoff will be negative. In fact, theoretically the null contract might even be optimal under certain conditions, e.g., when $c(y_i, z_i)$ is very large $\forall y_i \neq 0$. But it is inconceivable that a principal will hire a group of agents for nothing. To be more relevant for organizational design, for the optimal output monitoring contract, we will make the following assumption:

Assumption 2 *The optimal output monitoring contract is not a null contract.*

In the rest of the analysis, we will consider only this case. We therefore re-state Proposition 3 as follows:

Proposition 3′ (Information rent in optimal output monitoring) *Given Assumption 2, for any optimal output monitoring contract* $\{p_i^*(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i^*(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ *solving the program* $[\mathcal{P}_{out}]$*, there is at least one agent such that every type of him earns strictly positive information rent.*

Define

$$\hat{\delta}_i^{con}(\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i) \equiv \underset{\bar{\delta}_i(\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i)}{\operatorname{argmax}} E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \bar{\delta}_i(\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}),$$

where $\hat{z}_i \neq z_i$, i.e., $\hat{\delta}_i^{con}(\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i)$ is the optimal contribution level of agent i if his true type is z_i and he misreports his type to be $\hat{z}_i$. Thus, the IC(con) constraint can be replaced by the following:

$$E_{-i}\bar{\pi}_i(z_i, z_i, \bar{y}_i(z_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}), \quad \forall i, z_i, \hat{z}_i. \quad \text{(IC-type*(con))}$$

The following lemma will be useful to show the strict dominance of the contributions monitoring contract.

Lemma 2 *For any punishing contract* $\mathcal{M}_{con}$ *that is feasible, suppose there exists a particular type of an agent, not necessarily his lowest type, such that none of his IC-type*(con) and PC(con) constraints are binding. Then, there exists another feasible punishing contract which generates a strictly higher expected profit for the principal.*

The proof of Lemma 2 in the Appendix uses the following logic. With non-binding incentive and participation constraints, the particular type (of the agent) must be receiving a positive reward for some contingency. Now the principal can lower (only) this reward (leaving all other payments unchanged) appropriately without violating any of the IC-type*(con), PC(con) and LL(con) constraints. This new contract will be feasible and none of the contributions (of this or any other agent) will change. Thus, doing so leads to a higher expected profit for the principal.

We have our main result as follows:

Proposition 4 (Strict dominance) *Given Assumption 2, there always exists a contributions monitoring contract that is strictly superior to the optimal output monitoring contract.*

By Proposition 2, we could replicate the optimal output monitoring contract with a punishing contract. By Proposition 3′, we know that under the optimal output monitoring contract and the replicated punishing contract, for at least one agent, the lowest type of him earns strictly positive information rent. By making use of Lemma 2, we then show the existence of another feasible punishing contract which generates a strictly higher expected profit for the principal by extracting surplus from the agents. The detailed proof is in the Appendix. It may also be noted that for the dominance result, we do not require any existence result on optimal contributions monitoring contract.

Technically, LL(out) requires $p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}) \geq 0$ while LL(con) requires $\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) = \sum_x p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i}) f(x|\mathbf{y}^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})) \geq 0$ when $y_i = y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})$. As $\breve{p}_i(y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i}), \hat{z}_i, \hat{\mathbf{z}}_{-i})$ is constructed in expected form, LL(con) is a less stringent condition than LL(out). Even for some output monitoring contracts such that some payments $p_i(x, \hat{z}_i, \hat{\mathbf{z}}_{-i})$'s are negative (so that LL(out) is violated), their equivalent contributions monitoring contracts may well satisfy the limited liability condition, since the contribution-based payments are constructed taking expectation of the output-based payments.[12]

[12]Note that the actual implementation of the punishing contract involves all non-negative ex post payments.

Thus, under limited liability punishing contracts embrace more possibilities than output monitoring contracts. This makes the punishing contract more powerful than the output monitoring contract, yielding a beneficial improvement for the principal.

Proposition 4 highlights the fact that *unlimited agent liabilities*, an assumption implicit in McAfee and McMillan, is necessary for McAfee–McMillan type equivalence result, i.e., under all situations the two (optimal) monitoring mechanisms are equivalent in terms of the principal's payoff. In order to show the necessity of unlimited agent liabilities, all we require is to show that if unlimited liabilities fail then we cannot have equivalence. Proposition 4 proves the necessity of unlimited liabilities by showing a stronger result, that the contributions monitoring would strictly dominate output monitoring when agents' liabilities are limited.

3.4 McAfee–McMillan's mechanisms vs. ours

McAfee and McMillan showed equivalence between contributions and output monitoring, whereas we show strict dominance of contributions monitoring. Different from McAfee and McMillan, we make two assumptions using which we establish our dominance result: (i) limited liability of the agents, and (ii) a technical assumption, Assumption 1, placing a positive lower bound on the conditional probability of any given output. While both these assumptions are important in deriving positive rent for the lowest type used in our proof for the dominance result, it is the limited liability that is key to our dominance result. This can be seen from our numerical example, with Assumption 1 imposed: the equivalence between the two monitoring mechanisms holds without limited liability and breaks down with limited liability.

Laffont and Martimort (2002) discuss the effect of limited liability in a class of principal-agent problems, and the same should apply to the lowest-type agent in any team problem. They mention that "A limited liability constraint on ex post rents may reduce the efficiency of ex ante contracting" (see page 125), which implies an ex ante information rent, and with limited liability the inefficient type's expected utility becomes strictly positive. In McAfee and McMillan's optimal output monitoring mechanism, due to the absence of the limited liability restriction the lowest type of every agent could be pushed to receive zero information rent. When one replicates the outcome under optimal output monitoring contract by a punishing contract, the lowest type agents receive zero rent in the replicated contract. Then the principal does not have any room to improve beyond the replicating punishing contract. In contrast, as we have shown in Proposition $3'$, after imposing limited liability it is no longer possible to extract all the surplus from the lowest type of every agent.[13] With limited liability brought in as in our formulation, the optimal output monitoring contract ties the principal's hands more severely than the punishing contract, enabling a strict improvement by the latter mechanism.

What makes output monitoring catch up with contributions monitoring as one drops the requirement of limited liability? Under contributions monitoring the principal can identify who are the failing agents, so he does not have to penalize all team members uniformly while such punishment would be almost necessary for low performing teams under output monitoring. This implies limited liability will constrain output monitoring more severely than contributions monitoring. Removing limited liability restriction would therefore lift output monitoring much more than the contributions monitoring.

Besides the positive lower bound assumption on conditional probability of any output (noted above), there are other technical differences in the setup between the current paper and McAfee and McMillan (1991). The agents' type space and the set of possible outputs are discrete here and continuous in McAfee and McMillan. The discrete type space and the lower bound assumption are mainly to facilitate the proofs. In particular, the assumption of discrete types helps to establish Lemma 1 which shows that

[13] In fact, it is easy to see that in our model the lowest type of every agent would earn a positive rent under limited liability, so long as one justifiably ignores the extreme contract in which an agent receives zero payments for all output realizations. See the discussion following Assumption 1 and Proposition 3. Thus, a stronger version of Proposition $3'$ can be established.

an optimal output monitoring contract exists in the absence of limited liability, provides a basis for our exercise without having to derive explicitly the optimal output monitoring contract. In McAfee and McMillan, the existence was guaranteed by their construction of the optimal contract. The lower bound assumption is a sufficient condition for Proposition 3.

3.5 Remarks

Under limited liability, contributions monitoring strictly dominates output monitoring (Proposition 4). This shows how under limited liability one can never obtain even a semblance of McAfee–McMillan type equivalence.

We are not claiming that without limited liability we obtain McAfee and McMillan's equivalence. Rather, equivalence *may* happen without limited liability as illustrated in the numerical example, which is a weaker version of McAfee and McMillan's result or what we may call a possibility result.

Nor are we claiming that limited liability will necessarily bind whenever McAfee and McMillan's equivalence result holds. It may or may not be binding: "the optimal contract based on team output often (but not always) has each team member making an initial payment to the principal and then the principal paying the agents a sum greater than the value of the output" (page 563 of McAfee and McMillan, 1991). This clearly shows that in McAfee and McMillan's environment limited liability may be violated, for instance, when actual output is very low. As an alternative possibility, the authors write, "if the uncertainty about the agents' abilities were sufficiently dispersed, the marginal payments would become so small that they would sum to less than one, and the fixed payments would become positive ..." (page 562). This shows that there can be situations in McAfee and McMillan's environment when limited liability does not bind for their equivalence result.

So what we are suggesting is that the absence of limited liability could be critical to McAfee and McMillan's equivalence result, especially when limited liability is violated. By bringing in limited liability, McAfee–McMillan type equivalence can no longer hold in our discrete environment as shown in Proposition 4. Certainly, the absence of limited liability is critical in our formulation to have any possibility of equivalence, as shown in the numerical example.

4 Further discussions

We now discuss two variations of the model and their effects on our main results.

4.1 Payoff equivalence for deterministic output

Our dominance result, Proposition 4, depends crucially on the noise in output: a given profile of contributions may yield any of the possible output levels in X. If we dispense with this noise and make the technology deterministic, will the dominance result still hold? We cannot answer this question if output is discrete and yet contributions are continuous. Instead, in the Supplementary file we prove the payoff equivalence of the two mechanisms if output is continuous, with or without limited liability. While we use the assumption of discrete output for the existence of optimal output monitoring contract, our dominance result does not require discreteness of output.[14] So analyzing the continuous output model helps to illustrate why the noise in our model, together with limited liability, is important for our result. The simple intuition is that with deterministic output that is strictly increasing in each agent's contribution, any individual deviation from the recommended contribution when others follow their recommendations

[14]In fact, for any feasible output monitoring contract, except for the 'null contract', we can find a dominating contributions monitoring contract. Thus, the existence result is not essential for our dominance result.

can be recognized and penalized by the principal (zero payment to all is always an option[15]).

4.2 Positive fixed cost

If the cost function involves positive fixed cost, the equivalence of output and contributions monitoring in the absence of limited liability and the dominance of contributions monitoring with the imposition of limited liability are likely to hold, as we illustrate in a numerical example in the Supplementary file.

Without limited liability, the induced optimal contributions under both contributions and output monitoring are still the same as those without fixed cost, but the objective value is smaller. Thus, the positive fixed cost only causes a parallel shift of the optimal payment, as the principal needs to pay more to induce participation, but without affecting the desired contribution levels.

With the introduction of limited liability, while the induced contributions under contributions monitoring are still the same as those without fixed cost, they are different under output monitoring. In fact, with fixed cost, the objective value under output monitoring becomes closer to the one under contributions monitoring. Table 3 presents the ratios of principal's optimal payoffs under output monitoring and contributions monitoring with limited liability and fixed cost. As we can see, when fixed cost increases, the ratio also increases. This suggests that fixed cost would mitigate the dominance result since output monitoring is catching up with contributions monitoring as fixed cost increases. This pattern can also be seen clearly from Fig. 1.

Fixed Cost	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Payoffs under O/M	1.140	1.140	1.140	1.140	1.140	1.135	1.127	1.116	1.103	1.088	1.072
Payoffs under C/M	1.278	1.258	1.238	1.218	1.198	1.178	1.158	1.138	1.118	0.098	1.078
Ratio	0.892	0.906	0.921	0.936	0.951	0.963	0.973	0.980	0.986	0.991	0.995

Table 3: Ratios of principal's optimal payoffs under output monitoring and contributions monitoring with limited liability and fixed cost. (O/M stands for output monitoring, and C/M stands for contributions monitoring.)[16]

Positive fixed costs, e.g., the requirement to travel and work in an office environment, adapting to office norms (dress codes, after office drinks for team bonding), etc. extend the range of individual punishments that the principal can use to discipline the agents: having sunk in fixed costs the agents' ex-post profitability of potential deviations by not implementing principal-recommended contributions will be reduced. Under output monitoring, without fixed cost, if an agent shirks by putting in 0 contribution, he will get positive payoff in expectation, since the expected payment is positive due to limited liability. But with fixed cost, the agent may get negative payoff if he shirks, as the payment rule is set in a way to cover the fixed cost in expectation only if he follows the recommended contributions (on equilibrium path) to ensure participation. As fixed cost increases, the "punishment" in deviation becomes more and more severe, thus less and less additional rent (with respect to the case without fixed cost) is required to be awarded to the agent. Under contributions monitoring, since the lowest type agent gets zero rent without fixed cost, the additional expected payment has to cover the entire fixed cost, if any, to ensure participation. In other words, under contributions monitoring the agents must be guaranteed an additional higher payment separately to cover for their fixed costs and ensure participation, whereas under output monitoring fixed costs would already be reflected in the agents' information rents which obviates the need to pay the agents a great deal extra for the cost of participation. This explains why, as

[15] Unlike in contributions monitoring, under output monitoring the principal has to penalize all following a deviation because he won't be able to tell who has deviated.

[16] Since the fixed cost is very small, due to rounding off numbers, payoffs under output monitoring when fixed cost is 0, 0.01, 0.02, 0.03 or 0.04 become the "same" visually in the table.

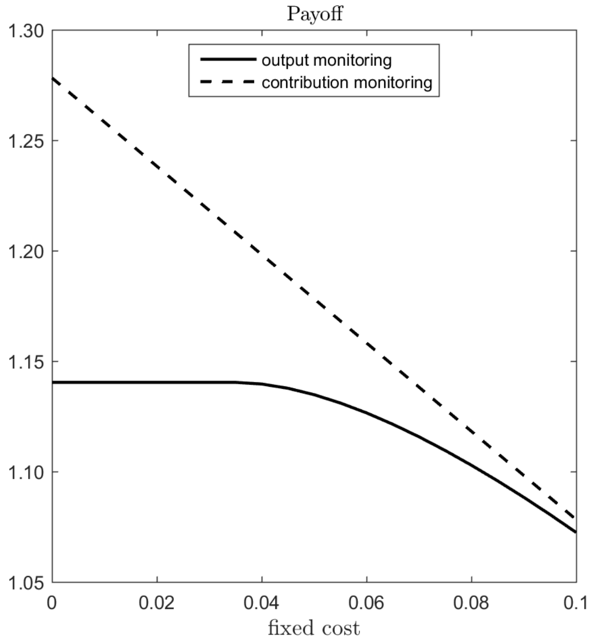

Figure 1: Principal's payoffs under output monitoring and contributions monitoring

fixed cost increases, the drop in principal's payoff under output monitoring is not as sharp and ultimately the difference in payoffs from the two monitoring mechanisms tends to narrow down, as shown in Fig. 1.

A Appendix

Proof of Lemma 1. First, we can see that the contract with the payment function $p_i(x, \hat{\mathbf{z}}) = 0$, $\forall i, x, \hat{\mathbf{z}}$ and equilibrium contribution level $y_i(\hat{\mathbf{z}}) = 0$, $\forall i, \hat{\mathbf{z}}$ satisfies all the feasibility constraints. Let $\tilde{V}$ denote the principal's expected profit under such contract.

Next, we are going to show the existence of optimal solution.

Since the domain of $p_i(x, \hat{\mathbf{z}})$ and $y_i(\hat{\mathbf{z}})$ are finite, choosing the set of such functions will be the same as choosing a finite number of vectors. Let V be the space of $n \times m^n \times (1 + |X|)$ dimensional vectors which satisfy all the feasibility constraints and yields principal the expected payoff at least $\tilde{V}$. Thus, each element $v_k \in V$ can be represented as

$$\begin{aligned} v_k(x, \hat{\mathbf{z}}) &= (y_k(\hat{\mathbf{z}}), p_k(x, \hat{\mathbf{z}})) \\ &= (y_{1k}(\hat{\mathbf{z}}), y_{2k}(\hat{\mathbf{z}}), ..., y_{nk}(\hat{\mathbf{z}}), p_{1k}(x, \hat{\mathbf{z}}), p_{2k}(x, \hat{\mathbf{z}}), ..., p_{nk}(x, \hat{\mathbf{z}})). \end{aligned}$$

Claim 1: The objective function in the program $[\mathcal{P}_{out}]$ *is continuous on* V.

This is trivial since the objective function only consists of simple arithmetic operators, so the proof is omitted.

Claim 2: V *is bounded.*

Since there are finite number of output x and type profiles $\mathbf{z}$, there always exists $\bar{x}$ and $\bar{\mathbf{z}}$ such that $U(x, \mathbf{z}) \leq U(\bar{x}, \bar{\mathbf{z}})$, $\forall x, \mathbf{z}$. As $p_i(x, \mathbf{z}) \geq 0$, $\forall i, x, \mathbf{z}$, the principal's maximum possible expected profit is $U(\bar{x}, \bar{\mathbf{z}})$. Since $\Pr(z_i = \theta_j) = q_j > 0$ $\forall j$, there always exists a $\underline{q} \in \{q_1, q_2, ..., q_m\}$ such that $q_j \geq \underline{q}$. Thus, the principal will choose a payment function such that $p_i(x, \mathbf{z}) \leq \frac{U(\bar{x}, \bar{\mathbf{z}}) - \tilde{V}}{a\underline{q}^n}$ $\forall i, x, \mathbf{z}$. Otherwise, his expected profit will be strictly smaller than $\tilde{V}$. By LL(out), we know that $p_i(x, \mathbf{z}) \geq 0$ $\forall i, x, \mathbf{z}$. Thus, the choice set of the function $p_i(x, \mathbf{z})$ is bounded.

Since $c(y_i, z_i) \to \infty$ as $y_i \to \infty$, $p_i(x, \mathbf{z})$ is bounded uniformly by $\frac{U(\bar{x},\bar{\mathbf{z}})-\tilde{V}}{aq^n}$ and $0 < f(x|\mathbf{y}) \leq 1$, we have $E_{-i}\big[\sum_x p_i(x, z_i, \mathbf{z}_{-i})f(x|\mathbf{y}(z_i, \mathbf{z}_{-i})) - c(y_i(z_i, \mathbf{z}_{-i}), z_i)\big] \to -\infty$ as $y_i(z_i, \mathbf{z}_{-i}) \to \infty$ for any z_i and $\mathbf{z}_{-i}$. So if $y_i(z_i, \mathbf{z}_{-i})$ is too large, then PC(out) cannot be satisfied. Thus, for every type profile $\mathbf{z}$, there exists an upper bar $\bar{y}_i(\mathbf{z})$ such that the original unconstrained maximization problem is equivalent to the problem with the addition of the constraint $y_i(\mathbf{z}) \leq \bar{y}_i(\mathbf{z})$. Therefore, the choice set of the function $y_i(\mathbf{z})$ of the original problem is bounded.

Claim 3: V is closed.

The proof is also trivial since all the constraints are weak inequalities.

Claim 4: The optimal solution exists.

Since V is closed and bounded, V is compact. Since the objective function is continuous on V, the optimal output monitoring contract exists. **Q.E.D.**

Proof of Proposition 1. Fix any feasible contributions monitoring contract $\{\bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), \bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$. Consider the proposed punishing contract with the target level of contribution $y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i}) = \bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$ and payment function

$$\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) = \begin{cases} \bar{p}_i(\bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i}), \hat{z}_i, \hat{\mathbf{z}}_{-i}), & \text{if} \quad y_i = \bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i}) \\ 0, & \text{otherwise.} \end{cases}$$

We are going to show that the proposed contract will implement the same output at an identical cost to the principal as the given contributions monitoring contract.

Clearly, $\breve{p}_i(\bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i}), \hat{z}_i, \hat{\mathbf{z}}_{-i}) \geq 0$ $\forall i, \hat{z}_i, \hat{\mathbf{z}}_{-i}$ since $\bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}) \geq 0$ $\forall i, y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}$. Thus, LL(con) is satisfied.

Also, PC(con) is satisfied as each agent's (ex-ante) equilibrium payoff (when he reports truthfully and follows the recommendation) is the same as the given contributions monitoring contract.

We now proceed to prove that IC(con) is also satisfied. Under the given contract $\{\bar{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), \bar{y}_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$, agent i's interim deviation payoff (after reporting $\hat{z}_i \neq z_i$ and receiving recommendation $\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i})$) when he contributes y_i is

$$\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i)\, \bar{p}_i(y_i, \hat{z}_i, \mathbf{z}_{-i}) - c(y_i, z_i),$$

where $\Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i)$ is his updated belief about other agents' types based on the received recommendation and his reported type.

Define

$$y_i^{\max} = \arg\max_{y_i} \sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i)\, \bar{p}_i(y_i, \hat{z}_i, \mathbf{z}_{-i}) - c(y_i, z_i).$$

Consider two cases:

(i) Suppose $y_i^{\max} = \bar{y}_i(\hat{z}_i, \mathbf{z}_{-i})$. Then $y_i^{\max}$ should still be the best deviation contribution under the punishing contract, since

$$\begin{aligned} &\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i)\, \bar{p}_i(y_i^{\max}, \hat{z}_i, \mathbf{z}_{-i}) - c(y_i^{\max}, z_i) \\ \geq &\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i)\, \bar{p}_i(\delta_i, \hat{z}_i, \mathbf{z}_{-i}) - c(\delta_i, z_i), \quad \forall \delta_i \neq \bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}) \\ \geq &\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}), \hat{z}_i) \times 0 - c(\delta_i, z_i), \quad \forall \delta_i \neq \bar{y}_i(\hat{z}_i, \mathbf{z}_{-i}). \end{aligned}$$

(ii) Suppose $y_i^{\max} \neq \bar{y}_i(\hat{z}_i, \mathbf{z}_{-i})$. Then the interim deviation payoff under the punishing contract will be smaller. This is because

$$\begin{aligned}
&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\,\bar{p}_i(y_i^{\max},\hat{z}_i,\mathbf{z}_{-i}) - c(y_i^{\max}, z_i) \\
\geq \max\Bigg\{ &\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\,\bar{p}_i(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i,\mathbf{z}_{-i}) - c(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}), z_i), \\
&\quad \sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\,\bar{p}_i(\delta_i,\hat{z}_i,\mathbf{z}_{-i}) - c(\delta_i, z_i)\Bigg\}, \quad \forall \delta_i \neq \bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}) \\
\geq \max\Bigg\{ &\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\,\bar{p}_i(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i,\hat{\mathbf{z}}_{-i}) - c(\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}), z_i), \\
&\quad \sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|\bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i) \times 0 - c(\delta_i, z_i)\Bigg\}, \quad \forall \delta_i \neq \bar{y}_i(\hat{z}_i,\mathbf{z}_{-i}).
\end{aligned}$$

Therefore, the interim deviation payoff under the punishing contract is always smaller than that under the given contributions monitoring contract, so does the ex-ante deviation payoff. Thus, IC(con) is satisfied.

Hence, the proposed punishing contract implements the same outcomes in the ex-ante as well as interim stages as in the contributions monitoring contract. **Q.E.D.**

Proof of Proposition 2. Fix any feasible output monitoring contract $\{p'_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i}), y'_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})\}$. Consider the proposed punishing contract with payment function $\breve{p}_i(y_i,\hat{z}_i,\hat{\mathbf{z}}_{-i})$ and the target contribution $y_i^c(\hat{z}_i,\hat{\mathbf{z}}_{-i}) = y'_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})$ $\forall i$. Next, following the same procedure as in the proof of Proposition 1, we are going to show that the punishing contract and the output monitoring contract are outcome equivalent.

Since $p'_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i}) \geq 0, \forall i, x, \hat{z}_i, \hat{\mathbf{z}}_{-i}$, we have

$$\sum_x p'_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i}) f(x|\mathbf{y}'(\hat{z}_i,\hat{\mathbf{z}}_{-i})) \geq 0, \quad \forall i, \hat{z}_i, \hat{\mathbf{z}}_{-i}.$$

Thus,

$$\breve{p}_i(y_i,\hat{z}_i,\hat{\mathbf{z}}_{-i}) \geq 0, \quad \forall i, y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i},$$

i.e., LL(con) is satisfied.

Also, PC(con) is satisfied as each agent's (ex-ante) equilibrium payoff (when he reports truthfully and follows the recommendation) is the same as the given output monitoring contract.

We now proceed to prove that IC(con) is also satisfied. Under the given contract $\{p'_i(x,\hat{z}_i,\hat{\mathbf{z}}_{-i}), y'_i(\hat{z}_i,\hat{\mathbf{z}}_{-i})\}$, agent i's interim deviation payoff (after reporting $\hat{z}_i \neq z_i$ and receiving recommendation $y'_i(\hat{z}_i,\mathbf{z}_{-i})$) when he contributes y_i is

$$\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y'_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x \bar{p}'_i(x,\hat{z}_i,\mathbf{z}_{-i}) f(x|y_i,\mathbf{y}'_{-i}(\hat{z}_i,\mathbf{z}_{-i}))\big] - c(y_i,z_i),$$

where $\Pr(\mathbf{z}_{-i}|y'_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)$ is his updated belief about other agents' type profile based on the received recommendation and his reported type.

Define

$$y_i^{\max} = \arg\max_{y_i} \sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y'_i(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x \bar{p}'_i(x,\hat{z}_i,\mathbf{z}_{-i}) f(x|y_i,\mathbf{y}'_{-i}(\hat{z}_i,\mathbf{z}_{-i}))\big] - c(y_i,z_i).$$

Consider two cases:

(i) If $y_i^{\max} = y'_i(\hat{z}_i,\mathbf{z}_{-i})$. Then $y_i^{\max}$ should still be the best deviation contribution under the contributions monitoring contract since

$$\begin{aligned}
&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\,\breve{p}_i(y_i^{\max},\hat{z}_i,\mathbf{z}_{-i}) - c(y_i^{\max},z_i)\\
=&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x p_i'(x,\hat{z}_i,\mathbf{z}_{-i})f\big(x|y_i^{\max},\mathbf{y}_{-i}'(\hat{z}_i,\mathbf{z}_{-i})\big)\big] - c(y_i^{\max},z_i)\\
\geq&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x p_i'(x,\hat{z}_i,\mathbf{z}_{-i})f\big(x|\delta_i,\mathbf{y}_{-i}'(\hat{z}_i,\mathbf{z}_{-i})\big)\big] - c(\delta_i,z_i), \quad \forall \delta_i \neq y_i'(\hat{z}_i,\mathbf{z}_{-i})\\
\geq&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i) \times 0 - c(\delta_i,z_i), \quad \forall \delta_i \neq y_i'(\hat{z}_i,\mathbf{z}_{-i}).
\end{aligned}$$

(ii) If $y_i^{\max} \neq y_i'(\hat{z}_i,\mathbf{z}_{-i})$. Then the interim deviation payoff under the punishing contract will be smaller. This is because

$$\begin{aligned}
&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x p_i'(x,\hat{z}_i,\mathbf{z}_{-i})f(x|y_i^{\max},y_{-i}'(\hat{z}_i,\mathbf{z}_{-i}))\big] - c(y_i^{\max},z_i)\\
\geq \max&\Big\{\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x p_i'(x,\hat{z}_i,\mathbf{z}_{-i})f\big(x|y_i'(\hat{z}_i,\mathbf{z}_{-i}),y_{-i}'(\hat{z}_i,\mathbf{z}_{-i})\big)\big] - c(y_i'(\hat{z}_i,\mathbf{z}_{-i}),z_i),\\
&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\big[\sum_x p_i'(x,\hat{z}_i,\mathbf{z}_{-i})f\big(x|\delta_i,\mathbf{y}_{-i}'(\hat{z}_i,\mathbf{z}_{-i})\big)\big] - c(\delta_i,z_i)\Big\}, \quad \forall \delta_i \neq y_i'(\hat{z}_i,\mathbf{z}_{-i})\\
\geq \max&\Big\{\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i)\breve{p}_i(y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i,\mathbf{z}_{-i}) - c(y_i'(\hat{z}_i,\mathbf{z}_{-i}),z_i),\\
&\sum_{\mathbf{z}_{-i}} \Pr(\mathbf{z}_{-i}|y_i'(\hat{z}_i,\mathbf{z}_{-i}),\hat{z}_i) \times 0 - c(\delta_i,z_i)\Big\}, \quad \forall \delta_i \neq y_i'(\hat{z}_i,\mathbf{z}_{-i}).
\end{aligned}$$

Therefore, the interim deviation payoff under the punishing contract is always smaller than that under the given output monitoring contract, so does the ex-ante deviation payoff. Thus, IC(con) is satisfied.

Hence, the proposed punishing contract implements the same outcomes in the ex-ante as well as interim stages as in the output monitoring contract. **Q.E.D.**

Proof of Proposition 3. First, we know that a higher type should earn no less payoff than a lower type, since the higher type can always mimic as the lower type and he has (weakly) lower cost. Thus, if the lowest type of agent i earns strictly positive information rent, the higher type of him must also earn strictly positive information rent.

Next, suppose for a feasible output monitoring contract $\{p_i(x,z_i,\mathbf{z}_{-i}),y_i(z_i,\mathbf{z}_{-i})\}$, there exists an agent i such that the lowest type of him earns zero information rent, i.e.,

$$E_{-i}\big[\sum_x p_i(x,\theta_1,\mathbf{z}_{-i})f(x|\mathbf{y}(\theta_1,\mathbf{z}_{-i})) - c(y_i(\theta_1,\mathbf{z}_{-i}),\theta_1)\big] = 0. \tag{A.1}$$

By IC(out), we have

$$E_{-i}\big[\sum_x p_i(x,\hat{z}_i,\mathbf{z}_{-i})f(x|\delta_i(y_i(\hat{z}_i,\mathbf{z}_{-i}),\theta_1,\hat{z}_i),\mathbf{y}_{-i}(\hat{z}_i,\mathbf{z}_{-i})) - c(\delta_i(y_i(\hat{z}_i,\mathbf{z}_{-i}),\theta_1,\hat{z}_i),\theta_1)\big] \leq 0,$$
$$\forall \hat{z}_i \neq \theta_1,\ \delta_i(y_i(\hat{z}_i,\mathbf{z}_{-i}),\theta_1,\hat{z}_i).$$

Thus,

$$E_{-i}\big[\sum_x p_i(x,\hat{z}_i,\mathbf{z}_{-i})f(x|0,\mathbf{y}_{-i}(\hat{z}_i,\mathbf{z}_{-i})) - c(0,\theta_1)\big] \leq 0, \quad \forall \hat{z}_i \neq \theta_1. \tag{A.2}$$

Since $p_i(x, \hat{z}_i, \mathbf{z}_{-i}) \geq 0\ \forall x, \hat{z}_i, \mathbf{z}_{-i}$ due to LL(out), we have

$$\sum_x p_i(x, \hat{z}_i, \mathbf{z}_{-i}) f(x|0, \mathbf{y}_{-i}(\hat{z}_i, \mathbf{z}_{-i})) - c(0, \theta_1) \geq 0, \quad \forall \hat{z}_i \neq \theta_1, \mathbf{z}_{-i},$$

that, together with (A.2) and the fact that $\Pr(\mathbf{z}_{-i}) > 0$ for all $\mathbf{z}_{-i}$, yields:

$$\sum_x p_i(x, \hat{z}_i, \mathbf{z}_{-i}) f(x|0, \mathbf{y}_{-i}(\hat{z}_i, \mathbf{z}_{-i})) - c(0, \theta_1) = 0, \quad \forall \hat{z}_i \neq \theta_1, \mathbf{z}_{-i},$$

implying $p_i(x, \hat{z}_i, \mathbf{z}_{-i}) = 0$, $\forall x, \hat{z}_i, \mathbf{z}_{-i}$, where $\hat{z}_i \neq \theta_1$. Also, from (A.1) it follows that $p_i(x, \theta_1, \mathbf{z}_{-i}) = 0$ $\forall x, \mathbf{z}_{-i}$.

For agent i whose true type is $\hat{z}_i$, given that $p_i(x, \hat{z}_i, \mathbf{z}_{-i}) = 0$, $\forall x, \hat{z}_i, \mathbf{z}_{-i}$, his best contribution level is 0, and thus, his interim and ex-ante expected payoffs are 0. Thus, if the lowest type of agent i earns zero information rent, every type of him earns zero information rent. **Q.E.D.**

Proof of Lemma 2. Take a feasible punishing contract $\{\breve{p}_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$. Suppose there exists an agent k with true type $\tilde{z}_k$ such that all of his IC-type*(con) and PC(con) constraints are non-binding, i.e.,

$$E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) > E_{-k}\bar{\pi}_k(\tilde{z}_k, \hat{z}_k, \hat{\delta}_k^{con}(y_k^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \mathbf{z}_{-k}), \qquad \forall \hat{z}_k \neq \tilde{z}_k,$$

and $E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) > 0$.

The second inequality above implies that, without loss of generality we can pick one type profile, say $\check{\mathbf{z}}_{-k}$, so that $\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \check{\mathbf{z}}_{-k}) > 0$. This also implies $\breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) > 0$.

Also define

$$\delta \equiv \min_{\hat{z}_k \neq \tilde{z}_k} \left\{ E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) - E_{-k}\bar{\pi}_k(\tilde{z}_k, \hat{z}_k, \hat{\delta}_k^{con}(y_i^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \mathbf{z}_{-k}) \right\} > 0,$$

which is the smallest difference between agent k's expected equilibrium payoff and expected deviation payoffs obtained from among all non-truthful type reports, and

$$\epsilon \equiv \frac{1}{2} \min \left\{ \delta, \bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \check{\mathbf{z}}_{-k}), E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) \right\} > 0.$$

Consider the new contract $\{\breve{p}'_i(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ with the following payment rule:

$$\breve{p}'_k(y_k, \hat{z}_k, \hat{\mathbf{z}}_{-k}) = \begin{cases} \breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \hat{z}_k, \hat{\mathbf{z}}_{-k}) - \epsilon, & \text{if } (\hat{z}_k, \hat{\mathbf{z}}_{-k}) = (\tilde{z}_k, \check{\mathbf{z}}_{-k}) \text{ and } y_k = y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}) \\ 0, & \text{if } (\hat{z}_k, \hat{\mathbf{z}}_{-k}) = (\tilde{z}_k, \check{\mathbf{z}}_{-k}) \text{ and } y_k \neq y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}) \\ \breve{p}_k(y_k, \hat{z}_k, \hat{\mathbf{z}}_{-k}), & \text{otherwise,} \end{cases}$$

and $\breve{p}'_\ell(y_\ell, \hat{z}_\ell, \hat{\mathbf{z}}_{-\ell}) = \breve{p}_\ell(y_\ell, \hat{z}_\ell, \hat{\mathbf{z}}_{-\ell})$ for $\ell \neq i$.

Under this new contract, only the payment to agent k is reduced by a fixed amount ϵ if the reported profile is $(\tilde{z}_k, \check{\mathbf{z}}_{-k})$ and agent k chooses $y_k = y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k})$. Now, check that under the new contract, LL(con) is still satisfied, i.e., $\breve{p}'_k(y_k, \hat{z}_k, \hat{\mathbf{z}}_{-k}) \geq 0$. Since $\breve{p}_k(y_k, \hat{z}_k, \hat{\mathbf{z}}_{-k})$ is feasible by assumption, $\breve{p}_k(y_k, \hat{z}_k, \hat{\mathbf{z}}_{-k}) \geq 0$. Also,

$$\begin{aligned} & \breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) - \epsilon \\ = \; & \breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) - \frac{1}{2} \min \left\{ \delta, \bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \check{\mathbf{z}}_{-k}), E_{-i}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) \right\} \\ \geq \; & \breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) - \frac{1}{2}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \check{\mathbf{z}}_{-k}) \end{aligned}$$

$$\begin{aligned}
&= \breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) - \frac{1}{2}\left[\breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) - c(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k)\right] \\
&= \frac{1}{2}\left[\breve{p}_k(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k, \check{\mathbf{z}}_{-k}) + c(y_k^c(\tilde{z}_k, \check{\mathbf{z}}_{-k}), \tilde{z}_k)\right] \\
&> 0.
\end{aligned}$$

Thus, the new contract $\{\breve{p}_i'(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ satisfies LL(con).

Next, check that under this new contract, PC(con) is still satisfied if agent i reports truthfully and follows the recommendation:

$$\begin{aligned}
&\sum_{\mathbf{z}_{-k}} \Pr(\mathbf{z}_{-k}|y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\breve{p}_k'(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \mathbf{z}_{-k}) - c(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k) \\
=&\sum_{\mathbf{z}_{-k}} \Pr(\mathbf{z}_{-k}|y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\breve{p}_k(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \mathbf{z}_{-k}) - \Pr(\check{\mathbf{z}}_{-k}|y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\epsilon - c(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k) \\
\geq& E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) - \Pr(\check{\mathbf{z}}_{-k}|y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) \\
=&\left[1 - \Pr(\check{\mathbf{z}}_{-k}|y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\right]E_{-k}\bar{\pi}_k(\tilde{z}_k, \tilde{z}_k, y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \mathbf{z}_{-k}) \\
\geq& 0.
\end{aligned}$$

Finally, check that under this new contract, IC-type*(con) is also satisfied:

$$\begin{aligned}
&E_{-k}\left[\breve{p}_k'(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \mathbf{z}_{-k}) - c(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\right] \\
&\quad - E_{-k}\left[\breve{p}_k'\left(\hat{\delta}_k^{con}(y_k^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \hat{z}_k, \mathbf{z}_{-k}\right) - c\left(\hat{\delta}_k^{con}(y_k^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \tilde{z}_k\right)\right], \quad \forall \hat{z}_k \neq \tilde{z}_k \\
=&E_{-k}\left[\breve{p}_k(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \mathbf{z}_{-k}) - c(y_k^c(\tilde{z}_k, \mathbf{z}_{-k}), \tilde{z}_k)\right] - \Pr(\check{\mathbf{z}}_{-k})\epsilon \\
&\quad - E_{-k}\left[\breve{p}_k(\hat{\delta}_k^{con}(y_k^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \hat{z}_k, \mathbf{z}_{-k}) - c(\hat{\delta}_k^{con}(y_k^c(\hat{z}_k, \mathbf{z}_{-k}), \tilde{z}_k, \hat{z}_k), \tilde{z}_k)\right], \quad \forall \hat{z}_k \neq \tilde{z}_k \\
\geq&\delta - \Pr(\check{\mathbf{z}}_{-k})\frac{1}{2}\delta \geq \frac{1}{2}\delta > 0.
\end{aligned}$$

Therefore, the new contract $\{\breve{p}_i'(y_i, \hat{z}_i, \hat{\mathbf{z}}_{-i}), y_i^c(\hat{z}_i, \hat{\mathbf{z}}_{-i})\}$ is feasible and induces same amount of contributions with less expected cost. Thus, the principal's ex-ante expected profit will be higher, given our assumption that each type profile occurs with positive probability (and thus the occurrence of $(\tilde{z}_k, \check{\mathbf{z}}_{-k})$ is a non-negligible event). **Q.E.D.**

Proof of Proposition 4. For the replicated punishing contract $\{\breve{p}_i(y_i, z_i, \mathbf{z}_{-i}), y_i^c(z_i, \mathbf{z}_{-i})\}$, without loss of generality, suppose the lowest type of agent i earns strictly positive information rent (by Proposition 3′). Thus, every type of agent i should earn strictly positive ex-ante expected payoff: the higher type can always mimic as the lowest type, choose the target contribution for the lowest type at a weakly lower cost, and earn a weakly higher payoff than the lowest type's equilibrium payoff.

Note that for any type z_i of agent i, his PC(con) is non-binding. If there is a particular type such that all of his IC-type*(con) are non-binding, by Lemma 2, we know that there exists another feasible punishing contract which generates a strictly higher expected profit for the principal. Thus, the strict dominance of contributions monitoring easily follows. So suppose for any type z_i, at least one of his IC-type*(con) is binding. This implies that for *every* z_i, at least one of his ex-ante deviation payoff is positive. Denote R to be the smallest positive ex-ante deviation payoff across all type pairs $(z_i, \hat{z}_i)$ (where z_i is the true type and $\hat{z}_i \neq z_i$ is the reported type). That is,

$$R \equiv \min_{z_i, \hat{z}_i} \left\{ E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}) \,:\, E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}) > 0 \right\}.$$

For any type $\hat{z}_i$, we know that when some other type z_i reports as type $\hat{z}_i$, either

$$E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}) \geq R,$$

or

$$E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}) = 0.$$

Next, observe that the following inequalities are always satisfied due to the monotonicity of cost of any contribution with respect to types:

$$E_{-i}\bar{\pi}_i(z_i, z_i, y_i^c(z_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\tilde{z}_i, z_i, \hat{\delta}_i^{con}(y_i^c(z_i, \mathbf{z}_{-i}), \tilde{z}_i, z_i), \mathbf{z}_{-i}) \quad \forall i, \text{ if } z_i > \tilde{z}_i; \tag{A.3}$$

$$E_{-i}\bar{\pi}_i(z_i, \tilde{z}_i, \hat{\delta}_i^{con}(y_i^c(\tilde{z}_i, \mathbf{z}_{-i}), z_i, \tilde{z}_i), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\tilde{z}_i, \tilde{z}_i, y_i^c(\tilde{z}_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \quad \forall i, \text{ if } z_i > \tilde{z}_i; \tag{A.4}$$

$$\text{and } E_{-i}\bar{\pi}_i(z_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), z_i, \hat{z}_i), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\tilde{z}_i, \hat{z}_i, \hat{\delta}_i^{con}(y_i^c(\hat{z}_i, \mathbf{z}_{-i}), \tilde{z}_i, \hat{z}_i), \mathbf{z}_{-i}), \quad \forall i, \ \hat{z}_i \neq z_i, \tilde{z}_i, \text{ if } z_i > \tilde{z}_i. \tag{A.5}$$

The first inequality says for each agent i, the higher type of him can always obtain a weakly higher equilibrium payoff than the deviation payoff of any lower type who wants to mimic him. The second inequality says for each agent i, when the higher type mimics a lower type, the higher type can always obtain a weakly higher deviation payoff than the equilibrium payoff of the lower type. The last inequality says for each agent i, the deviation payoff of higher type is always weakly higher than that of the lower type if they mimic as some other common type.

The rest of the proof is divided into two steps.

Step 1. Choose one $\hat{z}_i'$ for which there exists some z_i such that

$$E_{-i}\bar{\pi}_i(z_i, \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), z_i, \hat{z}_i'), \mathbf{z}_{-i}) \geq R.$$

The existence of such $\hat{z}_i'$ is guaranteed by our earlier argument above.

Let

$$\bar{z}_i = \min\left\{z_i \ : \ E_{-i}\bar{\pi}_i(z_i, \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), z_i, \hat{z}_i'), \mathbf{z}_{-i}) \geq R\right\}.$$

Thus, for all types $z_i < \bar{z}_i$,

$$E_{-i}\bar{\pi}_i(z_i, \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), z_i, \hat{z}_i'), \mathbf{z}_{-i}) = 0, \tag{A.6}$$

and for all types $z_i' > \bar{z}_i$, we know by (A.5) that

$$E_{-i}\bar{\pi}_i(z_i', \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), z_i', \hat{z}_i'), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\bar{z}_i, \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), \bar{z}_i, \hat{z}_i'), \mathbf{z}_{-i}) \geq R. \tag{A.7}$$

Consider the following two cases:

Case (i) $\bar{z}_i < \hat{z}_i'$.

Given that $E_{-i}\bar{\pi}_i(\bar{z}_i, \hat{z}_i', \hat{\delta}_i^{con}(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), \bar{z}_i, \hat{z}_i'), \mathbf{z}_{-i}) \geq R$, identify all z_{-i} profiles such that $\bar{\pi}_i(\bar{z}_i, \hat{z}_i', y_i^c(\hat{z}_i', \mathbf{z}_{-i}), \mathbf{z}_{-i}) > 0$, which also implies $\breve{p}_i(y_i^c(\hat{z}_i', z_{-i}), \hat{z}_i', \mathbf{z}_{-i}) > 0$. Now reduce the above payments $\breve{p}_i(y_i^c(\hat{z}_i', \mathbf{z}_{-i}), \hat{z}_i', \mathbf{z}_{-i})$ such that the expected reductions in the ex-ante deviation payoff of agent $\bar{z}_i$ is R, and his optimal contribution response following misreporting as $\hat{z}_i'$ remains the same as before the reduction.[17]

[17]The latter requirement is fulfilled so long as the downward-adjusted payment does not fall below the cost of the suggested contribution $y_i^c(\hat{z}_i', \mathbf{z}_{-i})$ by the true type $\bar{z}_i$.

Since $\hat{z}'_i > \bar{z}_i$, by (A.3), we know

$$E_{-i}\bar{\pi}_i(\hat{z}'_i, \hat{z}'_i, y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\bar{z}_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \bar{z}_i, \hat{z}'_i), \mathbf{z}_{-i}). \tag{A.8}$$

Thus, the expected reduction in the ex-ante equilibrium payoff for type $\hat{z}'_i$ agent is also R.[18] Before the reduction, we know $E_{-i}\bar{\pi}_i(\hat{z}'_i, \hat{z}'_i, y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq R$, so the ex-ante expected payoff for type $\hat{z}'_i$ agent is still non-negative after the reduction, i.e., PC(con) is still satisfied.

Case (ii) $\bar{z}_i > \hat{z}'_i$.

Given that $E_{-i}\bar{\pi}_i(\hat{z}'_i, \hat{z}'_i, y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq R$ (due to the fact that R is the smallest positive deviation payoff), identify all z_{-i} profiles such that $\bar{\pi}_i(\hat{z}'_i, \hat{z}'_i, y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) > 0$, which also implies $\breve{p}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \hat{z}'_i, \mathbf{z}_{-i}) > 0$. Now reduce the above payments $\breve{p}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \hat{z}'_i, \mathbf{z}_{-i})$ such that the expected reductions in the ex-ante equilibrium payoff of agent $\hat{z}'_i$ is R, and yet his optimal contribution remains the same as before the reduction.

Since $\bar{z}_i > \hat{z}'_i$, by (A.4), we know

$$E_{-i}\bar{\pi}_i(\bar{z}_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \bar{z}_i, \hat{z}'_i), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\hat{z}'_i, \hat{z}'_i, y^c_i(\hat{z}_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}).$$

Thus, the expected reduction in the ex-ante deviation payoff for type $\bar{z}_i$ agent when reporting as type $\hat{z}'_i$ is also R. ||

For all types $z_i < \bar{z}_i$, initially $E_{-i}\bar{\pi}_i(z_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), z_i, \hat{z}'_i), \mathbf{z}_{-i}) = 0$ (by (A.6)), and after the reduction in payment, those ex-ante deviation payoff of agent z_i when he reports as $\hat{z}'_i$ is still 0.

For all types $z'_i > \bar{z}_i$, initially $E_{-i}\bar{\pi}_i(z'_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), z'_i, \hat{z}'_i), \mathbf{z}_{-i}) \geq R$ (by (A.7)). By (A.5), we know

$$E_{-i}\bar{\pi}_i(z'_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), z'_i, \hat{z}'_i), \mathbf{z}_{-i}) \geq E_{-i}\bar{\pi}_i(\bar{z}_i, \hat{z}'_i, \hat{\delta}^{con}_i(y^c_i(\hat{z}'_i, \mathbf{z}_{-i}), \bar{z}_i, \hat{z}'_i), \mathbf{z}_{-i}).$$

Thus, the expected reduction in the deviation payoff of agent z'_i when he reports as $\hat{z}'_i$ is R.

Step 2. Repeat such reductions as in Step 1 for each possible type $\hat{z}''_i$ for which there exists z_i such that

$$E_{-i}\bar{\pi}_i(z_i, \hat{z}''_i, \hat{y}^{con}_i(z_i, \hat{z}''_i, \mathbf{z}_{-i}), \mathbf{z}_{-i}) \geq R.$$

Thus, we know that each such $\hat{z}''_i$ type's ex-ante equilibrium payoff is reduced by R, and the payoff is still non-negative. Also, for the ex-ante deviation payoff that is initially 0, it is still 0 after the reduction in payment; for the ex-ante deviation payoff that is initially positive, it is also reduced by R after the reduction in payment. Thus, every type's IC-type*(con) is still satisfied.

We have thus shown the existence of a feasible punishing contract which generates a higher ex-ante expected profit for the principal (by maintaining the same contributions level with a lower payment) than the replicated punishing contract $\{\breve{p}_i(y_i, z_i, \mathbf{z}_{-i}), y^c_i(z_i, \mathbf{z}_{-i})\}$. **Q.E.D.**

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[18]Note that the reductions in type $\bar{z}_i$'s deviation payment that were carried out earlier for $(\hat{z}'_i, z_{-i})$ reported profiles, the same reductions will happen when the true type is $\hat{z}'_i$ and the reported profiles are the same $(\hat{z}'_i, \mathbf{z}_{-i})$. That is, the same reductions apply to both sides of (A.8).

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