

Money Burning in Subjective Evaluation and Limited Liability: A Case for Pay for Performance[☆]

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Abstract

Research on subjective evaluation in principal-agent models have shown that the optimal contract should give uniform reward with zero money burning at all but the worst performance. In a static game involving two independent tasks with only high or low output possible, it is shown that when the agent is subjected to limited liability the optimal contract is more likely to exhibit ‘pay for performance’: full money burning if both tasks yield low outputs, partial money burning for mixed performance of low and high outputs, and zero money burning following high output in both tasks.

Key Words: Subjective evaluation, money burning, wage compression, pay for performance, limited liability.

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1. Introduction

This work complements two seminal works on subjective evaluation. In a static one-shot principal-agent model with agent risk aversion, MacLeod (2003) had shown that to maximize profits the principal ought to penalize the agent and burn money only when the performance is the worst possible, and for all other performance the rewards should be equalized. In a T rounds, repeated efforts model with binary outcomes in each period (success or failure) but agent risk neutral, Fuchs (2007) has shown that the principal should burn money only when the privately observed signals of agent performance in all T rounds are low. The combined message is that, if money burning is allowed, the principal should limit wasteful money burning to the worst incidence of performance. Equalizing of agent rewards in all but the worst performance is what we describe as a case of *extreme wage compression*. Extreme wage compression implies large penalties for the worst performance, so the agent’s liability may have to be unlimited.

We argue that when the agent is subjected to limited liability preventing principal from penalizing the agent too harshly (Sappington, 1983), the optimal contract is likely to exhibit a compensation ladder: full money burning, followed by partial money burning, finally zero money burning (Proposition 1). Thus, the money burning scheme is either of the *pay for performance* type with reward decreasing as performance drops (Holmstrom, 1979; Harris and Raviv, 1979), or one of *moderate wage compression* similar to Levin’s (2003) termination contract – wage compression around a non-extreme threshold performance.¹ Given perhaps greater prevalence of this threshold contracting in real life, this shift in results should improve our understanding of monitoring and subjective evaluation in principal-agent environments.

Our model is a static, multi-task formulation of Fuchs (2007). A principal hires an agent to work on two tasks in each of which the agent either exerts effort or shirks. The principal wants the agent to exert effort in both tasks at minimal cost. The principal burns money lowering the agent’s actual reward below an announced maximum if the agent’s performance in nonverifiable output does not prove satisfactory. Different from Fuchs’s model is the imposition of agent limited liability. Our principal minimizes reward costs (or maximizes profit).

The analysis and results appear in section 2. Proofs are relegated to an Appendix.

2. Main Result

A risk-neutral principal hires a risk-neutral agent to carry out two independent tasks. For each task the agent can either exert one unit of effort or shirk, $e_t \in \{0, 1\}$, with the effort costing the agent $c > 0$. The principal does not observe the agent’s effort but only privately learns about the agent’s binary output $y_t \in \{y_H, y_L\}$, which cannot be disclosed verifiably. The tasks are ex-ante identical with the probability of high or low output contingent only on effort:

$$\Pr[y_t = y_H \mid e_t = 0] = p_0, \quad \Pr[y_t = y_H \mid e_t = 1] = p_1,$$

with $0 < p_0 < p_1 < 1$.

Denote the principal’s expected wage payment by $\mathbb{E}(W)$. The principal’s objective is:

$$\max \sum_t \mathbb{E}(y_t \mid e_t) - \mathbb{E}(W).$$

Let $\mathbf{e} = (e_1, e_2)$ be the agent’s efforts in the two tasks. To solve the principal’s problem, the first step is to minimize his cost $\mathbb{E}(W)$ of inducing any effort profile \mathbf{e} ; then the second step is to determine the

¹Moderate wage compression typically involves, respectively, full and zero money burning below and above a threshold performance, and occasionally partial money burning at the threshold performance.

profit-maximizing \mathbf{e}^* . To simplify analysis, we assume the incremental expected output in any task following change from $\mathbf{e} = 0$ to $\mathbf{e} = 1$ is large enough so that the principal wants to uniquely implement $\mathbf{e}^* = (1, 1)$ at minimal wage costs.

At the end the principal submits a performance report, $\mathbf{s} = (\mathbf{y}_1, \mathbf{y}_2)$. The wage contract involves money burning: to dissuade misreporting principal must pay a fixed amount W independent of his report; on the other hand, to incentivize efforts the reward $w^\Lambda(\mathbf{s})$ and money burning $z(\mathbf{s})$ should be contingent on the report. Let $\Sigma = \{\mathbf{s}\} = \{(\mathbf{y}_1, \mathbf{y}_2)\}$ be the set of all performances, and bold symbols \mathbf{w}^Λ and \mathbf{z} be the vector of rewards and money burning corresponding to various performance reports \mathbf{s} . Formally, the payment scheme is defined as:

$$\begin{aligned} & (W, \mathbf{w}^\Lambda(\Sigma), \mathbf{z}(\Sigma)), \quad \text{where} \\ & W = w^\Lambda(\mathbf{s}) + z(\mathbf{s}) \quad \forall \mathbf{s} \in \Sigma. \end{aligned} \quad (1)$$

We may simply write $\omega = (W, \mathbf{z}(\Sigma))$ to refer to the incentive mechanism, where $\mathbf{z}(\Sigma) = \{z^{\text{HH}}, z^{\text{HL}}, z^{\text{LH}}, z^{\text{LL}}\}$.

Given the incentives, the agent's expected utility of exerting effort profile \mathbf{e} can be written as

$$\begin{aligned} V(\mathbf{e}) &= \mathbb{E}(w^\Lambda(\mathbf{s}) \mid \mathbf{e}) - c \sum_t e_t \\ &= W - \sum_{\mathbf{s} \in \Sigma} z(\mathbf{s}) \Pr(\mathbf{s} \mid \mathbf{e}) - c \sum_t e_t. \end{aligned} \quad (2)$$

Then the principal solves the following problem:

$$\begin{aligned} & \min_{W, \mathbf{z}(\Sigma)} W & (\mathcal{P}_1) \\ \text{subject to} & \quad \text{(Incentive Compatibility)} & \mathbf{e}^* \in \arg \max V(\mathbf{e}), & (3) \\ & \quad \text{(Limited Liability)} & W - z(\mathbf{s}) \geq 0 \quad \forall \mathbf{s} \in \Sigma, & (4) \\ & & \mathbf{z}(\Sigma) \geq \mathbf{0}. & (5) \end{aligned}$$

Incentive compatibility (3) and limited liability (4) will guarantee agent's participation constraint $V(\mathbf{e}^*) \geq 0$.

Sequence of events: A contract $(W, \mathbf{z}(\Sigma))$ is signed between parties. Then for each task $t = 1, 2$ the agent decides whether to exert effort or shirk and the principal observes the output. Finally, the principal reports the output profile \mathbf{s} and makes the payment $w^\Lambda(\mathbf{s})$.

We now derive the optimal money burning contract for a profit-motivated principal. The principal will evaluate the agent only after observing the overall performance.

Lemma 1 *The optimal money burning mechanism $\mathbf{z}(\Sigma)$ implementing full efforts satisfy the following properties:*

- (i) *The principal does no better than to set $z^{\text{LH}} = z^{\text{HL}}$;*
- (ii) *Money burning is weakly decreasing in outputs, i.e.,*

$$z^{\text{LL}} \geq z^{\text{LH}} = z^{\text{HL}} \geq z^{\text{HH}}, \quad (6)$$

$$\text{and} \quad z^{\text{HH}} = 0. \quad (7)$$

Constraint (5) is guaranteed by conditions (6)–(7), and constraint (4) in problem (\mathcal{P}_1) can be simplified to:

$$W - z^{\text{LL}} \geq 0. \quad (8)$$

Solving the problem subject to effort incentive constraint (3), and constraints (6)–(8) yields the following result.

Proposition 1 (Optimal contract: complete characterization) *Under subjective evaluation a profit-seeking principal typically relies on the pay for performance incentives or moderate wage compression, although in some situations extreme wage compression is still a possibility. More specifically, the principal's optimal contract when the agent is subjected to limited liability is as follows (Fig. 1):*

- (I) [Moderate compression] If $p_1 > \frac{1}{2}$ and $p_1 + p_0 > 1$, then $W = \frac{2c}{(p_1+p_0)(p_1-p_0)}$, $z^{HH} = 0$, $z^{LH} = z^{HL} = z^{LL} = \frac{2c}{(p_1+p_0)(p_1-p_0)}$.
- (II) (a) [Pay for performance/moderate compression] If $p_1 > \frac{1}{2}$ and $p_1 + p_0 = 1$, then $W = \frac{2c}{p_1-p_0}$, $z^{HH} = 0$, $z^{LH} = z^{HL} \in [\frac{c}{p_1-p_0}, \frac{2c}{p_1-p_0}]$, $z^{LL} = \frac{2c}{p_1-p_0}$.
- (b) [Pay for performance] If $p_1 > \frac{1}{2}$ and $p_1 + p_0 < 1$, then $W = \frac{2c}{p_1-p_0}$, $z^{HH} = 0$, $z^{LH} = z^{HL} = \frac{c}{p_1-p_0}$, $z^{LL} = \frac{2c}{p_1-p_0}$.
- (c) [Pay for performance/extreme compression] If $p_1 = \frac{1}{2}$, then $W = \frac{2c}{p_1-p_0}$, $z^{HH} = 0$, $z^{LH} = z^{HL} \in [0, \frac{c}{p_1-p_0}]$, $z^{LL} = \frac{2c}{p_1-p_0}$.
- (III) [Extreme compression] If $p_1 < \frac{1}{2}$, then $W = \frac{c}{(1-p_1)(p_1-p_0)}$, $z^{HH} = z^{LH} = z^{HL} = 0$, $z^{LL} = \frac{c}{(1-p_1)(p_1-p_0)}$.

Thus, the optimal contract admits several possibilities. In part (II) of the contract, optimal incentives are of the pay-for-performance variety. Extreme wage compression happens in part (III) where $p_1 \leq 1/2$. Such a case can be associated with the agent being assigned to a rather hard task where failure is more likely than success even in the case of high effort.

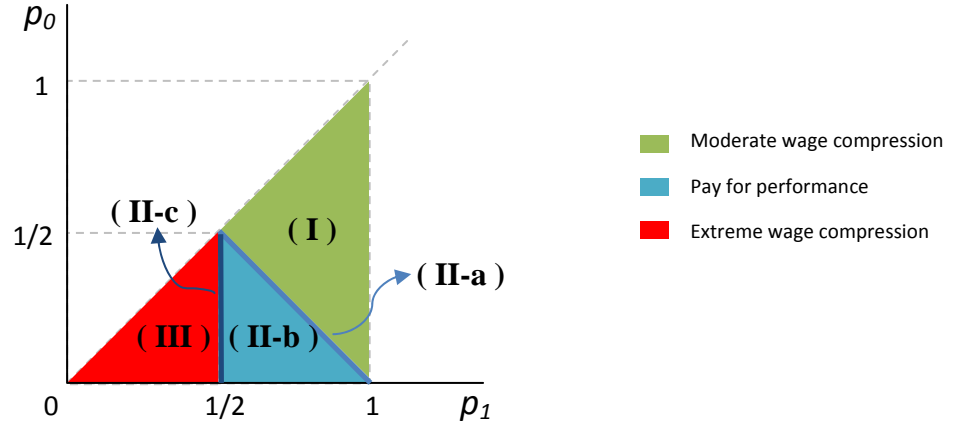


Figure 1: Optimal contract

Why extreme wage compression need no longer be optimal can be understood as follows. In Fuchs (2007), the agent had to be induced to exert efforts but the principal's main concern was to do so at minimal expected money burning. Our principal, on the other hand, is interested in minimizing his own reward cost that equals the *maximum* of the burnt money. Since minimization of expected money burning is not necessarily the dual of the principal's profit maximization problem, solution to Fuchs's implementation program does not necessarily minimize principal's costs. The main difference comes from the imposition of agent limited liability. Without this constraint, the principal could freely transfer rents from the agent and thus maximization of social surplus also meant maximization of profits. Social surplus is maximized when money burnt (a deadweight loss) is minimized; to do that, the principal could

transfer any positive money burning upon high performance to that of low performance scenarios. That is, Fuchs's principal would penalize the agent only for *all* low outputs, which increases the absolute level of money burning (although with lower incidence) and thus pushes up principal's reward costs. But high reward costs are recoverable by asking the agent to make the necessary transfers so long as the agent's reservation utility is met. However with limited liability such punishment and transfer schemes often do not work, necessitating spreading of penalty and money burning to moderate performances (outputs in between the worst and best ones).²

The intuition for performance pay can also be understood as follows, in addition to the limited liability based explanation provided above. When $p_1 > \frac{1}{2}$ and $p_1 + p_0 \leq 1$, it implies that the difference in the probabilities of generating a high (output) performance when the agent exerts effort vs. when he shirks is going to be non-trivial ($p_1 > \frac{1}{2}$ but $p_0 < \frac{1}{2}$). Thus the choice of effort or shirking is very likely to be reflected in the performance: outputs, although stochastic, are informative and hence the scope for nuanced rewards/punishment; all money is burnt if both periods see low outputs, whereas partial money is burnt upon a combination of one low and one high output.

Finally, the intuition for extreme wage compression is more subtle. Here realization of high output is not very informative as it does not suggest a strong evidence of agent's effort (recall, $p_1 \leq \frac{1}{2}$). In contrast, low output would imply a high chance that the agent did *not* exert effort: $p_0 < \frac{1}{2}$. Thus, rather than the realization of high output, its *absence* is more indicative of lack of effort. With $p_0 < p_1$, principal can rely on the outputs' informativeness (monotone likelihood ratio property) to determine money burning. Principal chooses to burn money only when output in both tasks are low; when only one output is low, it could be that the agent did exert effort in that task yet he was unlucky (recall $p_1 \leq \frac{1}{2}$) and principal does not want to wrongfully penalize the agent.

To summarize, the first case burns money whenever the report contains at least one low performance, with penalty (money burning) strictly increasing in the number of low performances. This represents the pay-for-performance principle of Holmstrom (1979) and Harris and Raviv (1979). And the second case is same as the extreme wage compression result of Fuchs (2007).

Appendix A.

Proof of Lemma 1. The agent's payoffs from various effort choices are as follows:

$$\begin{aligned} V(1,1) &= W - p_1 p_1 z^{HH} - p_1(1-p_1)z^{HL} - (1-p_1)p_1 z^{LH} - (1-p_1)(1-p_1)z^{LL} - 2c, \\ V(1,0) &= W - p_1 p_0 z^{HH} - p_1(1-p_0)z^{HL} - (1-p_1)p_0 z^{LH} - (1-p_1)(1-p_0)z^{LL} - c, \\ V(0,1) &= W - p_0 p_1 z^{HH} - p_0(1-p_1)z^{HL} - (1-p_0)p_1 z^{LH} - (1-p_0)(1-p_1)z^{LL} - c, \\ V(0,0) &= W - p_0 p_0 z^{HH} - p_0(1-p_0)z^{HL} - (1-p_0)p_0 z^{LH} - (1-p_0)(1-p_0)z^{LL}. \end{aligned}$$

For full efforts implementation, the agent's incentive compatibility (IC) conditions $V(1,1) - V(1,0) \geq 0$, $V(1,1) - V(0,1) \geq 0$, and $V(1,1) - V(0,0) \geq 0$ can thus be written as, respectively:

$$p_1(z^{HL} - z^{HH}) + (1-p_1)(z^{LL} - z^{LH}) \geq \frac{c}{p_1 - p_0}, \quad (\text{A.1})$$

$$p_1(z^{LH} - z^{HH}) + (1-p_1)(z^{LL} - z^{HL}) \geq \frac{c}{p_1 - p_0}, \quad (\text{A.2})$$

$$p_1(z^{HL} - z^{HH}) + (1-p_1)(z^{LL} - z^{LH}) + p_0(z^{LH} - z^{HH}) + (1-p_0)(z^{LL} - z^{HL}) \geq \frac{2c}{p_1 - p_0}. \quad (\text{A.3})$$

²In MacLeod (2003)'s model, an agent's performance could be tracked by observing an imperfect signal conditional on efforts. In his analysis limited liability was not an issue because of an Inada condition stipulating agent ruin near zero consumption. If, however, the Inada condition is dropped, limited liability will have a bite and an effect similar to spreading of money burning beyond the lowest signal of performance will arise.

Rewrite (A.1)–(A.3) respectively as:

$$p_1(z^{\text{HL}} + z^{\text{LH}}) - z^{\text{LH}} - p_1 z^{\text{HH}} + (1 - p_1)z^{\text{LL}} \geq \frac{c}{p_1 - p_0}, \quad (\text{A.4})$$

$$p_1(z^{\text{HL}} + z^{\text{LH}}) - z^{\text{HL}} - p_1 z^{\text{HH}} + (1 - p_1)z^{\text{LL}} \geq \frac{c}{p_1 - p_0}, \quad (\text{A.5})$$

$$[p_0 + p_1 - 1](z^{\text{HL}} + z^{\text{LH}}) - [p_0 + p_1]z^{\text{HH}} + [2 - p_0 - p_1]z^{\text{LL}} \geq \frac{2c}{p_1 - p_0}. \quad (\text{A.6})$$

Also, rewrite $V(1, 1)$ as $V(1, 1) = W - p_1 p_1 z^{\text{HH}} - (1 - p_1)(1 - p_1)z^{\text{LL}} - p_1(1 - p_1)[z^{\text{HL}} + z^{\text{LH}}] - 2c$.

The principal's objective is to implement full efforts at minimal W . So at full efforts implementation, \mathbf{z} must satisfy (A.4)–(A.6), limited liability conditions, and the agent's participation constraint $V(1, 1) \geq 0$.

It should now be clear that the principal can optimally set $z^{\text{HL}} = z^{\text{LH}}$. Suppose not so that $z^{\text{HL}} \neq z^{\text{LH}}$. Then reset at $\hat{z}^{\text{HL}} = \hat{z}^{\text{LH}} = (z^{\text{HL}} + z^{\text{LH}})/2$, so that (A.4)–(A.6) continue to be satisfied and the value of $V(1, 1)$ remains unchanged. The limited liability condition will also be satisfied, since $W \geq \max\{z^{\text{HL}}, z^{\text{LH}}\} \geq (z^{\text{HL}} + z^{\text{LH}})/2$. This implies the principal's implementation cost, W , cannot go up in the modified mechanism. This establishes part (i).

Keeping in mind the result that $z^{\text{LH}} = z^{\text{HL}}$, we can establish the monotonicity result in part (ii) if we can rule out the following two possibilities:

$$z^{\text{LL}} < z^{\text{LH}}, \quad (\text{A.7})$$

$$z^{\text{LH}} < z^{\text{HH}}. \quad (\text{A.8})$$

Suppose, on the contrary, (A.7) holds. Let us now increase z^{LL} by a small $\epsilon > 0$ such that $z^{\text{LL}} + \epsilon < z^{\text{LH}}$. This will relax all the ICs, (A.1)–(A.3). We will now be able to adjust $\max\{z^{\text{LH}} = z^{\text{HL}}, z^{\text{HH}}\}$ downwards by a small $\delta > 0$ such that the adjusted value still exceeds $z^{\text{LL}} + \epsilon$ and all the ICs continue to remain non-binding. At this stage we need to make sure that after the above two adjustments the new $V(1, 1) > 0$. Then we can lower W slightly, satisfying the limited liability constraints and the final $V(1, 1)$ is still non-negative. The lowered W , by construction, will satisfy the limited liability constraints.

If $V(1, 1)$ can be shown to be strictly positive before the ϵ adjustment to z^{LL} , then the final $V(1, 1)$ after all the adjustments detailed above will be strictly positive, because the ϵ and δ adjustments can be kept suitably small. The required contradiction is now complete since $V(1, 1) \geq V(0, 0) > 0$, given that by hypothesis $z^{\text{LL}} < \max\{z^{\text{LH}} = z^{\text{HL}}, z^{\text{HH}}\}$, and $0 < p_0 < 1$ and the limited liability constraints on the initial incentives.

Finally, suppose (A.8) holds. The required contradiction is now much simpler: lower z^{HH} towards $z^{\text{LH}} = z^{\text{HL}}$, which will relax all the ICs (A.4)–(A.6), satisfy the limited liability constraints, and improve $V(1, 1)$. The principal can then lower W slightly and still satisfy all the ICs and limited liability constraints while maintaining $V(1, 1) > 0$. This completes the monotonicity result.

The claim $z^{\text{HH}} = 0$ follows directly from the IC conditions, because choosing $z^{\text{HH}} > 0$ will make the ICs more demanding and also at the same time lower $V(1, 1)$. **Q.E.D.**

Proof of Proposition 1. Applying Lemma 1, the principal's problem (\mathcal{P}_1) can be written as:

$$\begin{aligned} & \min_{\{W, z^{\text{HH}}, z^{\text{HL}}, z^{\text{LH}}, z^{\text{LL}}\}} W \\ \text{subject to} & \quad (\text{A.1}), (\text{A.2}), (\text{A.3}) \\ & \quad W - z^{\text{LL}} \geq 0, \quad (\text{A.9}) \\ & \quad z^{\text{LL}} \geq z^{\text{LH}} = z^{\text{HL}} \geq z^{\text{HH}} = 0. \quad (\text{A.10}) \end{aligned}$$

Clearly, to achieve the objective of minimizing W , it must be set equal to z^{LL} binding the constraint (A.9). Now applying $W = z^{\text{LL}}$, $z^{\text{LH}} = z^{\text{HL}}$ and $z^{\text{HH}} = 0$, we can simplify the set of constraints

to:

$$p_1 z^{\text{HL}} + (1 - p_1)(W - z^{\text{HL}}) \geq \frac{c}{p_1 - p_0}, \quad (\text{A.11})$$

$$p_1 z^{\text{HL}} + (1 - p_1)(W - z^{\text{HL}}) + p_0 z^{\text{HL}} + (1 - p_0)(W - z^{\text{HL}}) \geq \frac{2c}{p_1 - p_0}, \quad (\text{A.12})$$

$$W \geq z^{\text{HL}} \geq 0. \quad (\text{A.13})$$

Now our task is to find the minimum value of W , subject to (A.11), (A.12) and (A.13). Rewrite the first two inequalities as follows:

$$W \geq \frac{1 - 2p_1}{1 - p_1} \cdot z^{\text{HL}} + \frac{c}{(1 - p_1)(p_1 - p_0)}, \quad (\text{A.14})$$

$$W \geq \frac{1 - 2[\frac{1}{2}(p_1 + p_0)]}{1 - \frac{1}{2}(p_1 + p_0)} \cdot z^{\text{HL}} + \frac{c}{[1 - \frac{1}{2}(p_1 + p_0)](p_1 - p_0)}. \quad (\text{A.15})$$

Below we derive the optimal contract for different partitions of p_0 and p_1 by identifying the feasible set in the (W, z^{HL}) space and then choosing the (W, z^{HL}) pair with the minimal W .

(1) The region $p_1 > \frac{1}{2}$ and $p_1 + p_0 > 1$.

The boundaries of the constraints (A.14) and (A.15) will be linear and negatively sloped, with the first one steeper than the latter as illustrated in Fig. A.2. (From here onwards, when referring to the constraints we will mean their corresponding boundaries.) It is now easy to see that W is minimal when (A.15) intersects with (A.13) (the 45-degree line). Thus the optimal contract is as follows:

$$W = \frac{2c}{(p_1 + p_0)(p_1 - p_0)}, \quad z^{\text{HH}} = 0, \quad z^{\text{LH}} = z^{\text{HL}} = z^{\text{LL}} = \frac{2c}{(p_1 + p_0)(p_1 - p_0)}.$$

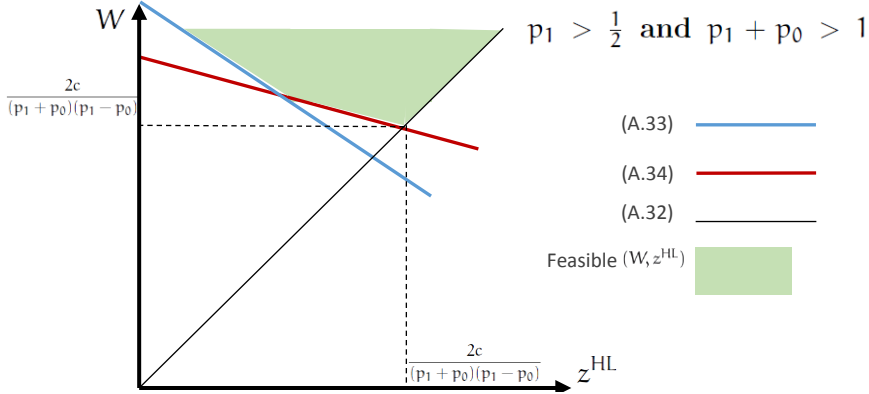


Figure A.2: Part (I)

(2) The region $p_1 > \frac{1}{2}$ and $p_1 + p_0 = 1$.

In this case, (A.14) is negatively sloped while (A.15) is horizontal, so the Northeast segment satisfy the two constraints. Then the 45-degree line (A.13) determines the range of z^{HL} that also belongs to the Northeast segment identified above (see Fig. A.3). Hence, the optimal contract is as follows:

$$W = \frac{2c}{p_1 - p_0}, \quad z^{\text{HH}} = 0, \quad z^{\text{LH}} = z^{\text{HL}} \in \left[\frac{c}{p_1 - p_0}, \frac{2c}{p_1 - p_0} \right], \quad z^{\text{LL}} = \frac{2c}{p_1 - p_0}.$$

(3) The region $p_1 > \frac{1}{2}$ and $p_1 + p_0 < 1$.

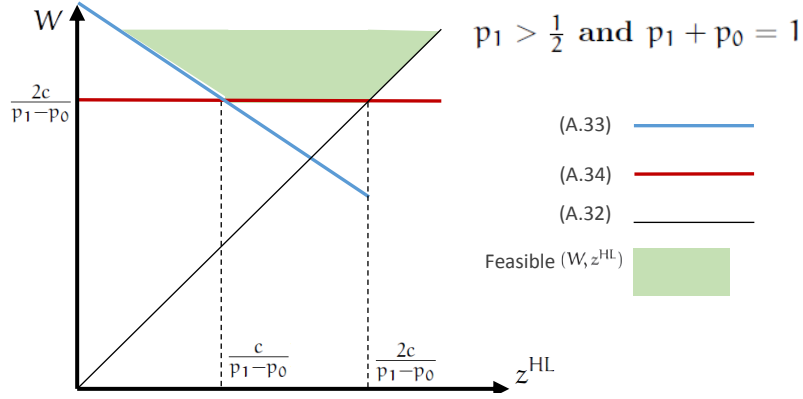


Figure A.3: Part (II-a)

In this case, (A.14) is negatively sloped while (A.15) is upward sloping (but flatter than the 45-degree line); see Fig. A.4. At the point of intersection between the two lines, (A.13) is not binding, hence the minimal W is given by the solution of (A.14) and (A.15) and the optimal contract is unique:

$$W = \frac{2c}{p_1 - p_0}, \quad z^{HH} = 0, \quad z^{LH} = z^{HL} = \frac{c}{p_1 - p_0}, \quad z^{LL} = \frac{2c}{p_1 - p_0}.$$

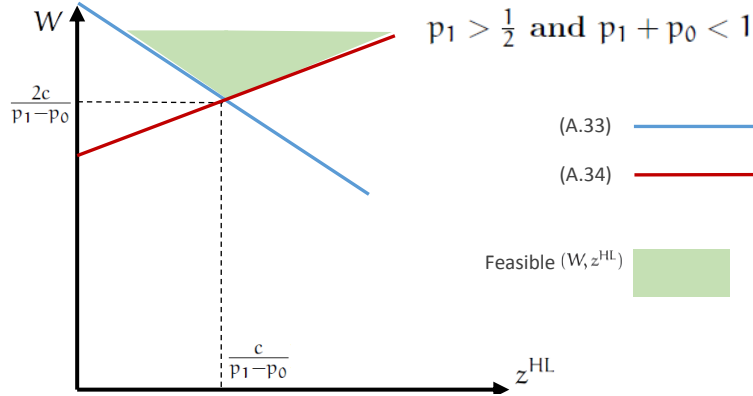


Figure A.4: Part (II-b)

(4) **The region $p_1 = \frac{1}{2}$.**

(A.14) and (A.15) are drawn in Fig. A.5, and it is straightforward to determine the optimal contract ((A.13) will not be binding):

$$W = \frac{2c}{p_1 - p_0}, \quad z^{HH} = 0, \quad z^{LH} = z^{HL} \in [0, \frac{c}{p_1 - p_0}], \quad z^{LL} = \frac{2c}{p_1 - p_0}.$$

(5) **The region $p_1 < \frac{1}{2}$.**

In this case, both (A.14) and (A.15) are positively sloped (Fig. A.6). With (A.13) not binding, the minimum W is obtained at $z^{HL} = 0$ and the optimal contract is:

$$W = \frac{c}{(1 - p_1)(p_1 - p_0)}, \quad z^{HH} = z^{LH} = z^{HL} = 0, \quad z^{LL} = \frac{c}{(1 - p_1)(p_1 - p_0)}. \quad \mathbf{Q.E.D.}$$

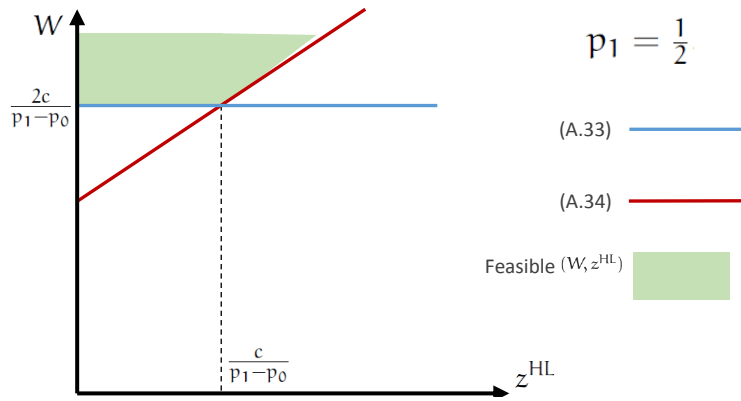


Figure A.5: Part (II-c)

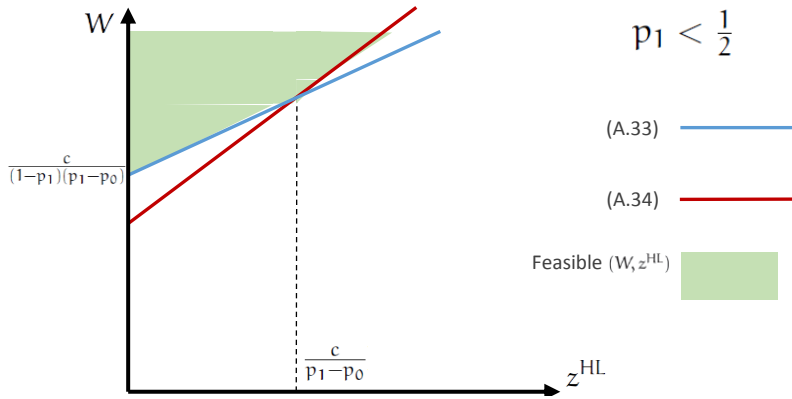


Figure A.6: Part (III)

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