# A technical appendix for multihoming and compatibility*

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July 19, 2005

*We would like to thank two anonymous referees and our editor, Simon Anderson, for their very helpful comments.

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# A Proof of Existence of Equilibrium with Multihoming

In this appendix, we derive sufficient conditions for the existence of the multihoming equilibrium that we have utilized in Doganoglu and Wright (2005). We briefly summarize the model here once again so that the reader can follow our arguments in a self contained fashion.

There are two symmetric firms denoted 1 and 2 which provide a service to consumers at the constant marginal cost, $f$. Consumers can subscribe to a service from either firm 1, firm 2, or both. Subscribing to a service gives consumers network benefits that are linear in the number of other agents that the consumer can access through the service. There are two types of consumers according to their marginal valuation of the network size, denoted $b$. A fraction $\lambda$ of consumers value the network benefits highly (high types) and have $b = b_H$. The remaining $1 - \lambda$ consumers do not value the network benefits highly (low types) and have $b = b_L \geq 0$. Naturally, we assume that $b_H > b_L$ and $0 < \lambda < 1$. Furthermore, we adopt

**Assumption 1** $t > \lambda b_H + (1 - \lambda) b_L = \beta$,

which simply states that transport costs are higher than average marginal network benefits.

The net utility of a consumer of type $b$ located at $x \in [0, 1]$ when she purchases from firm $i$ is given by

$$U_i(x, b, N_i) = v - p_i - t_i(x) + bN_i$$

for $i = 1, 2$, where $v$ is the intrinsic benefit[1] of the service, $p_i$ is the (uniform) subscription price of firm $i$, transportation costs $t_i(x)$ equal $tx$ for firm 1 and $t(1 - x)$ for firm 2, and $N_i$ represents the total number of consumers that can be reached by subscribing to firm $i$. When the same consumer multihomes, subscribing to both firms, the net utility she gets is

$$U(x, b, N) = v - p_1 - p_2 - t + b = U_{12}(b),$$

given that multihoming ensures all consumers can be reached and the total distance of travelling to both firms is always unity.

In Doganoglu and Wright (2005), we concentrate on an equilibrium where all high types multihome and all low types singlehome implying $s_i = 1$ and $N_i = \lambda + (1 - \lambda) n_i$ for $i = 1, 2$. Subscribing to firm $i$ exclusively allows a consumer to reach all high types and a share $n_i$ of low types. As a result, the share of singlehoming consumers that join firm 1 is found by solving $U_1(n_1, b_L, N_1) = U_2(n_1, b_L, N_2)$ for $n_1$ which implies

$$n_1 = \frac{1}{2} + \frac{p_2 - p_1}{2\left(t - (1 - \lambda) b_L\right)}$$

and $n_2 = 1 - n_1$. Assumption 1, is sufficient to ensure the market share equation is well behaved. Furthermore, it is easy to verify using the expression for $n_1$ that the total demand as well as the network size of firm $i$ is given by

$$N_1 = \frac{1}{2} + \frac{\lambda}{2} + \frac{(1 - \lambda)(p_2 - p_1)}{2(t - (1 - \lambda) b_L)}.$$

[1]We assume that the intrinsic benefit, $v$, is sufficiently high that all consumers subscribe to at least one firm.

The profits of network $i$ are

$$\pi_i = (p_i - f)N_i.$$

Substituting the share function into profits for $i = 1, 2$, taking the first order conditions, solving out for prices, and simplifying implies candidate equilibrium prices of

$$p_M^* \equiv p_1^* = p_2^* = f + \frac{1+\lambda}{1-\lambda}\left(t - (1-\lambda)\, b_L\right). \quad \text{(A-1)}$$

Given Assumption 1, candidate equilibrium prices exceed costs $f$.

**Proposition A-1** *There is a set of parameters for which prices given in (A-1) constitute a pure strategy Nash equilibrium.*

The set of parameters which support the prices in (A-1) as an equilibrium outcome, satisfy Assumption 1 and require $b_H$ to be sufficiently large but not too large. However, it turns out that the main constraint for the existence of the equilibrium that we are considering is that the costs of providing the service for each consumer are not too high. Higher values of the per-customer cost imply higher levels of the candidate equilibrium prices. In this case, given that firm 2 charges the candidate equilibrium price, firm 1 finds it profitable to lower its price to a level where some of the high types stop subscribing to firm 2. This has a positive impact on the demand of firm 1 from the low types due to network effects. Thus, whenever $\lambda$ is sufficiently small, a price cut inducing some high types to stop multihoming increases the demand from low types enough that firm 1 finds it profitable to deviate.

**Proof.**

In order to show that the candidate equilibrium prices given in (A-1) indeed constitute an equilibrium, we first need to characterize the rational expectations demands faced by both firms. Then, using these demand functions, we find restrictions on parameters which rule out deviations (including non infinitesimal deviations) of each firm from the prices in (A-1).

Let us first establish that low types never prefer to multihome as long as $t > b_L$ which is a consequence of our maintained assumption $t > \beta$. Suppose prices are non-negative, then a necessary condition for low types to multihome is that $tx < b_L(1 - N_1)$ and $t(1 - x) < b_L(1 - N_2)$. With some multihoming, $N_1 > 0$, $N_2 > 0$, and $N_1 + N_2 > 1$, so that there is no $x$ in $[0, 1]$ for which these conditions can all hold. Hence, when we derive the demand functions below, we assume that low types always singlehome.

We define several variables below to facilitate computation.

1. $c_1 = t - (1-\lambda)\, b_L$
2. $c_2 = t - \lambda b_H - (1-\lambda)\, b_L$
3. $c_3 = 2t(t - (1-\lambda)\, b_L) - \lambda\,(1-\lambda)\; b_L b_H$
4. $c_4 = t - b_L$
5. $c_5 = (1-\lambda)\, b_H$

Note given Assumption 1 all these expressions are positive.

We consider rational expectations demands such that given prices, consumers have beliefs about network sizes which lead them to make subscription decisions that justify these beliefs. There are several cases to consider in terms of fixing particular expectations on network sizes. We will derive only the demand for firm 1 since the demand for firm 2 is given by symmetry. Taking $p_2$ as given, we vary $p_1$ to trace out the relevant demand curves.

**Case 1.** $s_1 = 1$**,** $s_2 = 1$**,** $n_1 = n_{11}, \quad n_2 = 1 - n_1, \quad 0 < n_{11} < 1$

This is the case in the main text where the high types all multihome. Solving $U_1(n_{11}, b_L, N_1) = U_2(1 - n_{11}, b_L, N_2)$, we get that

$$n_{11} = \frac{1}{2} + \frac{p_2 - p_1}{2c_1}.$$

**Case 2.** $s_2 = s_{22}$**,** $s_1 = 1$**,** $0 < s_{22} < 1$**,** $n_1 = n_{12}, \quad n_2 = 1 - n_1, \quad 0 < n_{12} < 1$

In this case, firm 1 sells to all high types, while firm 2 sells to some of each. Solving $U_1(1 - s_{22}, b_H, N_1) = U_{12}(b_H)$ and $U_1(n_{12}, b_L, N_1) = U_2(1 - n_{12}, b_L, N_2)$ implies

$$\begin{aligned} n_{12} &= \frac{c_3 - tc_4}{c_3} + \frac{\lambda b_L p_2}{c_3} + \frac{t(p_2 - p_1)}{c_3} \\ s_{22} &= \frac{c_3 c_4}{c_3} - \frac{2c_1 p_2}{c_3} + \frac{c_3\,(p_1 - p_2)}{c_3}. \end{aligned}$$

**Case 3.** $s_1 = s_{13}$**,** $s_2 = 1$**,** $0 < s_{13} < 1$**,** $n_1 = n_{13}, \quad n_2 = 1 - n_1, \quad 0 < n_{13} < 1$

This is where firm 2 sells to all high types, while firm 1 sells to some of each. The solution of $U_2(s_{13}, b_H, N_2) = U_{12}(b_H)$ and $U_1(n_{13}, b_L, N_1) = U_2(1 - n_{13}, b_L, N_2)$ is

$$\begin{aligned} n_{13} &= \frac{tc_4}{c_5} - \frac{\lambda b_L p_1}{c_5} + \frac{t(p_2 - p_1)}{c_5} \\ s_{13} &= \frac{c_5 c_4}{c_3} - \frac{2c_1 p_1}{c_3} + \frac{c_5\,(p_2 - p_1)}{c_3}. \end{aligned}$$

**Case 4.** $s_2 = 0$**,** $s_1 = 1$**,** $n_1 = n_{14}, \quad n_2 = 1 - n_1, \quad 0 < n_{14} < 1$

This is where firm 1 sells to all high types, while firm 2 sells to no high types but some low types. The solution of $U_1(n_{14}, b_L, N_1) = U_2(1 - n_{14}, b_L, N_2)$ is

$$n_{14} = \frac{1}{2} + \frac{1}{2}\frac{\lambda b_L}{c_1} + \frac{p_2 - p_1}{2c_1}.$$

Naturally, these demand curves are valid only for some prices, and the rational expectations demands switch as we change prices. There are a number of cutoff values of prices which play an important role in the following. The first set of cutoff values are important values of $p_1$, where a shift in the demand curve occurs and are functions of $p_2$. Cutoffs which involve $p_2$:

$$\begin{aligned}
d_1 &= \lambda b_L + p_2 - c_1 && p_1 > d_1 \Rightarrow n_{14} < 1 \\
d_2 &= \big(p_2\,(2\,c_1 + c_5) - c_5 c_4\big)/c_5 && p_1 > d_2 \Rightarrow s_{22} > 0 \\
d_3 &= d_2 + c_3/c_5 && p_1 < d_3 \Rightarrow s_{22} < 1 \\
d_4 &= \big(c_5\,(c_4 + p_2) - c_3\big)/\big(2\,c_1 + c_5\big) && p_1 > d_4 \Rightarrow s_{13} < 1 \\
d_5 &= \big(c_5\,(c_4 + p_2)\,\big)/\big(2\,c_1 + c_5\big) && p_1 < d_5 \Rightarrow s_{13} > 0
\end{aligned}$$

First, notice that $d_3 > d_2 > d_1$, and $d_5 > d_4$ as long as $p_2 > 0$. In the paper, we concentrate on an equilibrium where $s_1 = 1$ and $s_2 = 1$, where all high types multihome. This is only possible whenever $d_4 > d_3$. However, whether this inequality holds or not depends on the value of $p_2$. Let $e_1 = (1-\lambda)b_H/2 - t$. It is easy to verify that $d_4 > d_3$ if and only if $p_2 < e_1$.

Notice that for $p_1 < d_4$, the high type consumer located at 1 is best off with multihoming, while this is the case for the consumer located at 0 whenever $p_1 > d_3$. Thus, as long as $p_2 < e_1$ and $d_3 < p_1 < d_4$, all high types prefer multihoming to singlehoming. We can write down the demand function for firm 1 whenever $0 < p_2 < e_1$; that is, whenever multihoming of the all high types is possible. Table 2 presents the ranges of prices where a particular set of demands are valid.

Table 1: Rational expectations demands of firm 1 when $0 < p_2 < e_1$.

| | $0 < p_2 < e_1$ |
|---|---|
| $0 < p_1 < d_1$ | $s_1 = 1, s_2 = 0, n_1 = 1$ |
| $d_1 < p_1 < d_2$ | $s_1 = 1, s_2 = 0, n_1 = n_{14}$ |
| $d_2 < p_1 < d_3$ | $s_1 = 1, s_2 = s_{22}, n_1 = n_{12}$ |
| $d_3 < p_1 < d_4$ | $s_1 = 1, s_2 = 1, n_1 = n_{11}$ |
| $d_4 < p_1 < d_5$ | $s_1 = s_{13}, s_2 = 1, n_1 = n_{13}$ |
| $p_1 > d_5$ | $s_1 = 0, s_2 = 1, n_1 = 0$ |

Recall that the demand configurations we are interested in involve all high types multihoming while low types subscribe to one of the two firms. Table 2 implies that for this configuration to be a rational expectations demand, we need the prices to be less than $e_1$. Given (A-1), this requires

$$f < \frac{(1-\lambda)\,b_H}{2} + (1+\lambda)\,b_L - \frac{2t}{1-\lambda} \equiv x_0.$$

Let $(d_3^*, d_4^*)$ be simply defined as $(d_3, d_4)$ evaluated at $p_2^*$. The best response of firm 1 to the price $p_2^*$ is to match this price if firm 1 considers prices such that $d_3^* < p_1 < d_4^*$. However, we need to also consider the possibility the best response occurs when $p_1 < d_3^*$ or $p_1 > d_4^*$.

Let us first look at the prices such that $p_1 > d_4^*$. In this case, firm 1 will face $s_1 = s_{13}$ and $n_1 = n_{13}$, and its profit will be given by

$$\pi_{13} = (p_1 - f)\,(\lambda s_{13} + (1-\lambda) n_{13})\,,$$

which attains an unconstrained maximum at

$$p_{13} = \frac{1}{2} f + \frac{1}{2} \frac{(\lambda c_5 + (1-\lambda)t)(c_4 + p_2)}{\lambda (c_1 + c_5) + t}.$$

If $p_{13}^* < d_4^*$, where $p_{13}^*$ is $p_{13}$ evaluated at $p_2^*$, and given that $\pi_{13}$ is continuous at $d_4$, firm 1 will have no profitable deviations whenever $p_1 > d_4^*$. After substituting in all the variables, it is easy to verify that $d_4^* > p_{13}^*$ whenever

$$f < \frac{(b_H (\lambda c_1 + \lambda c_5 + t) - c_3)((1-\lambda) c_4 + (1+\lambda) c_1)}{(1-\lambda) c_3 + 2 c_1 (\lambda c_1 + \lambda c_5 + t)} - \frac{2(\lambda c_1 + \lambda c_5 + t) c_3}{(1-\lambda) c_3 + 2 c_1 (\lambda c_1 + \lambda c_5 + t)} \equiv x_1.$$

If $x_1 > x_0$, we obtain the desired result. Evaluating this difference yields

$$x_1 - x_0 = \frac{1}{2} \frac{(1+\lambda)\lambda (t + c_1)(2 c_1 + c_5)^2}{(1-\lambda)((1-\lambda) c_3 + 2 c_1 (\lambda c_1 + \lambda c_5 + t))} > 0.$$

Therefore, there is no profitable deviation of firm 1 where $p_1 > d_4^*$.

Similarly, a condition for the candidate equilibrium prices to constitute an equilibrium is that there is no profitable deviation such that $p_1 < d_3^*$. In this case, the relevant rational expectations demand is such that $s_1 = 1$, $s_2 = s_{22}$ and $n_1 = n_{12}$. The profit function of firm 1 is given by

$$\pi_{12} = (p_1 - f)(\lambda + (1-\lambda) n_{12}),$$

which is maximized at

$$p_{12} = \left(\frac{\lambda b_L}{2t} + \frac{1}{2}\right) p_2 + \frac{f}{2} - \frac{c_4}{2} + \frac{c_3}{2(1-\lambda)t}.$$

The first condition to check is whether $p_{12}^* - d_3^* > 0$, where $p_{12}^*$ is $p_{12}$ evaluated at $p_2 = p_2^*$. Then

$$p_{12}^* - d_3^* = \frac{1}{2} \frac{(4 t c_1 - \lambda b_L c_5)((b_H - t - f)(1-\lambda) - (1+\lambda) c_1)}{t (1-\lambda) c_5} - \frac{(1+\lambda) c_1}{1-\lambda},$$

which is positive whenever

$$f < b_H - t - \frac{(1+\lambda) c_1 (4 t c_1 + (2t - \lambda b_L) c_5)}{(4 t c_1 - \lambda b_L c_5)(1-\lambda)} \equiv x_2.$$

Unfortunately, $x_0 > x_2$, since

$$x_0 - x_2 = \frac{1}{2} \frac{\lambda b_L {c_5}^2 (1+\lambda)}{(4 t c_1 - \lambda b_L c_5)(1-\lambda)} > 0.$$

Thus, there are costs $f$ where multihoming of all the high types arises, but where firm 1's profits are maximized in a region where this is not a rational expectations demand. To rule out such deviations, we need the maximum deviation profit to be less than that of the profit that may be obtained in the candidate equilibrium; that is, $\pi_M^* \geq \pi_{12}(p_{12}^*, p_2^*)$. With the notation above,

$$\pi_M^* = \frac{1}{2} \frac{(1+\lambda)^2 c_1}{1-\lambda},$$

while

$$\pi_{12}(p_{12}^*, p_2^*) = \frac{1}{4} \frac{((1-\lambda)(\lambda b_L f - t c_4) + (1+\lambda)(t + \lambda b_L) c_1 + c_3)^2}{(1-\lambda) t c_3}$$

and $\pi_M^* - \pi_{12}(p_{12}^*, p_2^*)$ is a quadratic (concave) function of $f$. Hence, $\pi_M^* \geq \pi_{12}(p_{12}^*, p_2^*)$ between the two values of $f$ which satisfy $\pi_M^* = \pi_{12}(p_{12}^*, p_2^*)$. Computing both roots yield one negative and one positive root. The relevant root is

$$x_3 = \frac{\sqrt{2tc_3c_1}\,(1+\lambda)}{\lambda\, b_L\,(1-\lambda)} - \frac{(1+\lambda)\,(2t+\lambda\, b_L)\,c_1}{\lambda\, b_L\,(1-\lambda)} + b_H - t.$$

To facilitate further computations, let

$$x_3^1 = \frac{\sqrt{2tc_3c_1}\,(1+\lambda)}{\lambda\, b_L\,(1-\lambda)}$$

and

$$x_3^2 = -\frac{(1+\lambda)\,(2t+\lambda\, b_L)\,c_1}{\lambda\, b_L\,(1-\lambda)} + b_H - t.$$

We can now show that $x_3 > x_2$, since $x_3 - x_2 = x_3^1 + x_3^2 - x_2 = x_3^1 - (x_2 - x_3^2) > 0$, if and only if $(x_3^1)^2 - (x_2 - x_3^2)^2 > 0$ and

$$(x_3^1)^2 - (x_2 - x_3^2)^2 = \frac{2\,tb_H^2 c_3 c_1\,(1+\lambda)^2}{(4tc_1 - \lambda b_L c_5)^2} > 0.$$

Similarly, $x_0 > x_3$, since $x_0 - x_3^1 - x_3^2 = (x_0 - x_3^2) - x_3^1 > 0$, if and only if $(x_0 - x_3^2)^2 - (x_3^1)^2 > 0$ and

$$(x_0 - x_3^2)^2 - (x_3^1)^2 = \frac{1}{4}(1+\lambda)^2 b_H^2 > 0.$$

Thus, we have $x_2 < x_3 < x_0$. As long as $f < x_2 < x_3$, profit of firm 1 is increasing at $p_1 = d_3^*$, and the candidate equilibrium delivers a higher payoff than deviating to $p_1 = d_3^*$. When, $x_2 < f < x_3$, even though the profit function of firm 1 has a local maximum, for $p_1 < d_3^*$, this delivers a lower payoff than the candidate equilibrium. Therefore, as long as $f < x_3 \equiv f_0$, firm 1 has no incentive to deviate to a lower price such that $p_1 < d_3^*$, and our candidate equilibrium prices given in (A-1) constitute a Nash Equilibrium with rational expectations.

Let

$$f_0 = \frac{(1+\lambda)\left(\sqrt{2t\,(t-(1-\lambda)\,b_L)\,c} - (t-(1-\lambda)\,b_L)\,(2\,t+\lambda b_L)\right)}{\lambda\,(1-\lambda)\,b_L} + b_H - t$$

with $c = 2\,t\,(t-(1-\lambda)\,b_L) - \lambda\,(1-\lambda)\,b_L b_H$, and define $\mathcal{P} \equiv \{\lambda, b_L, b_H, f, t \mid b_H > b_L \geq 0, b_H > 2t/\,(1-\lambda)\,, t - \lambda b_H - (1-\lambda) b_L > 0, f \geq 0, f_0 > 0, f \leq f_0\}$. Thus, a sufficient condition for (A-1) to constitute an equilibrium is that the model parameters belong to the set $\mathcal{P}$. Note that since $\mathcal{P}$ is characterized by sufficient conditions, the equilibrium indeed exists for a larger parameter set. ■

# B Multihoming in asymmetric two-sided markets

In this appendix, we extend our model of symmetric two-sided markets in section 3 of Doganoglu and Wright (2005) to the case with asymmetry. We allow the intrinsic utility, marginal costs of production, transportation costs and the distribution of types to differ across the two sides of the market. We consider two groups of agents $A$ and $B$, each of which consists of high and low types, just as in our one-sided model. Suppose there is measure 1 of agents from group $A$ and likewise for group $B$. Consistent with the standard assumption of two-sided markets, it is assumed each group values the number of agents belonging to the other group, but not the number of agents within the same group. In particular, agents get benefits $bn$ of subscribing to a service which allows them to access $n$ agents from the other group. A measure $\lambda_j$ agents from group $j$ are high types (in which case $b = b_H$) and a measure $1 - \lambda_j$ of agents from group $j$ are low types (in which case $b = b_L$). Without loss of generality, assume $\lambda_A \geq \lambda_B$ and $b_H > b_L$ so that agents from group $A$ are more likely (or equally likely) to be high types compared to agents from group $B$. We define the average values of the network benefit parameter $b$ as $\beta_A = \lambda_A b_H + (1 - \lambda_A)\, b_L$ for group $A$ and $\beta_B = \lambda_B b_H + (1 - \lambda_B)\, b_L$ for group $B$, so that $\beta_A \geq \beta_B$. We allow for different transportation costs for each group, and parallel to the Assumption (1) in the main text, we assume $t_A > \beta_A$ and $t_B > \beta_B$. The measure of high types from group $j$ that subscribe to firm $i$ is denoted $s_{ij}$ and the measure of low types from group $j$ that subscribe to firm $i$ is denoted $n_{ij}$.

The net utility of an agent with network benefits parameter $b$ from group $j$ located at $x \in [0, 1]$ when she purchases from firm $i$ is given by

$$U_{ij}\,(x, b, N_{ik}) = v_j - p_{ij} - t_{ij}\,(x) + bN_{ik},$$

for $i = 1, 2$ and $j = A, B$, where $v_j$ is the intrinsic benefit[2] of the service for side $j$ customers, transportation costs, $t_{ij}\,(x)$, are given by $t_j x$ for firm 1 and $t_j(1 - x)$ for firm 2, $N_{ik}$ represents the total number of agents from group $k = A, B \neq j$ that can be reached by subscribing to firm $i$, and $p_{ij}$ is the subscription price set by firm $i$ for group $j$. Consistent with the literature on two-sided markets, firms are assumed to be able to set different prices to the two different groups. When the same agent multihomes, subscribing to both firms, the net utility she gets can be written as

$$U_j(x, b, N_k) = v_j - p_{1j} - p_{2j} - t_j + b,$$

for $j = A, B$. Assume the cost of handling each user in group $A$ is $f_A$ and the cost in group $B$ is $f_B$. Finally, to make results comparable to equivalent one-sided markets, assume it costs each platform $F$ to make itself compatible with the rival's platform for each group, i.e. a total cost of $2F$.

## B.1 The benchmark without multihoming

In the one-sided model, we considered the benchmark case in which multihoming was not allowed. In this setting we showed that having both high and low types did not change standard results. Prices, profits and welfare would be the same as if all agents had the same network benefits $\beta$ (where

[2] We assume that the intrinsic benefits, $v_j$, $j = A, B$, are sufficiently high for both sides of the market that all consumers subscribe to at least one firm.

$\beta = \lambda b_H + (1-\lambda)\, b_L)$. This was true both with and without compatibility. It is straightforward to show the same result holds in the two-sided market setting presented above. As a result, equilibrium prices are given by, $p_A^N = f_A + t_A - \beta_B$ for group $A$, and $p_B^N = f_B + t_B - \beta_A$ for group $B$. The result is identical to that obtained in Armstrong (2005, section 4), who analyzes a two-sided market with two competing incompatible platforms in the same setting, but in which there is only one type of agent in each group. These results, together with the fact $\beta_A \geq \beta_B$, imply $A$ types will be charged more than $B$ types, reflecting that there is a greater fraction of high types in group $A$ who care a lot about connecting with $B$ types than vice-versa. Given users split evenly between the two symmetric platforms, the results also imply the platforms' equilibrium profits are the sum of their profits from two equivalent one-sided markets[3] (and likewise for welfare). It follows the profit of each platform is

$$\pi_N^* = \frac{t_A + t_B - \beta_A - \beta_B}{2} \tag{B-1}$$

and welfare is

$$W_N^* = v_A + v_B + \frac{\beta_A}{2} + \frac{\beta_B}{2} - f_A - f_B - \frac{t_A}{4} - \frac{t_B}{4} - (1-\alpha)\,(t_A + t_B - \beta_A - \beta_B)\,. \tag{B-2}$$

When the platforms are made compatible, the prices will just be the normal Hotelling prices as consumers get the same network benefits regardless of the platform they join. Thus, again, profits and welfare are the sum of profits and welfare from two equivalent one-sided markets, these being

$$\pi_C^* = \frac{t_A + t_B}{2} - 2F \tag{B-3}$$

and

$$W_C^* = v_A + v_B + \beta_A + \beta_B - f_A - f_B - \frac{t_A}{4} - \frac{t_B}{4} - 4F - (1-\alpha)\,(t_A + t_B - 4F)\,. \tag{B-4}$$

From these results, it is clear that the incentive for firms or a social planner to make their services compatible in the absence of multihoming remains unchanged by the move to a two-sided market setting. Firms prefer compatibility if

$$F \leq \frac{\beta_A}{4} + \frac{\beta_B}{4} \equiv \tilde{F}_N^{\Pi}$$

while the social planner only prefers compatibility if

$$F \leq \frac{\beta_A}{4} + \frac{\beta_B}{4} + \frac{\beta_A + \beta_B}{8\alpha} \equiv \tilde{F}_N^W\,.$$

Comparing the two results, we have that $\tilde{F}_N^{\Pi} - \tilde{F}_N^W > (\beta_A + \beta_B)/8\alpha > 0$. Thus, firms will achieve compatibility not only when it is socially optimal to do so but also in some instances

[3] That is, one market with $v = v_A$, $f = f_A$, $t = t_A$, and $\lambda = \lambda_A$, and one market with $v = v_B$, $f = f_B$, $t = t_B$, and $\lambda = \lambda_B$.

when it is socially inferior to incompatibility. Once again, our benchmark model does not exhibit a "compatibility problem". Instead, firms have excessive incentives towards compatibility. Our conclusions regarding the benchmark model from section 2.3 where we looked at the one-sided market case extends naturally to two-sided markets case. As in Proposition 1, we obtain that absent multihoming, there will be an excessive tendency towards compatibility in a two-sided markets setting.

## B.2 Results with multihoming

As in section 2.4, we consider only the case in which, in equilibrium, all high types choose to multihome and all low types choose to singlehome. This implies $s_{ij} = 1$ and $N_{ij} = \lambda_j + (1-\lambda_j)\, n_{ij}$ for $i = 1, 2$ and $j = A, B$. As a result, the proportion of low types that join firm 1 is found by solving

$$
\begin{aligned}
U_{1A}\left(n_{1A}, b_L, N_{1B}\right) &= U_{2A}\left(n_{1A}, b_L, N_{2B}\right) \\
U_{1B}\left(n_{1B}, b_L, N_{1A}\right) &= U_{2B}\left(n_{1B}, b_L, N_{2A}\right).
\end{aligned}
$$

This implies market shares

$$
\begin{aligned}
n_{1A} &= \frac{1}{2} + \frac{t_B\left(p_{2A} - p_{1A}\right) + \left(1-\lambda_B\right) b_L \left(p_{2B} - p_{1B}\right)}{2\left(t_A t_B - \left(1-\lambda_A\right)\left(1-\lambda_B\right) b_L^2\right)} \\
n_{1B} &= \frac{1}{2} + \frac{t_A\left(p_{2B} - p_{1B}\right) + \left(1-\lambda_A\right) b_L \left(p_{2A} - p_{1A}\right)}{2\left(t_A t_B - \left(1-\lambda_A\right)\left(1-\lambda_B\right) b_L^2\right)},
\end{aligned}
$$

where $n_{2A} = 1 - n_{1A}$ and $n_{2B} = 1 - n_{1B}$. Since $t_A t_B > \beta_A \beta_B$, it follows that $t_A t_B > \left(1-\lambda_A\right)\left(1-\lambda_B\right) b_L^2$ and the market share equations are well behaved.

Firm $i$ obtains profits of

$$
\pi_i = \left(p_{iA} - f_A\right)\left(\lambda_A + (1-\lambda_A) n_{iA}\right) + \left(p_{iB} - f_B\right)\left(\lambda_B + (1-\lambda_B) n_{iB}\right).
$$

Substituting the share functions into the profit function for $i = 1, 2$, and simultaneously solving the resulting first order conditions implies equilibrium prices of

$$
p_A^M = f_A + \frac{\left(1+\lambda_A\right) t_A}{1-\lambda_A} - \left(1+\lambda_B\right) b_L \qquad \text{(B-5)}
$$

$$
p_B^M = f_B + \frac{\left(1+\lambda_B\right) t_B}{1-\lambda_B} - \left(1+\lambda_A\right) b_L \qquad \text{(B-6)}
$$

for group $A$ and $B$ respectively. Note these prices are the natural extension of the prices in (A-1) to two-sided markets. As we have derived for the one sided case in Appendix A, there will be some parameter restrictions required for this equilibrium to exist. We assume these restrictions hold, and instead focus on the properties of this equilibrium.[4]

[4] In particular, we require that all high types prefer to multihome and all low types prefer to singlehome. The former is ensured if $b_H$ is sufficiently high, while the latter condition holds given $t_A > b_L$ and $t_B > b_L$. However, one also needs parameter conditions to ensure neither firm wants to lower its price (non-infinitesimally) to one group in order to convince high types from that group to stop multihoming, which would boost demand from the low types from the other group.

One of the central points of interest in two-sided markets is to explain the determinants of the structure of fees across the two groups. Given group $A$ has a higher fraction of high types ($\lambda_A > \lambda_B$), the platforms will charge $A$ types more than $B$ types, other things equal. This was also a property of the equilibrium price structure without multihoming, reflecting the fact that the side which enjoys greater cross-group externalities will be charged more. Here we find this is also the side in which more agents will choose to multihome.

It is straightforward to see that the equilibrium prices on side $A$ are higher with multihoming. On the other hand, the prices charged on side $B$ may be less than the prices charged in the absence of multihoming. Even though the effects of multihoming on the structure of prices is interesting in itself, for our analysis of the compatibility choice of platforms, the relevant quantity is the change in profits. Similar to Proposition 2, we find

**Proposition B-2** *The equilibrium profits are higher when consumers are able to multihome compared to when they are not.*

**Proof.** It is easily checked that

$$\begin{aligned}\pi_M^* - \pi_N^* &= \frac{3}{2}\frac{\lambda_A\left(t_A-(1-\lambda_A)\,b_L\right)}{1-\lambda_A}+\frac{3}{2}\frac{\lambda_B\left(t_B-(1-\lambda_B)\,b_L\right)}{1-\lambda_B}\\ &\quad+\frac{1}{2}\frac{\lambda_A^2 t_A}{1-\lambda_A}+\frac{1}{2}\frac{\lambda_B^2 t_B}{1-\lambda_B}\\ &\quad+\frac{1}{2}\lambda_B\left(b_H-b_L\lambda_A\right)+\frac{1}{2}\lambda_A\left(b_H-b_L\lambda_B\right)>0.\end{aligned}$$

■

Equilibrium welfare after simplifying terms can be written as

$$\begin{aligned}W_M^* &= v_A+v_B-(1+\lambda_A)\,f_A-(1+\lambda_B)\,f_B\\ &\quad+(\lambda_A+\lambda_B)\,b_H+(1-\lambda_A\lambda_B)\,b_L-\frac{(1+3\lambda_A)\,t_A}{4}-\frac{(1+3\lambda_B)\,t_B}{4} \qquad \text{(B-7)}\\ &\quad-(1-\alpha)\left(\left(\frac{(1+\lambda_A)\,t_A}{1-\lambda_A}-(1+\lambda_B)\,b_L\right)(1+\lambda_A)+\left(\frac{(1+\lambda_B)\,t_B}{1-\lambda_B}-(1+\lambda_A)\,b_L\right)(1+\lambda_B)\right)\end{aligned}$$

Similar to the case of a one-sided market, we get:

**Proposition B-3** *Assume a multihoming equilibrium exists. Then there exist a $\hat{\hat{\alpha}} \in [0,1]$ such that welfare is higher under multihoming ($W_M \geq W_N$) whenever the social planner values profits sufficiently ($\alpha \geq \hat{\hat{\alpha}}$), and welfare is higher without multihoming ($W_M < W_N$) whenever the social planner puts sufficiently low weight on profits ($\alpha < \hat{\hat{\alpha}}$).*

**Proof.** The first thing to notice is $\frac{d}{d\alpha}W_M(\alpha) - W_N(\alpha) = 2(\pi_M^* - \pi_N^*)$. Hence, due to Proposition

(B-2), the welfare difference is increasing in $\alpha$. For $\alpha = 0$ we have,

$$\begin{aligned}
W_M(0) - W_N(0) \quad = \quad & -\frac{1}{2}\lambda_B b_H - \frac{1}{2}\lambda_A b_H - f_A\lambda_A - f_B\lambda_B \\
& -\frac{7}{2}\frac{\lambda_A\left(t_A - (1-\lambda_A)\,b_L\right)}{1-\lambda_A} - \frac{7}{2}\frac{\lambda_B\left(t_B - (1-\lambda_B)\,b_L\right)}{1-\lambda_B} \\
& -\frac{1}{4}\frac{\lambda_A\left(t_A - \lambda_B(1-\lambda_A)\,b_L\right)}{1-\lambda_A} - \frac{1}{4}\frac{\lambda_B\left(t_B - \lambda_A(1-\lambda_B)\,b_L\right)}{1-\lambda_B} \\
& -\frac{1}{4}\frac{{\lambda_A}^2\left(t_A - (1-\lambda_A)\,b_L\right)}{1-\lambda_A} - \frac{1}{4}\frac{{\lambda_B}^2\left(t_B - (1-\lambda_B)\,b_L\right)}{1-\lambda_B} - \frac{1}{4}b_L\left(-\lambda_A + \lambda_B\right)^2 < 0,
\end{aligned}$$

which implies $W_M(0) < W_N(0)$. For $\alpha = 1$,

$$W_M(1) - W_N(1) = -\lambda_A f_A - \lambda_B f_B - \frac{3}{4}t_A - \frac{3}{4}t_B + \frac{1}{2}(\lambda_A + \lambda_B)b_H + \frac{1}{2}b_L(\lambda_A + \lambda_B - \lambda_A\lambda_B).$$

Hence, $W_M(1) > W_N(1)$ whenever

$$\lambda_A f_A + \lambda_B f_B \le -\frac{3}{4}t_A - \frac{3}{4}t_B + \frac{1}{2}(\lambda_A + \lambda_B)b_H + \frac{1}{2}b_L(\lambda_A + \lambda_B - \lambda_A\lambda_B) = \Upsilon_1$$

Notice that A-side high type user will multihome whenever

$$f_A \le -2\frac{t_A}{1-\lambda_A} + (1+\lambda_B)\,b_L + \frac{1}{2}b_H\,(1-\lambda_B)$$

and B-side high type user will multihome whenever

$$f_B \le -2\frac{t_B}{1-\lambda_B} + (\lambda_A+1)\,b_L + \frac{1}{2}b_H\,(1-\lambda_A)$$

or, combining both inequalities, whenever

$$\begin{aligned}
\lambda_A f_A + \lambda_B f_B \quad \le \quad & \lambda_A\Big(-2\frac{t_A}{1-\lambda_A} + (1+\lambda_B)\,b_L + \frac{1}{2}b_H\,(1-\lambda_B)\Big) \\
& +\lambda_B\Big(-2\frac{t_B}{1-\lambda_B} + (\lambda_A+1)\,b_L + \frac{1}{2}b_H\,(1-\lambda_A)\Big) \equiv \Upsilon_2
\end{aligned}$$

If we can show $\Upsilon_1 > \Upsilon_2$, then we will have the desired result.

$$\begin{aligned}
\Upsilon_1 - \Upsilon_2 \quad = \quad & \frac{1}{4}\frac{\lambda_A\left(5\,t_A + 3\,\lambda_A t_A - 4\,b_L\lambda_B + 4\,b_L\lambda_B\lambda_A - 2\,b_L + 2\,b_L\lambda_A\right)}{1-\lambda_A} \\
& +\frac{1}{4}\frac{\lambda_B\left(5\,t_B + 3\,\lambda_B t_B - 4\,b_L\lambda_A + 4\,b_L\lambda_B\lambda_A - 2\,b_L + 2\,b_L\lambda_B\right)}{1-\lambda_B} \\
& +\lambda_A\lambda_B\left(b_H - b_L\right) \\
> \quad & \frac{1}{4}\frac{\lambda_B\left(5\,t_A + 3\,\lambda_A t_A - 4\,b_L\lambda_B + 4\,b_L\lambda_B\lambda_A - 2\,b_L + 2\,b_L\lambda_A\right)}{1-\lambda_B} \\
& +\frac{1}{4}\frac{\lambda_B\left(5\,t_B + 3\,\lambda_B t_B - 4\,b_L\lambda_A + 4\,b_L\lambda_B\lambda_A - 2\,b_L + 2\,b_L\lambda_B\right)}{1-\lambda_B} \\
& +\lambda_A\lambda_B\left(b_H - b_L\right) \\
= \quad & \frac{1}{4}\frac{\lambda_B\left(5\,t_A + 3\,\lambda_A t_A - 2\,b_L\lambda_B + 8\,b_L\lambda_B\lambda_A - 4\,b_L - 2\,b_L\lambda_A + 5\,t_B + 3\,\lambda_B t_B\right)}{1-\lambda_B} \\
& +\lambda_A\lambda_B\left(b_H - b_L\right) > 0.
\end{aligned}$$

The first inequality is due to the fact that $\lambda/(1-\lambda)$ is increasing in $\lambda$. The final inequality follows from $3\lambda_A t_A > 2\lambda_A b_L$, $3\lambda_B t_B > 2\lambda_B b_L$, and $5t_A + 5t_B > 4b_L$.

Given that $W_M(0) - W_N(0) < 0$, $W_M(1) - W_N(1) > 0$ and $W_M(\alpha) - W_N(\alpha)$ increasing in $\alpha$, there must be an $\hat{\hat{\alpha}}$ which satisfies $W_M(\hat{\hat{\alpha}}) = W_N(\hat{\hat{\alpha}})$. Thus, for $\alpha \geq \hat{\hat{\alpha}}$, we have $W_M \geq W_N$ and, for $\alpha < \hat{\hat{\alpha}}$, we have $W_M < W_N$. ■

## B.3 Multihoming and compatibility in two-sided markets

Similar to the one-sided case, given that firms can increase their profits with multihoming, we expect their excessive incentives towards compatibility to reduce. Firms will make their networks compatible when $\pi_M^* \geq \pi_C^*$, which in turn implies that compatibility is privately beneficial only if

$$F \leq -\frac{1}{4}\frac{\lambda_A(3+\lambda_A)t_A}{1-\lambda_A} - \frac{1}{4}\frac{\lambda_B(3+\lambda_B)t_B}{1-\lambda_B} + \frac{1}{2}b_L(1+\lambda_A)(1+\lambda_B) \equiv \tilde{F}_M^{\Pi}.$$

Once again for small values of $b_L$ firms will never achieve compatibility, even if it is costless. In this case, firms also have no incentive to support exclusivity.

On the other hand, the social planner will compare $W_C^*$ with $W_M^*$ in making a decision regarding compatibility. It is straightforward to see that the social planner will impose compatibility if

$$\begin{aligned} F \leq \; & \frac{1}{4\alpha}\Bigg[(1-\lambda_A)(1-\lambda_B)b_L + \lambda_A f_A + \lambda_B f_B + \frac{3}{4}\lambda_A t_A + \frac{3}{4}\lambda_B t_B \\ & +2(1-\alpha)\left(\frac{1}{2}\frac{\lambda_A(\lambda_A+3)t_A}{1-\lambda_A} + \frac{1}{2}\frac{\lambda_B(3+\lambda_B)t_B}{1-\lambda_B} - b_L(\lambda_A+1)(1+\lambda_B)\right)\Bigg] \equiv \tilde{F}_M^W. \end{aligned}$$

**Proposition B-4** *There exists a $\bar{\bar{b}}_L$ such that (i) whenever $b_L < \bar{\bar{b}}_L$, firms have an insufficient incentive to choose compatibility (that is, $\tilde{F}_M^{\Pi} < \tilde{F}_M^W$ ). (ii) When $b_L > \bar{\bar{b}}_L$, firms have an excessive incentive to choose compatibility (that is, $\tilde{F}_M^W < \tilde{F}_M^{\Pi}$), and (iii) if in addition $\alpha < \hat{\hat{\alpha}}$ defined in Proposition B-3, this excessive incentive of firms to choose compatibility is reduced as a result of the ability of consumers to multihome (that is, $\tilde{F}_N^W < \tilde{F}_M^W < \tilde{F}_M^{\Pi} < \tilde{F}_N^{\Pi}$).*

**Proof.** We have

$$\begin{aligned} \tilde{F}_M^{\Pi} - \tilde{F}_M^W = \; & \frac{1}{4}\frac{b_L(1+\lambda_A\lambda_B+3(\lambda_A+\lambda_A))}{\alpha} \\ & +\frac{1}{16}\frac{(\lambda_A+15)\lambda_A t_A}{\alpha(-1+\lambda_A)} + \frac{1}{16}\frac{(15+\lambda_B)\lambda_B t_B}{(-1+\lambda_B)\alpha} \\ & -\frac{1}{4}\frac{\lambda_A f_A}{\alpha} - \frac{1}{4}\frac{\lambda_B f_B}{\alpha} \end{aligned}$$

which is negative whenever

$$b_L \leq \frac{1}{1+\lambda_A\lambda_B+3(\lambda_A+\lambda_A)}\left[\frac{1}{4}\frac{(\lambda_A+15)\lambda_A t_A}{1-\lambda_A} + \frac{1}{4}\frac{(15+\lambda_B)\lambda_B t_B}{1-\lambda_B} + \lambda_A f_A + \lambda_B f_B\right] \equiv \bar{\bar{b}}_L,$$

since $\tilde{F}_M^{\Pi} - \tilde{F}_M^W$ is increasing in $b_L$. Thus, $\tilde{F}_M^{\Pi} < \tilde{F}_M^W$ if and only if $b_L \leq \bar{\bar{b}}_L$, proving the results in (i) and (ii).

On the other hand, for $b_L > \bar{\bar{b}}_L$, we have $\tilde{F}_M^W < \tilde{F}_M^{\Pi}$, while $\alpha < \hat{\hat{\alpha}}$ implies $W_M < W_N$ from Proposition B-3, so that $\tilde{F}_N^W < \tilde{F}_M^W$. From Proposition B-2 we have $\pi_M^* > \pi_N^*$, so that $\tilde{F}_M^{\Pi} < \tilde{F}_N^{\Pi}$.

Combining these results gives $\tilde{F}_N^W < \tilde{F}_M^W < \tilde{F}_M^\Pi < \tilde{F}_N^\Pi$, which implies a reduction in the excessive tendency towards compatibility proving (iii) in the proposition. ■

We can once again show that $\tilde{F}_M^\Pi - \tilde{F}_M^W < \tilde{F}_N^\Pi - \tilde{F}_N^W$ independent of the parameter values. Moreover, we have that $0 < \tilde{F}_M^\Pi - \tilde{F}_M^W < \tilde{F}_N^\Pi - \tilde{F}_N^W$ whenever $b_L > \bar{\bar{b}}_L$. Hence, similar to the one-sided case, the range of fixed costs where social and private incentives for achieving compatibility do not agree becomes smaller in the presence of multihoming. We thus have:

**Proposition B-5** *Whenever $b_L > \bar{\bar{b}}_L$, the ability of consumers to multihome means there is a smaller range of fixed costs of achieving compatibility for which firms have an excessive incentive towards compatibility.*

**Proof.** We know that for $b_L > \bar{\bar{b}}_L$, we have $\tilde{F}_M^\Pi > \tilde{F}_M^W$ from Proposition B-4. We also know $\tilde{F}_N^\Pi \geq \tilde{F}_N^W$ from our analysis on the benchmark model. Unfortunately, whenever $\alpha > \hat{\alpha}$, $[\tilde{F}_M^W, \tilde{F}_M^\Pi]$ is not a subset of $[\tilde{F}_N^W, \tilde{F}_N^\Pi]$. Nevertheless, regardless of parameter values, it is straightforward to verify that

$$\begin{aligned}
\tilde{F}_N^\Pi - \tilde{F}_N^W - \tilde{F}_M^\Pi + \tilde{F}_N^W \quad = \quad & \frac{1}{16}\frac{(\lambda_A + 1)\lambda_A t_A}{\alpha(1-\lambda_A)} + \frac{1}{16}\frac{(1+\lambda_B)\lambda_B t_B}{(1-\lambda_B)\alpha} + \frac{1}{4}\frac{\lambda_A f_A}{\alpha} + \frac{1}{4}\frac{\lambda_B f_B}{\alpha} \\
& + \frac{1}{8}\frac{\lambda_A(b_H - b_L\lambda_B)}{\alpha} + \frac{1}{8}\frac{\lambda_B(b_H - b_L\lambda_A)}{\alpha} \\
& + \frac{7}{8}\frac{\lambda_A(t_A - (1-\lambda_A)b_L)}{(1-\lambda_A)\alpha} + \frac{7}{8}\frac{\lambda_B(t_B - (1-\lambda_B)b_L)}{(1-\lambda_B)\alpha} \quad > \quad 0.
\end{aligned}$$

Thus the range of fixed costs where social and private incentives diverge is smaller with multihoming. ■

# References


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