Supplementary Appendix
for
Exclusive dealing with network effects

Toker Doganoglu[†]
Julian Wright [‡]

[†]Department of Business and Economics, University of Southern Denmark, Odense, Denmark and Faculty of Arts and Sciences, Sabanci University, Turkey.
[‡]Contact author. Department of Economics, National University of Singapore: e-mail: jwright@nus.edu.sg.

Supplementary Appendix

Introduction

In this supplementary appendix, we provide the full detailed analysis referred to in our paper titled "Exclusive dealing with network effects". We also present a few variations of our models to illustrate the robustness of our results to changes in the model assumptions. Section S-1 presents a formal statement of the proof of lemma 1. The proof includes the derivation of the equilibrium choices of optimal coordinators. In section S-2, we present a detailed description of how the incumbent can use fully discriminatory offers to block entry by a more efficient entrant. In Section S-3, we investigate and confirm the robustness of our main results in the one-sided model when the asymmetry between the firms arises due to differences in stand alone benefits rather than marginal network benefits. In section S-4, we consider a two sided model where one side cannot multihome and investigate the role of exclusive contracts in this context.

S–1 Detailed proof of lemma 1

Provided $p_I \leq \min\{\beta + p_E, v + \beta\}$, there is a consistent demand configuration, denoted configuration $\mathcal{I}$, in which all unattached consumers buy from I in stage 2. Likewise, provided $p_E \leq \min\{(\alpha+\beta)(1-n_1) - \beta n_1 + p_I, v + (\alpha+\beta)(1-n_1)\}$, there is a consistent demand configuration in which all unattached consumers buy from E, denoted configuration $\mathcal{E}$. Subject to prices being in the range where both configurations $\mathcal{I}$ and $\mathcal{E}$ apply, optimal coordination by consumers implies they will buy from I in stage 2 if $p_I < p_E + \beta n_1 - \alpha(1-n_1)$, will buy from E if $p_I > p_E + \beta n_1 - \alpha(1-n_1)$, and are indifferent if $p_I = p_E + \beta n_1 - \alpha(1-n_1)$. Provided that $n_1 > \bar{n}_1$, I can charge a price at least as high as E and still get all the consumers. As competition forces E's price to cost, the equilibrium is obtained when $p_I = c + \beta n_1 - \alpha(1-n_1) > c$ and $p_E = c$, with all consumers buying from I. In contrast, whenever $n_1 < \bar{n}_1$, E can charge a price that is higher than I's and still serve all the free consumers. In this case, equilibrium is established when $p_I = c$ and $p_E = c + \alpha(1-n_1) - \beta n_1$ with all free consumers buying from E. Finally, if $n_1 = \bar{n}_1$, then both firms will compete price down to cost, with all consumers buying from E.

S–2 Fully discriminatory sequential introductory offers

To demonstrate how powerful full price discrimination can be, consider the extreme case where the incumbent can make sequential offers to consumers in the first stage, with each offer in the sequence depending on the uptake of previous offers. That is, suppose the incumbent can order consumers and make a sequence of offers, one to each consumer in turn in stage 1. For simplicity, let us assume that the second stage competition is modeled as before. The first consumer that is made an offer knows that if she rejects, I has a feasible strategy to convince all the remaining consumers to accept its offer. Thus, the first consumer is willing to pay almost $c + \beta$ to sign. In general, assume that the incumbent orders the consumers by a variable $t \in [0,1]$ in an arbitrary way and consider the pricing function of the incumbent for the consumer at t, when y preceding consumers have already accepted the deal that is given by $p_I(t,y) = \max[c, c + \beta(1+y-t) - (\alpha + \beta)(t-y)]$. It can be verified that this ensures all consumers agree to sign with the incumbent and gives it the same profits as if expectations favored it, β.

S–3 The multihoming model with $v > c > 0$

We first consider the case where exclusive offers are not allowed in either stage. We proceed by assuming that $n_1 > 0$ consumers have accepted a non-exclusive offer from I in stage 1 and characterize the demand as a function of both prices (which are restricted to be non-negative as a negative price will imply a firm makes a loss in stage 2). Given that both firms' offers are non-exclusive, there are five different consistent demand configurations in the second stage: (i) unattached buy from I and attached do *not* buy from E (configuration $\mathcal{I}$) when $p_I \leq \min(\beta + p_E, v + \beta)$; (ii) unattached buy from E and attached also buy from E (configuration $\mathcal{E}$) when $p_E \leq \alpha + \beta(1 - n_1)$; (iii) unattached multihome and attached do *not* buy from E when $p_I \leq \beta n_1$ and $p_E = \alpha(1 - n_1)$; (iv) unattached buy from E and attached do *not* buy from E when $\beta n_1 \leq p_I$ and $(\alpha + \beta)(1 - n_1) \leq p_E \leq (\alpha + \beta)(1 - n_1) + \min(v, p_I - \beta n_1)$; and (v) no one buys from either firm in stage 2 (configuration $\varnothing$) when $p_I > v + \beta n_1$ and $p_E > v$.

The configuration in (ii) yields a higher joint surplus than configuration (iii) or (iv) when they are simultaneously equilibrium demand configurations. Similarly, configuration (i) yields a higher joint surplus than configuration (iv) when they represent consistent demand configurations. There are parameters where configuration (iv) is the unique consistent demand configuration, namely whenever $p_I > v + \beta n_1$ but $\alpha + \beta(1 - n_1) < p_E < v$. In this case, I does not make any sales and obtains a zero profit in the second stage. It is easy to see that this cannot be part of an equilibrium as by reducing its price to $p_I = \beta n_1$, I can induce configuration $\mathcal{I}$. Both configurations $\mathcal{I}$ and $\mathcal{E}$ remain when $p_I \leq \min(\beta + p_E, v + \beta)$ and $p_E \leq \alpha + \beta(1 - n_1)$. The expectation rule implies configuration $\mathcal{E}$ arises if $p_E \leq \alpha + (1 - n_1)p_I$ and otherwise configuration $\mathcal{I}$ arises. If $p_I > \beta$ and $p_E > \alpha + \beta(1 - n_1)$, then configuration $\varnothing$ is the only consistent demand configuration. This defines demand uniquely for any given set of prices.

An important point to note is that even when I sells an introductory offer to all consumers in stage 1, by charging $p_E = \alpha$ the entrant can make all the second stage sales implying a comparative advantage for E regardless of the value of n_1, provided $\alpha \geq c$. Thus, whenever $\alpha \geq c$, any equilibrium must involve configuration $\mathcal{E}$ being played. In this case, I will charge $p_I = c$, and as a best response E can make sales to both attached and unattached by charging $p_E = \alpha + (1 - n_1)c$. As long as, $\alpha + (1 - n_1)c \geq c$, E makes positive profits, and these prices can be sustained in an equilibrium. This requires that $\bar{\bar{n}}_1 \leq \frac{\alpha}{c}$ which is trivially satisfied for $\alpha \geq c$.

When $\alpha < c$, by selling a non-exclusive offer to sufficiently many consumers in the first stage, I can ensure that it makes all the second stage sales. That is by selecting $n_1 > \bar{\bar{n}}_1$, I can make all the sales in the second stage to $1 - n_1$ consumers by charging $p_I = \frac{c-\alpha}{1-n_1}$ while E charges $p_E = c$. In this case the highest price that I can charge for its non-exclusive offer in the first stage is c, as consumers by rejecting the offer of I and buying from E in the second stage can guarantee a surplus of $v + \beta - c$. Thus, the total profit of I is obtained only through the second stage sales and is given by $\left(\frac{\alpha-c}{1-n_1} - c\right)(1 - n_1)$ which is an increasing function of n_1. Notice that, from the conditions of configuration $\mathcal{I}$, the highest price I can charge in the second stage and still make sales is given by $c + \beta < v + \beta$ when $p_E = c$. Thus, I will select n_1 such that $\frac{\alpha-c}{1-n_1} = \beta + c$ which in turn implies that the optimal choice of I is given by $\bar{\bar{n}}_1^* = \frac{\alpha+\beta}{c+\beta}$ (and is less than one whenever $\alpha < c$).

We can thus conclude that whenever $\alpha \geq c$, the results we have found in the paper remain the same, thereby extending the proposition 2 in the main paper. On the other hand, whenever $\alpha < c$, the multihoming equilibrium ceases to exist and the incumbent can block the sales of a more efficient entrant by selling to sufficiently many consumers in the first stage (i.e., $\bar{\bar{n}}_1^*$) even when these offers are non-exclusive.

Proposition S-1 *When neither firm can offer exclusive deals at either stage, multihoming is possible and $\alpha \geq c$, then no one buys from the incumbent in stage 1; in stage 2 they all buy, but only from the entrant. This is true even if the incumbent can limit the number of consumers who can obtain its first stage offers. The outcome is efficient. On the other hand, whenever $\alpha < c$, the incumbent will sell nonexclusively to $\bar{\bar{n}}_1^* = \frac{\alpha+\beta}{c+\beta}$ consumers in the first stage, and make all the sales to the remaining consumers in the second stage at a price of $c+\beta$. The entrant cannot make any sales and the outcome is inefficient.*

As can be seen from the proposition above, for sufficiently low values of the marginal cost c, having the possibility of making introductory offers is not sufficient for the incumbent to profitably block a more efficient from making sales. However, for larger values of c, the incumbent can block sales to a more efficient entrant profitably, by selling introductory deals to sufficiently many consumers in the first stage.

Next consider the model where exclusive deals are allowed. Since lemma 1 and proposition 1 in the main paper are formulated with $v > c > 0$, they continue to apply. This, in turn, implies that when exclusive deals are allowed there is a subgame perfect equilibrium where both firms use exclusive deals in either stage, and the incumbent makes all the sales. That is, the singlehoming equilibrium we presented in proposition 3 of the main paper continue to exist.

Now, let us investigate the second stage competition between the firms assuming that n_1 consumers accepted an exclusive offer from the incumbent in the first stage. Moreover, suppose that both firms use nonexclusive offers in the second stage. In this case, since only unattached consumers have a choice to make in stage 2, there are only four possible consistent demand configurations: unattached consumers multihome (configuration $\mathcal{M}$) if $p_I \leq \beta n_1$ and $p_E \leq \alpha(1 - n_1)$; unattached consumers buy from I (configuration $\mathcal{I}$) if $p_I \leq \min(p_E + \beta, v + \beta)$ and $p_E \geq 0$; unattached consumers buy from E (configuration $\mathcal{E}$) if $p_I \geq \beta n_1$ and $p_E \leq (\alpha + \beta)(1 - n_1) + \min(p_I - \beta n_1, v)$; and finally, unattached consumers do not buy from either firm (configuration $\emptyset$) when $p_I > v + \beta n_1$ and $p_E > v$.

Whenever multihoming is an equilibrium, it also yields the highest joint surplus, and thus will be selected by optimally coordinating consumers—the unattached consumers, in this case. On the other hand, whenever multihoming is not an equilibrium and both configurations $\mathcal{I}$ and $\mathcal{E}$ are consistent demand configurations, optimal coordinators buy from E whenever $p_E \leq (\alpha + \beta)(1 - n_1) - \beta + p_I$, and I otherwise. The selected configurations define demand uniquely for given prices.

Given these demands, it is straightforward to see that the unique equilibrium with non-exclusive offers in the second stage arises when $p_I = \beta n_1$ and $p_E = \alpha(1 - n_1)$, provided both these quantities are larger than the marginal cost, c. At these prices, a price cut does not increase the demand faced by a firm, while a price increase implies the loss of all demand. For any pair of prices in the multihoming region a firm would find it profitable to increase its price to the equilibrium level. For prices outside the multihoming region, one of the firms always would reduce its price until it moves inside the multihoming region or obtains all the demand from the unattached consumers.

As mentioned above, for this equilibrium to exist, we need both prices to exceed the marginal cost. It is clear that for some values of n_1 this would not be the case. For the incumbent's price to be above the marginal cost level, we need $n_1 \geq \frac{c}{\beta} = n_1^L$ while for the entrant's price to be higher than marginal cost we need $n_1 \leq 1 - \frac{c}{\alpha} = n_1^U$. Thus, for the firms to obtain positive profits under multihoming, it is necessary that $n_1^L \leq n_1^U$, or equivalently $c \leq \frac{\alpha\beta}{\alpha+\beta}$. Note that, whenever $c \leq \frac{\alpha\beta}{\alpha+\beta}$, $n_1^L \leq \bar{n}_1 = \frac{\alpha}{\alpha+\beta} \leq n_1^U$. Thus for $n_1 > n_1^U$, the incumbent will make all the sales at a price $p_I = c + \beta - (\alpha+\beta)(1-n_1)$ while the entrant charges $p_E = c$, regardless of these prices being exclusive or non-exclusive. On the other hand, for $n_1 < n_1^L$, the entrant will make all the sales at a price $p_E = c + (\alpha+\beta)(1-n_1) - \beta$ while the incumbent charges $p_I = c$, regardless of being exclusive or nonexclusive. Therefore, to make sales at all in the second stage the incumbent must select $n_1 > n_1^L$ in the first period. We can write the profit function of the incumbent as a function of n_1 when $c \leq \frac{\alpha\beta}{\alpha+\beta}$ as follows

$$\pi_I(n_1) = \begin{cases} 0, & n_1 < n_1^L \\ (\beta n_1 - c)(1-n_1) \equiv f(n_1), & n_1^L \leq n_1 \leq n_1^U \\ (\beta - (\alpha+\beta)(1-n_1))(1-n_1) \equiv g(n_1), & n_1^U < n_1 \leq 1. \end{cases}$$

It is easy to verify that $f(n_1)$ obtains its unconstrained maximum at $\hat{\hat{n}}_1 = \frac{1}{2} + \frac{c}{2\beta}$ and $g(n_1)$ obtains its unconstrained maximum at $n_1^* = \frac{1}{2} + \frac{\alpha}{2(\alpha+\beta)}$. Whenever, $c \leq \frac{\alpha\beta}{2(\alpha+\beta)} \equiv c_1$, it is easy to verify that $n_1^* \leq n_1^U$, and $g(n_1)$ is decreasing in n_1 at $n_1 = n_1^U$. Moreover, for $c \leq c_1$, $\hat{\hat{n}}_1 \leq n_1^U$, thus $\pi_I(n_1)$ is maximized at $\hat{\hat{n}}_1$. For $c_1 < c < \frac{\alpha\beta}{2\beta+\alpha}$, $\hat{\hat{n}}_1 < n_1^U$ and $n_1^* > n_1^U$, hence the profit maximizing choice of the incumbent can be found by comparing $f(\hat{\hat{n}}_1)$ and $g(n_1^*)$. Straightforward but cumbersome algebra confirms that $f(\hat{\hat{n}}_1) \geq g(n_1^*)$ whenever $c \leq \beta\big(1 - \sqrt{\frac{\beta}{\alpha+\beta}}\big) \equiv c_2$. Note that $c_1 \leq c_2 \leq \frac{\alpha\beta}{2\beta+\alpha} \equiv c_3$. For $c_2 < c \leq c_3$, we have $f(\hat{\hat{n}}_1) \leq g(n_1^*)$, thus $\pi_I(n_1)$ is maximized at n_1^*. Furthermore, $f(n_1)$ is increasing at $n_1 = n_1^U$ and $n_1^* \geq n_1^U$ whenever $c \geq c_3$. Thus, for $c \geq c_3$, the incumbent's profit, $\pi_I(n_1)$, is maximized at n_1^*. The firms may deviate by switching to exclusive prices when the rival is charging a non-exclusive price. Whenever the first stage choice of the incumbent is n_1^* and $c > c_2$, then regardless of the nature of the prices, the second stage equilibrium involves singlehoming. Hence, no firm can profit by switching to a exclusive price. When the first stage choice of the incumbent is $\hat{\hat{n}}_1$, neither firm can increase its profit by switching to an exclusive price as the highest price the incumbent can charge exclusively is $p_I = \beta\hat{\hat{n}}_1$, while the highest exclusive price the entrant can charge is $p_E = \alpha(1-\hat{\hat{n}}_1)$.

Let us summarize the results above. Suppose $c \leq \frac{\alpha\beta}{\alpha+\beta}$. Whenever $c \leq c_2$, the profit of the incumbent is maximized by selling $\hat{\hat{n}}_1 = \frac{1}{2} + \frac{c}{2\beta}$ consumers an exclusive deal in the first stage, and allowing the remaining $1 - \hat{\hat{n}}_1$ consumers to multihome by paying $p_I = \beta\hat{\hat{n}}_1$ and $p_E = \alpha(1-\hat{\hat{n}}_1)$. When $c_2 < c \leq \frac{\alpha\beta}{\alpha+\beta}$, the profit of the incumbent is maximized by selling $n_1^* = \frac{1}{2} + \frac{\alpha}{2(\alpha+\beta)}$ consumers an exclusive deal in the first stage, and then the incumbent sells to all the remaining consumers charging a price $p_I = c + \frac{\beta}{2}$.

Since depending on the value of the marginal cost, c, the cases that need to be considered are numerous, we will restrict our attention to the case where $c < \frac{\alpha\beta}{\alpha+\beta}$. In this case, the incumbent cannot make any sales in the second stage unless it makes its first stage offers exclusive, since according to proposition S-1 all the second stage sales will be made by the entrant if the first stage offers were non-exclusive. Furthermore, $\bar{n}_1 \geq \hat{\hat{n}}_1$ if $c \leq \frac{(\alpha-\beta)\beta}{\alpha+\beta}$. We can thus state the following proposition regarding the subgame perfect equilbria of the full game:

Proposition S-2 *When firms can employ exclusive deals, consumers can multihome, the incumbent can limit the number of consumers who can obtain its first stage offers and* $c \leq \frac{\alpha\beta}{\alpha+\beta}$*, the incumbent will always use exclusive first stage offers.*

There are two possible subgame perfect exclusive dealing equilibria:

a. **Singlehoming equilibrium:** *The incumbent offers an exclusive deal to $n_1^* > \bar{n}_1$ consumers in stage 1. In stage 2 equilibrium prices are $p_I^* = c + \frac{\beta}{2}$ and $p_E^* = c$, with both firms making their offers exclusive. The incumbent sells to all consumers.*

b. **Multihoming equilibrium:** *The incumbent offers an exclusive deal to $\hat{\hat{n}}_1 = \frac{1}{2} + \frac{c}{2\beta}$ of the consumers in stage 1. In stage 2 equilibrium prices are $p_I^* = \beta\hat{\hat{n}}_1$ and $p_E^* = \alpha(1 - \hat{\hat{n}}_1)$, with both firms making their offers non-exclusive. All unattached consumers multihome in stage 2.*

Define $c_2 = \left(1 - \sqrt{\frac{\beta}{\alpha+\beta}}\right)$. Whenever $c > c_2$, the unique subgame perfect equilibrium is the singlehoming equilibrium. Whenever $c \leq c_2$ and $c \leq \frac{(\alpha-\beta)\beta}{\alpha+\beta}$, then the multihoming equilibrium is the unique subgame perfect equilibrium. Finally, whenever $c \leq c_2$ but $c > \frac{(\alpha-\beta)\beta}{\alpha+\beta}$, both singlehoming and multihoming equilibria arise as subgame perfect equilibrium outcomes.

As can be seen from the proposition above, for sufficiently small values of c (namely $c < c_2$) multihoming in the second stage arises as a subgame perfect equilibrium outcome. However, for larger values of the marginal cost, one cannot sustain multihoming in equilibrium, and thus regardless of the nature of the second period prices, the incumbent can make all the second stage sales by selling exclusive deals to sufficiently many consumers in the first stage.

S–4 Proof of robustness with asymmetries in stand-alone benefits

In this subsection, we show that asymmetries in stand-alone benefits yield qualitatively similar results to those in our main paper, where asymmetries were instead in marginal network benefits. Focusing on our one-sided model, we consider a variant of our model where both I and E offer the same marginal network benefits but that E's product provides a higher stand-alone benefit. Specifically, we assume that both firms offer a network benefit given by βx when x consumers subscribe to its network. We assume that I's product delivers a utility only due to network effects, and as a result has a stand-alone benefit normalized to zero. On the other hand, E's product offers a positive stand-alone benefit v_E, which consumers receive if they subscribe to E's network. The rest of the model is assumed to remain the same as in the main text.

Let us first consider the case where the incumbent has made exclusive sales to n_1 consumers in the first stage, and characterize the equilibrium outcome in the second stage competition assuming multihoming is not possible (this also covers the case where at least one firm offers an exclusive price in stage 2). In this case, there are $1 - n_1$ free consumers which either buy from the entrant or the incumbent.

We can then characterize the second stage equilibrium outcome with at least one firm offering an exclusive price as follows.

Lemma S-1 *Define $\tilde{n}_1 = \frac{v_E}{\beta}$. Suppose multihoming is not possible. If $n_1 > \tilde{n}_1$, then the incumbent makes all the sales in stage 2 and the equilibrium prices are given by $p_I = -v_E + \beta n_1$ and $p_E = 0$. If, on the other hand, $n_1 \leq \tilde{n}_1$, then the entrant makes all the sales in stage 2 and equilibrium prices are $p_I = 0$ and $p_E = v_E - \beta n_1$.*

Proof. As long as $\beta - p_I > v_E - p_E$, buying from I is a rational demand configuration. On the other hand, whenever $v_E + \beta(1 - n_1) - p_E > \beta n_1 - p_I$, it is rational for all free consumers to buy from E. Optimal coordinators will buy from E whenever $v_E + \beta(1 - n_1) - p_E \geq \beta - p_I$, or equivalently, $p_E \leq v_E - \beta n_1 + p_I$. This formulation implies that whenever $v_E - \beta n_1 \geq 0$, it is

E that has a competitive advantage (with I's price driven down to cost), while the competitive advantage belongs to I otherwise (with E's price driven down to cost). The prices in the lemma then follow from the Bertrand-like competition. As a result, I can make sales in the second period only when $n_1 > \tilde{n}_1$. ■

Given this result, we can turn our attention to the first stage. Clearly, I will make an introductory/exclusive offer to $n_1 > \tilde{n}_1$ consumers in order to make sales in the second stage. In order for this to be possible, we need the stand-alone surplus of E to be not too large. Namely, it is necessary that $v_E \leq \beta$, since otherwise, whatever the size of the consumer population which purchases exclusively from I in the first stage, all the optimally coordinating free consumers will buy from E in the second stage. Here, we can state a proposition that is parallel to proposition 1 in the main paper.

Proposition S-3 *Assume $v_E < \beta$. If multihoming is not possible and the incumbent can limit the number of consumers buying in stage 1, then $n_1^* = \frac{1}{2} + \frac{v_E}{2\beta}$ consumers will buy from the incumbent in stage 1 at the price $p_1 = 0$, and the rest of the consumers will buy from the incumbent in stage 2 at the price $p_I = \frac{\beta - v_E}{2}$. The entrant makes no sales. The outcome is inefficient.*

Proof. If consumers who are offered an exclusive deal in the first stage reject the offer of I, they will all buy from E and obtain a surplus of β. Buying from I in the first stage exclusively yields a surplus of $\beta - p_1$. Therefore, for I's offer to be accepted, it must be that $p_1 \leq 0$. Hence, I will make sales to $n_1 > \tilde{n}_1$ consumers in the first stage at a zero price, and sell to the remaining consumers in the second stage at a price of $\beta n_X - v_E$. The profit of following this strategy is given by $(\beta n_X - v_E)(1 - n_X)$ and is maximized by

$$n_1^* = \frac{1}{2} + \frac{\tilde{n}_1}{2}$$

which is less than one whenever $\tilde{n}_1 < 1$. Note also that as long as $v_E < \beta$, $n_1^* > \tilde{n}_1$. ■

If both firms employ non-exclusive prices in the second stage, then a possibility for multihoming arises. However, depending on the nature of first period contracts, there are several possible multihoming scenarios. The first one requires I to make introductory but non-exclusive offers to n_1 consumers in the first stage. Let us first characterize the demand as a function of both prices, and then determine the equilibrium prices in stage 2. We restrict our attention to non-negative prices given a negative price will imply a firm makes a loss in stage 2. Following similar arguments as in the proof of proposition 2, one can show that both attached and unattached consumers buy from E if $p_E \leq v_E + (1 - n_1)p_I$ and the unattached consumers buy from I while the attached consumers do not buy otherwise. If $p_I > \beta$ and $p_E > v_E + \beta(1 - n_1)$, then the only consistent demand configuration is the one where neither the attached nor the unattached buy from either firm. This defines demand uniquely for any given set of prices. Given this demand, I charging a price of zero, $p_I = 0$, while E charging its stand-alone benefit advantage, $p_E = v_E$, constitutes a second stage equilibrium outcome. In equilibrium, I makes zero profits while E obtains a profit of v_E. Note that this is an equilibrium outcome regardless of the size of n_1. All consumers obtain a surplus of β.

Now consider consumers deciding whether to buy from I in stage 1. If they do not do so, they know they can get a surplus of β buying from E in stage 2. The best each consumer can do buying from I is if they all do so, which will only make them better off if $p_1 < 0$. However, this implies a loss for I, thus it will not make such an offer and no one will buy from I in either stage. The equilibrium outcome is efficient in this case. Thus, the statement we made in proposition

2 remains true even when the asymmetry between the net benefits offered by I and E is due to differences in stand-alone values.

Proposition S-4 *When neither firm can offer exclusive deals at either stage and multihoming is possible, then no one buys from the incumbent in stage 1; in stage 2 they all buy, but only from the entrant. This is true even if the incumbent can limit the number of consumers who can obtain its first stage offers. The outcome is efficient.*

Given that introductory offers cannot help I, there is a genuine role for exclusive deals when E offers a product that has a higher stand-alone value, just as in the main text where E instead offers a higher marginal network benefit. To explore this, let us suppose that n_1 consumers have accepted an exclusive deal from I in the first stage. In this case, only the free consumers can make a choice in the second stage, while the attached consumers are prohibited to consider any offer from E due to the exclusivity clause. An immediate result is that, by selling to $n_1 > \tilde{n}_1$ consumers, and offering an exclusive price in the second stage, I can make all the sales. Thus, there exists a subgame perfect equilibrium, where I optimally sells through an exclusive deal to $n_1^* = \frac{1}{2} + \frac{\tilde{n}_1}{2}$ consumers, and the remaining $1 - n_1^*$ consumers also buy from I at a price $p_I = \beta n_1^* - v_E = \frac{\beta - v_E}{2} > 0$, whenever $\beta > v_E$ due to lemma S-1.

However, as in the main text, there exists another second stage equilibrium where the free consumers are allowed to multihome. Given that the attached consumers are bound by exclusive deals to I, free consumers in the second stage can buy from I only, E only or multihome. Free consumers multihoming is a consistent demand configuration whenever $p_I \leq \beta n_1$ and $p_E \leq v_E$. Whenever $p_I > \beta n_1$ and $p_E \leq v_E + \beta(1 - n_1)$ there is a consistent demand configuration where the free consumers buy only from E. On the other hand, whenever $p_E > v_E$ and $p_I \leq \beta$, all free consumers buying from I constitutes a consistent demand configuration. Note once again that, whenever multihoming is a consistent demand configuration, optimally coordinating consumers coordinate on this equilibrium. For prices where multihoming ceases to be an equilibrium, consumers coordinate on the equilibrium where they all buy from E whenever

$$p_E \leq v_E - \beta n_X + p_I \tag{S-1}$$

and I otherwise.

Suppose E charges $p_E = v_E$ non-exclusively in stage 2 and consider I's options. I can charge $p_I = \beta n_1$ non-exclusively inducing all consumers to multihome. Clearly, I cannot increase its profits by reducing its price from this level. On the other hand, increasing its price beyond this implies that everybody will just buy from E. Hence, the best non-exclusive response of I against E's non-exclusive price is to charge $p_I = \beta n_1$. Alternatively, I can consider charging an exclusive second period price. I can sell to all free consumers using an exclusive price whenever this price satisfies $p_I < \beta n_1$. Such a deviation yields a profit that is no higher than it obtains with the non-exclusive price $p_I = \beta n_1$. Now assume I charges $p_I = \beta n_1$ non-exclusively and consider E's options. Clearly, by charging a non-exclusive price $p_E \leq v_E$, the entrant can induce all free consumers to multihome, and hence its optimal price in the multihoming region is $p_E = v_E$. If E charges a higher price, then all the free consumers will purchase from I, making such a deviation not profitable. The best exclusive offer E can make is to sell to all free consumers exclusively for $p_E = v_E$. Obviously, E is indifferent between charging a price of $p_E = v_E$ exclusively or non-exclusively. Therefore, the non-exlusive offers with prices $p_I = \beta n_1$ and $p_E = v_E$ is an equilibrium outcome of the second stage competition. All free consumers multihome.

Expecting the second stage equilibrium to be the multihoming equilibrium with non-exclusive prices, consumers facing an exclusive offer in the first stage demand at least a surplus of β, a surplus which they would obtain if they collectively rejected I's offer and bought from E in the second stage. Therefore the first stage offer of I must satisfy $\beta - p_1 \geq \beta$ to be accepted. Consequently, I will make exclusive offers to n_1 consumers at a zero price and sell its subscriptions non-exclusively in the second stage at a price of $p_I = \beta n_1$. The profit of this strategy is $\beta n_1(1-n_1)$, which is maximized whenever $n_1^* = \frac{1}{2}$. Therefore, there exists another subgame perfect equilibrium where I sells using exclusive contracts to half of the consumers in the first stage at a price of zero, and the remaining half of the consumers multihome in the second stage paying $p_I = \frac{\beta}{2}$ to I and $p_E = v_E$ to the entrant.

Whenever v_E, the utility advantage of E, is sufficiently high, namely $v_E > \frac{\beta}{2}$, the multihoming equilibrium is the unique subgame perfect equilibrium. Whenever, $v_E \leq \frac{\beta}{2}$, on the other hand, both subgame perfect equilibria exist. However, the profits of both firms are higher in the multihoming equilibrium. Moreover, a forward induction argument favors the multihoming equilibrium, as in the proof of proposition 3 in the main paper. Apart from a few quantitative changes in the conditions, and expressions determining equilibrium prices, the results of proposition 3 remain qualitatively the same.

Proposition S-5 *When firms can employ exclusive deals, consumers can multihome and the incumbent can limit the number of consumers who can obtain its first stage offers, the incumbent will always use exclusive first stage offers.*

For $v_E \leq \frac{\beta}{2}$, there are two subgame perfect exclusive dealing equilibria:

a. **Singlehoming equilibrium:** *The incumbent offers an exclusive deal to $n_1^* > \tilde{n}_1$ consumers in stage 1. In stage 2 equilibrium prices are $p_I^* = \frac{\beta}{2} - v_E$ and $p_E^* = 0$, with both firms making their offers exclusive. The incumbent sells to all consumers.*

b. **Multihoming equilibrium:** *The incumbent offers an exclusive deal to half of the consumers in stage 1. In stage 2 equilibrium prices are $p_I^* = \frac{\beta}{2}$ and $p_E^* = v_E$, with both firms making their offers non-exclusive. All unattached consumers multihome in stage 2.*

A forward induction argument selects the multihoming equilibrium.

For $\frac{\beta}{2} < v_E \leq \beta$, the multihoming equilibrium described is the unique subgame perfect equilibrium.

S–5 Proof of claim in footnote 18

We suppose buyers cannot multihome and reconsider propositions 4 and 5. First, suppose offers cannot be exclusive. In case no consumers sign in stage 1, then second stage competition is the same as before except that equilibria in which $p_E^S > \alpha_S$ are now also possible since I can no longer bribe buyers to multihome in order to deviate from any such equilibrium. This means $\alpha_B - \beta_S \leq p_E^B \leq \alpha_B$ and $\alpha_S \leq p_E^S \leq \alpha_S + \beta_S$ such that $p_E^B + p_E^S = \alpha_B + \alpha_S$. Thus, buyers can expect surplus of between β_B and $\beta_B + \beta_S$ if they wait until stage 2. Depending on which equilibrium is expected to be played in stage 2, I will make a corresponding introductory offer to them such that $-\beta_S \leq p_1^B \leq 0$ which they will accept since their surplus from accepting is $\beta_B - p_1^B$. I makes a non-negative profit. Thus, in case buyers singlehome proposition 4 changes quite dramatically.

Proposition S-6 *If platforms cannot discriminate amongst users of the same type, cannot make their offers exclusive at either stage and only sellers can multihome, then, in equilibrium, buyers will accept the incumbent's introductory offers in stage 1 and sellers have all their surplus exploited. The outcome is inefficient.*

Now suppose offers can be exclusive at either stage. If I signs up sellers at stage 1, buyers will pay β_B in stage 2. Sellers obtain a surplus of $\beta_S - p_1^S$, while I obtains $\beta_B + p_1^S$. To find p_1^S we determine what the seller can expect if it doesn't sign the exclusive offer. The analysis follows as in the proof of proposition 5. If $\beta_S \geq \beta_B$ then we require p_1^S to be between $-\beta_B$ and 0 depending on which equilibrium is played in stage 2. This gives I a profit of between 0 and β_B. If $\beta_S < \beta_B$ then we require p_1^S to be between $-\beta_S$ and 0 depending on which equilibrium is played in stage 2. This gives I a profit of between 0 and β_S. Depending on the equilibrium selection, I may be better off selling to buyers in stage 1 as in the proposition stated above, or may be better off signing up sellers exclusively in stage 1, as in proposition 5. In either case, consumers will buy from I rather than E, which is inefficient.

Proposition S-7 *If platforms cannot price discriminate amongst agents of the same type, can make exclusive offers at either stage to sellers, and assuming only sellers can multihome, then in equilibrium all consumers will buy from the incumbent's platform. The outcome is inefficient.*