

Naked Exclusion and the Anticompetitive Accommodation of Entry: Two Examples

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Abstract

In this appendix, specific results for the linear demand specification and Hotelling specification are established, corresponding to the paper: “Naked Exclusion and the Anticompetitive Accommodation of Entry”

1 Representative consumer (linear demand)

Consider the standard exogenous differentiation model with linear demands, with consumers’ utility from the two products defined as

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{\beta(q_1^2 + q_2^2 + 2\gamma q_1 q_2)}{2}, \quad (1)$$

where q_i is the amount consumed of good i . This provides a convenient way to parameterize the degree of competition between manufacturers. Specifically, inverse demand functions are given by $p_i = \alpha - \beta(q_i + \gamma q_j)$, where $0 \leq \gamma < 1$ serves as a measure of the degree of differentiation, and $\alpha > c_I$. The higher γ is, the more homogenous are the two products.

When the prices for the two goods are sufficiently close, both manufacturers will enjoy positive demands. However, when the prices for the two goods strongly diverge, the higher priced good will attract no demand, while the lower priced good will attract the entire market demand. Specifically, for any price p_j , when $p_i \geq \alpha(1 - \gamma) + \gamma p_j$, manufacturer i receives no demand, while for $0 < p_i \leq (-\alpha(1 - \gamma) + p_j) / \gamma$, manufacturer i captures the entire market. Thus, firm i ’s demand function is

$$q_i = \frac{\alpha(1 - \gamma) - p_i + \gamma p_j}{\beta(1 - \gamma^2)} \quad \text{if} \quad \frac{-\alpha(1 - \gamma) + p_j}{\gamma} < p_i < \alpha(1 - \gamma) + \gamma p_j \quad (2)$$

$$= \frac{\alpha - p_i}{\beta} \quad \text{if} \quad 0 < p_i \leq \frac{-\alpha(1 - \gamma) + p_j}{\gamma} \quad (3)$$

$$= 0 \quad \text{if} \quad p_i \geq \alpha(1 - \gamma) + \gamma p_j. \quad (4)$$

Now consider the equilibrium profits arising under the different subgames considered in the companion paper.

- If the entrant E does not enter (for instance, since the incumbent I signs up all retailers), I will sell to retailers at the monopoly price, which also becomes the retail price

$$p_M = \frac{\alpha + c}{2},$$

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implying quantity

$$q_M = \frac{\alpha - c}{2\beta}$$

and profit

$$\Pi_M = \frac{(\alpha - c)^2}{4\beta}.$$

Retailers and the entrant make no profit. Clearly none of the firms' profits depend on γ , consistent with A1.

- If E enters, and has two or more retailers to sell through, then we have a normal duopoly pricing problem where the retail price equals the wholesale price given retailers pass-through wholesale prices directly and make no profit. Then duopoly prices are

$$p_D = \frac{(1 - \gamma)\alpha + c}{2 - \gamma},$$

implying quantity

$$q_D = \frac{\alpha - c}{\beta(1 + \gamma)(2 - \gamma)}$$

and profit for each manufacturer of

$$\Pi_D = \frac{(1 - \gamma)(\alpha - c)^2}{\beta(1 + \gamma)(2 - \gamma)^2}.$$

Retailers make no profit. It is straightforward to check that Π_D is decreasing in γ so A2 holds.

- If E enters, and has only one retailer to sell through, then the equilibrium is determined as follows. The retailer selling E 's good sets p_E to maximize

$$(p_E - w_E) \left(\frac{\alpha(1 - \gamma) - p_E + \gamma p_I}{\beta(1 - \gamma^2)} \right)$$

which gives the best response

$$p_E = \frac{(1 - \gamma)\alpha + \gamma p_I + w_E}{2}.$$

E sets its wholesale price taking into account how this affects its demand through the free retailer's pricing of its product, but taking as given the price of I 's good p_I . This implies

$$w_E = \frac{(1 - \gamma)\alpha + \gamma p_I + c}{2}.$$

In contrast, I 's good is priced at $p_I = w_I$ given there are two or more retailers selling it. Given unobservable offers, I then sets its wholesale price taking as given p_E conjectured in equilibrium. This implies

$$p_I = w_I = \frac{(1 - \gamma)\alpha + \gamma p_E + c}{2}.$$

Solving these four first order conditions gives

$$\begin{aligned} w_E &= c + \frac{2(2 - \gamma - \gamma^2)(\alpha - c)}{8 - 3\gamma^2} \\ p_E &= c + \frac{3(1 - \gamma)(2 + \gamma)(\alpha - c)}{8 - 3\gamma^2} \\ p_I &= w_I = c + \frac{(1 - \gamma)(4 + 3\gamma)(\alpha - c)}{8 - 3\gamma^2}, \end{aligned}$$

implying demand for I 's good is

$$q(p_I, p_E) = \frac{(4 + 3\gamma)(\alpha - c)}{\beta(1 + \gamma)(8 - 3\gamma^2)}$$

and demand for E 's good is

$$q(p_E, p_I) = \frac{(2 + \gamma)(\alpha - c)}{\beta(1 + \gamma)(8 - 3\gamma^2)}.$$

Corresponding profits are

$$\Pi_I = \frac{(1 - \gamma)(\alpha - c)^2(4 + 3\gamma)^2}{\beta(1 + \gamma)(8 - 3\gamma^2)^2}$$

for I ,

$$\Pi_E = \frac{2(1 - \gamma)(2 + \gamma)^2(\alpha - c)^2}{\beta(1 + \gamma)(8 - 3\gamma^2)^2}$$

for E , and

$$\pi_f = \frac{(1 - \gamma)(2 + \gamma)^2(\alpha - c)^2}{\beta(1 + \gamma)(8 - 3\gamma^2)^2}$$

for the free retailer. It is straightforward to check that Π_I , Π_E and π_f are decreasing in γ over $[0, 1]$ so that $A2$ holds. Moreover, $\Pi_I > \Pi_D$, so I can always do better accommodating entry by costlessly signing up $N - 1$ retailers than not signing up any retailers.

Now combining the above results, we have

$$\begin{aligned} \Delta(\gamma) &= \Pi_M - N\pi_f - \Pi_I \\ &= \left(\frac{1}{4} - \left((4 + 3\gamma)^2 + (2 + \gamma)^2 N \right) \frac{(1 - \gamma)}{(1 + \gamma)(8 - 3\gamma^2)^2} \right) \frac{(\alpha - c)^2}{\beta}, \end{aligned}$$

so γ_N is solution to $\Delta(\gamma) = 0$ for a given N . This implies, for instance:

$$\begin{aligned} N &= 2 \rightarrow \gamma_N = 0.5861 \\ N &= 3 \rightarrow \gamma_N = 0.6868 \\ N &= 4 \rightarrow \gamma_N = 0.7473 \\ N &= 5 \rightarrow \gamma_N = 0.7878 \\ N &= 10 \rightarrow \gamma_N = 0.8810 \\ N &= 20 \rightarrow \gamma_N = 0.9362 \\ N &= 50 \rightarrow \gamma_N = .9732 \\ N &= 100 \rightarrow \gamma_N = .9864 \end{aligned}$$

Solving this relationship instead for N as a function of γ gives

$$N(\gamma) = \frac{\gamma(32 + 12\gamma - 12\gamma^2 + 9\gamma^3 + 9\gamma^4)}{4(1 - \gamma)(2 + \gamma)^2}.$$

For example, if $\gamma = 0.5$, I will always sign all retailers to foreclose the market; if $\gamma = 0.75$, it will foreclose if there are no more than four retailers, but it will accommodate if there are five or more retailers; while if $\gamma = 0.9$, I will foreclose as long as there are no more than twelve retailers. As the products become homogenous ($\gamma \rightarrow 1$), the critical number of retailers needed to make accommodation the preferred

strategy tends to infinity. I prefers to foreclose the market by signing up all retailers, even if there are large number of them.

The price increases resulting from anticompetitive accommodation relative to when exclusive deals are banned are:

$$\begin{aligned} p_I - p_D &= \frac{2\gamma(1-\gamma)(\alpha-c)}{(2-\gamma)(8-3\gamma^2)} > 0 \\ p_E - p_D &= \frac{4(1-\gamma)(\alpha-c)}{(2-\gamma)(8-3\gamma^2)} > 0 \end{aligned}$$

Note that the price of E 's good is always higher than I 's, and both are above the competitive level p_D .

Finally, consider welfare under the different possible outcomes. Welfare in the benchmark case (i.e. when exclusive deals are banned) equals

$$\begin{aligned} W_D &= U(q_D, q_D) - 2cq_D - F \\ &= \frac{(3-2\gamma)(\alpha-c)^2}{\beta(1+\gamma)(2-\gamma)^2} - F. \end{aligned}$$

Welfare under exclusive dealing and foreclosure equals

$$\begin{aligned} W_M &= U(q_M, 0) - cq_M \\ &= \frac{3(\alpha-c)^2}{8\beta}. \end{aligned}$$

Finally, welfare under anticompetitive accommodation equals

$$\begin{aligned} W_A &= U(q_I, q_E) - c(q_I + q_E) - F \\ &= \frac{(38 + 20\gamma - 21\gamma^2 - 12\gamma^3)(\alpha-c)^2}{\beta(1+\gamma)(8-3\gamma^2)^2} - F. \end{aligned}$$

We have $W_D > W_A$. Moreover, $W_D > W_M$ provided

$$F < \frac{(12 - 16\gamma + 9\gamma^2 - 3\gamma^3)(\alpha-c)^2}{8\beta(1+\gamma)(2-\gamma)^2}. \quad (5)$$

However, for entry to be profitable we require

$$F < \frac{(3-2\gamma)(\alpha-c)^2}{\beta(1+\gamma)(2-\gamma)^2}$$

which ensures the previous constraint on F holds. This implies $W_D > W_M$ whenever entry would arise absent exclusive deals. In other words, welfare is always higher when exclusive dealing is banned in a setting where there is a rival firm that wants to enter.

2 Hotelling model of demand

Consider next the standard Hotelling model of demand, with manufacturers (or their products) located on either end of a unit interval, with a measure one of consumers uniformly located along the unit interval and with linear transport cost of reaching the firms. Consumers only want to buy one unit (from one of the firms). We assume a consumer located at x gets utility $v - tx$ buying product I from the incumbent and $v - t(1-x)$ buying product E from the entrant, where we assume that $v > c + 21t/10$ to

ensure the market is covered under inter-brand competition (this simplifies the analysis and presentation). Consumers' outside option is to get zero utility.

When only product I is available, then I faces a demand equal to

$$q_M = \frac{v - p_I}{t},$$

where t is the transport cost parameter which is inversely related to γ . This specification can be mapped to our general model with some appropriate inverse function, such as $t = -\ln \gamma$. When γ is close to zero, this implies t is large, meaning products are highly differentiated. When γ is close to one, this implies t is close to zero, meaning products are almost homogenous. When both products are available, then demand is given by the usual Hotelling expressions

$$q_I = \frac{1}{2} + \frac{p_E - p_I}{2t}$$

and

$$q_E = \frac{1}{2} + \frac{p_I - p_E}{2t}.$$

Now consider the equilibrium profits arising under the different subgames.

- If E does not enter (for instance, since I signs up all retailers), I will sell to retailers at the monopoly price, which also becomes the retail price. The monopolist will choose to sell to consumers on the left hand segment of the line interval with a standard monopoly wholesale price (and so retail price) equal to

$$p_M = \frac{v + c}{2},$$

implying quantity

$$q_M = \frac{v - c}{2t}$$

and profit

$$\Pi_M = \frac{v^2 - c^2}{4t}.$$

Retailers and E make no profit, while I 's profit is decreasing in t (increasing in γ), consistent with A1.

- If E enters, and has two or more retailers to sell through, then we have a normal duopoly pricing problem where the retail price equals the wholesale price given retailers pass-through wholesale prices directly and make no profit. Then duopoly prices are the normal Hotelling prices

$$p_D = c + t,$$

given market coverage (which is true given $v > c + 3t/2$). Each firm equally shares the market so

$$q_D = \frac{1}{2}$$

and profit for each manufacturer of

$$\Pi_D = \frac{t}{2}.$$

Retailers make no profit. Clearly Π_D is increasing in t and so decreasing in γ , consistent with A2.

- If E enters, and has only one retailer to sell through, then the equilibrium is determined as follows. The retailer selling E 's good sets p_E to maximize

$$(p_E - w_E) \left(\frac{1}{2} + \frac{p_I - p_E}{2t} \right)$$

which gives the best response

$$p_E = \frac{t + p_I + w_E}{2}.$$

E sets its wholesale price taking into account how this affects its demand through the free retailer's pricing of its product. This implies

$$w_E = \frac{t + p_I + c}{2}.$$

Note in contrast $p_I = w_I$ given there are two or more retailers selling I 's good. Given unobservable offers, I then sets its wholesale price taking as given p_E conjectured in equilibrium. This implies

$$p_I = w_I = \frac{t + p_E + c}{2}.$$

Solving these four first order conditions gives

$$\begin{aligned} w_E &= c + \frac{6t}{5} \\ p_E &= c + \frac{9t}{5} \\ p_I &= w_I = c + \frac{7t}{5}, \end{aligned}$$

implying demand for I is

$$q(p_I, p_E) = \frac{7}{10}$$

and demand for E is

$$q(p_E, p_I) = \frac{3}{10}.$$

Note the utility of the marginal consumer is $v - c - 21t/10$, which we have assumed is positive to ensure market coverage under inter-brand competition.¹ Corresponding profits are

$$\Pi_I = \frac{49t}{50}$$

for I ,

$$\Pi_E = \frac{27t}{50}$$

for E , and

$$\pi_f = \frac{9t}{50}$$

for retailers. Clearly Π_I , Π_E and π_f are increasing in t and so decreasing in γ , so that $A2$ holds. Moreover, $\Pi_I > \Pi_D$, so I can always do better accommodating entry by costlessly signing up $N - 1$ retailers.

¹The alternative for I is to set a lower wholesale price so its retailers monopolize the market. This requires setting $p_I = c + 4t/5$, which gives a profit of $4t/5$ compared to $49t/50$ when it does not monopolize the market, thereby ruling out the possibility of such monopolization.

Now combining the above results, we have

$$\begin{aligned}\Delta(t) &= \Pi_M - N\pi_f - \Pi_I \\ &= \frac{v^2 - c^2}{4t} - \frac{9t}{50}N - \frac{49t}{50}.\end{aligned}$$

implying

$$N(t) = \frac{25(v^2 - c^2)}{18t^2} - \frac{49}{9}$$

as the critical number of retail firms above which I prefers to accommodate and below which it prefers to sign all retailers and foreclose the market. Thus, when t is small, I always prefers to foreclose the market unless there are very many retailers, while when t is sufficiently large (but recall $v > c + 21t/10$ so it cannot be too large relative to $v - c$), then I may prefer to accommodate even if there are only a few retailers.

The price increases resulting from anticompetitive accommodation relative to when exclusive deals are banned are:

$$\begin{aligned}p_I - p_D &= \frac{2t}{5} > 0 \\ p_E - p_D &= \frac{4t}{5} > 0\end{aligned}$$

Note that the price of E 's good is always higher than I 's, and both are above the competitive level p_D .

Finally, consider welfare under the different possible outcomes. Welfare effects arise from three sources — higher transport costs when the market is not equally divided between the two firms, fixed costs of the rival's entry and some consumers not purchasing in the monopoly case.

Welfare in the benchmark case (i.e. when exclusive deals are banned) equals

$$W_D = v - c - \frac{t}{4} - F.$$

Welfare under exclusive dealing and foreclosure (i.e. monopoly) equals

$$W_M = \frac{3(v - c)^2}{8t}.$$

Finally, welfare under anticompetitive accommodation equals

$$W_A = v - c - \frac{29t}{100} - F.$$

Clearly, $W_A < W_D$. However, depending on parameter values it is possible for $W_M < W_D$ or $W_M > W_D$. Banning exclusive dealing is not necessary good for welfare in this case, although it is still desirable for all consumers (it lowers prices and decreases transportation costs for some consumers).