Technical Appendix to Exclusive Dealing with Imperfect Downstream Competition: Observable Contracts

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In this appendix, we restate Proposition 4 in Abito and Wright (2006) in the case upstream firms offer observable contracts in their stage 3 competition. As stated in Abito and Wright, results are very similar although the set of values of (γ, θ) that give rise to unique exclusion is somewhat expanded.

Proposition 1 *Suppose the fixed costs of entry are low enough so that the entrant can enter even if just one retailer is available, that upstream firms are restricted to linear wholesale prices and final demand is linear. Provided downstream competition is not too strong ($\gamma < 0.78$), then we have a unique entry equilibrium. Provided downstream competition is sufficiently strong ($\gamma > 0.94$), then we have a unique exclusion equilibrium. For an intermediate level of downstream competition, we have a unique exclusion equilibrium provided the entrant's cost advantage is not too strong ($\theta < 0.68$ is sufficient) and a unique entry equilibrium provided the entrant's cost advantage is sufficiently strong ($\theta > 0.72$ is sufficient), with the critical value of θ decreasing in γ for intermediate cases. Figure 1a illustrates.*

Proof. The analysis for the $S = 0$ and $S = 2$ subgames is identical to that in the case of unobservable contracts. Note that in the $S = 1$ subgame, entry is possible given the assumption on fixed costs in the

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proposition. In this subgame, the best stage 3 offer that I can give the free retailer is c_I. E will always at least match this to attract the free retailer. The question is, in competition with I's signed retailer (retailer 1), will E want to set a lower wholesale price to allow retailer 2 to take a greater share of the market? There are three different regions of γ for which prices and profits are determined differently. These reflect whether I's signed retailer is monopolized by the free retailer or whether the two retailers share the market, and in the latter case, whether the constraint on E's wholesale price caused by competition with I for the free retailer is binding or not.

Let γ_L and γ_H be defined such that for $\gamma < \gamma_L$, E sets its wholesale price offer to the free retailer at c_I and for $\gamma > \gamma_H$, E induces its retailer to monopolize the downstream market. Specifically, to derive these critical values of γ, we need to first derive the unique intersection of best response functions from the respective optimization problems of I and E:

$$\max_{w_1} (w_1 - c_I)\, q_1 \left(p_1 (w_1, w_2), p_2 (w_2, w_1)\right)$$

$$\max_{w_2} (w_2 - c_E)\, q_2 \left(p_2 (w_2, w_1), p_1 (w_1, w_2)\right).$$

where

$$p_i (w_i, w_j) = \frac{(2+\gamma)(1-\gamma)\alpha + 2w_i + gw_j}{4-\gamma^2}$$

is the equilibrium price in the retailing pricing subgame given both retailers are active.[1] The simultaneous solution to the above problem is given by the pair (w_1^*, w_2^*) which denotes the optimal wholesale prices assuming no constraints are active. The critical value γ_L is derived by solving $w_2^* = c_I$ for γ, which implies

$$\gamma_L = -\frac{2-\theta+\sqrt{36-52\theta+17\theta^2}}{16(1-\theta)} + \frac{1}{8(1-\theta)}\sqrt{\frac{276-540\theta+265\theta^2}{2} + \frac{(2-\theta)\left(228-436\theta+209\theta^2\right)}{2\sqrt{\left(36-52\theta+17\theta^2\right)}}},$$

[1] Since stage 3 offers are observed, the upstream firms take into account how each retailers prices will respond to the wholesale price it offers. This is the main change from the unobservable case, where the price of the rival retailer was taken as fixed at its conjectured equilibrium level.

where we have used the definition that $c_I = \theta(\alpha + c_E)/2 + (1-\theta)c_E$. For lower levels of downstream competition, E would like to set a higher wholesale price if not for the constraint imposed by competition with I. Therefore, for $\gamma \leq \gamma_L$, E sets its wholesale price at c_I to the free retailer in equilibrium.

The critical value γ_H is derived by solving $w_1^* = c_I$ for γ, which implies

$$\begin{aligned}\gamma_H &= -\frac{1+\sqrt{9-8\theta+2\theta^2}}{4(2-\theta)}+ \\ &\quad \frac{1}{2(2-\theta)}\sqrt{\frac{69-68\theta+17\theta^2}{2}+\frac{\sqrt{(9-8\theta+2\theta^2)}}{2}}.\end{aligned}$$

Monopolization starts at the value of γ such that even if I charges c_I and its signed retailer follows suit, then if E sets its wholesale price according to market sharing, the free retailer is indifferent between setting a retail price that monopolizes the downstream market and setting a wholesale price that implies market sharing. For higher values of γ, the free retailer will strictly prefer monopolization. For lower values of γ, I can still supply some demand at a wholesale price above cost.

Note that $\gamma_L < \gamma_H$ with both critical values decreasing in θ. Moreover, $\gamma_L \in [0,1]$ and $\gamma_H \in [0.89, 1]$ for $\theta \in [0,1]$. Define three cases as follows: (i) $\gamma < \gamma_L$, (ii) $\gamma_L \leq \gamma \leq \gamma_H$ and (iii) $\gamma > \gamma_H$.

Consider case (i). As noted above, in equilibrium $w_2 = c_I$. I will therefore set w_1 to maximize $(w_1 - c_I)\, q_1(p_1(w_1, c_I),\ p_2(c_I, w_1))$. In the resulting equilibrium,

$$\Pi_{I|S=1}^{(i)} = \frac{(2+\gamma)(1-\gamma)(\alpha-c_I)^2}{4\beta(1+\gamma)(2-\gamma)(2-\gamma^2)}$$

and

$$\Pi_{E|S=1}^{(i)} = \frac{(4+\gamma-2\gamma^2)(\alpha-c_I)(c_I-c_E)}{2\beta(1+\gamma)(2-\gamma)(2-\gamma^2)}.$$

The signed retailer earns (before compensation)

$$\pi_{1|S=1}^{(i)} = \frac{(1-\gamma)(\alpha-c_I)^2}{4\beta(1+\gamma)(2-\gamma)^2}$$

while the free retailer earns

$$\pi_{2|S=1}^{(i)} = \frac{(4+\gamma-2\gamma^2)^2(1-\gamma)(\alpha-c_I)^2}{4\beta(1+\gamma)(2-\gamma)^2(2-\gamma^2)^2}.$$

Consider case (ii). We still have sharing as before but E's wholesale price is no longer constrained by c_I — it is set to maximize $(w_2 - c_E)\, q_2\, (p_2\, (w_2, w_1)\, , p_1\, (w_1, w_2))$. Equilibrium wholesale prices are determined by the simultaneous solution to the unconstrained maximization problems of I and E. In the resulting equilibrium,

$$\Pi_{I|S=1}^{(ii)} = \frac{\left(2-\gamma^2\right)\left(\left(\left(8-9\gamma^2+2\gamma^4\right)(\alpha-c_I)-\left(\gamma\left(2-\gamma^2\right)\right)(\alpha-c_E)\right)\right)^2}{\beta\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(16-17\gamma^2+4\gamma^4\right)^2}$$

and

$$\Pi_{E|S=1}^{(ii)} = \frac{\left(2-\gamma^2\right)\left(-\left(\gamma\left(2-\gamma^2\right)\right)(\alpha-c_I)+\left(\left(8-9\gamma^2+2\gamma^4\right)\right)(\alpha-c_E)\right)^2}{\beta\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(16-17\gamma^2+4\gamma^4\right)^2}$$

The signed retailer earns (before compensation) $\pi_{1|S=1}^{(ii)} = \Pi_{I|S=1}^{(ii)}\left(2-\gamma^2\right)/\left(4-\gamma^2\right)$ while the free retailer earns $\pi_{2|S=1}^{(ii)} = \Pi_{E|S=1}^{(ii)}\left(2-\gamma^2\right)/\left(4-\gamma^2\right)$.

Consider case (iii). For this case, E induces the free retailer to monopolize the downstream market. Given that the signed retailer sets its price equal to its cost w_1, E sets the highest wholesale price such that the free retailer has an incentive to monopolize the downstream market. Under a monopolization equilibrium, we know that $w_1 = c_I$ and thus the monopolization wholesale price is $w_2 = \left(\left(2-\gamma^2\right)c_I - \alpha(1-\gamma)(2+\gamma)\right)/\gamma$. This wholesale price induces the free retailer to set its retail price equal to $(c_I - \alpha(1-\gamma))/\gamma$ which is sufficient for monopolization. Both I and the signed retailer earn zero profits. E earns

$$\Pi_{E|S=1}^{(iii)} = \frac{(\alpha-c_I)\left(\gamma(\alpha-c_E)-\left(2-\gamma^2\right)(\alpha-c_I)\right)}{\beta\gamma^2}$$

while the free retailer earns

$$\pi_{2|S=1}^{(iii)} = \frac{\left(1-\gamma^2\right)(\alpha-c_I)^2}{\beta\gamma^2}.$$

Now consider the retailers' signing decisions. For $k = i, ii, iii$, an exclusion equilibrium exists if $\pi_{i|S=2} + \Pi_{I|S=2}/2 \geq \pi_{2|S=1}^{(k)}$ since then a retailer does worse rejecting an exclusive contract when the other retailer signs, while an entry equilibrium arises if $\pi_{i|S=0} > \pi_{1|S=1}^{(k)} + \Pi_{I|S=2}/2$ since then a retailer does worse signing a contract when the other does not. Substituting in the above profit expressions (and those from the proof of Proposition 2 in the main paper), and for each value of γ determining the range of θ for which each equilibrium exists, gives a complete picture of the implied equilibria as a function of $0 \leq \gamma < 1$ and $0 < \theta < 1$. This is shown in figure 1a below, which confirms the statement of the proposition. (In the statement of the proposition we have rounded critical parameter values to two decimal places.) ■

Presented below are the figures 1a and 2b for the observable contracts case (they correspond to figures 1 and 2 in the unobservable case in Abito and Wright, 2006):

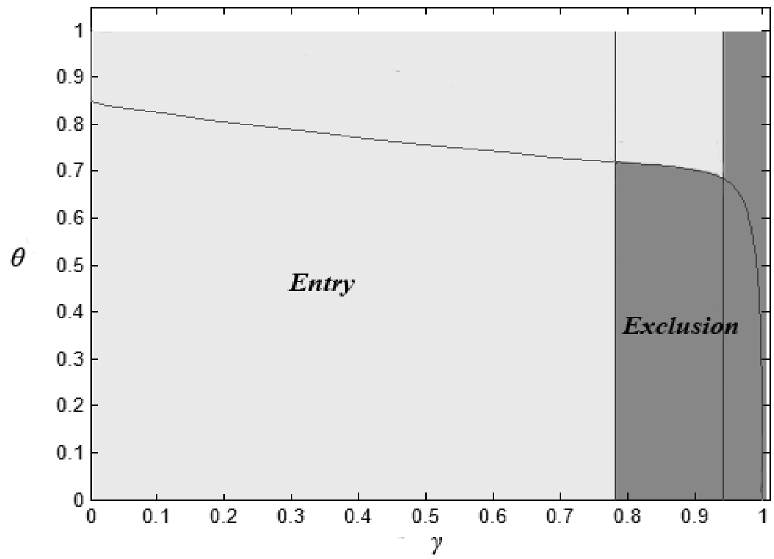

Figure 1a

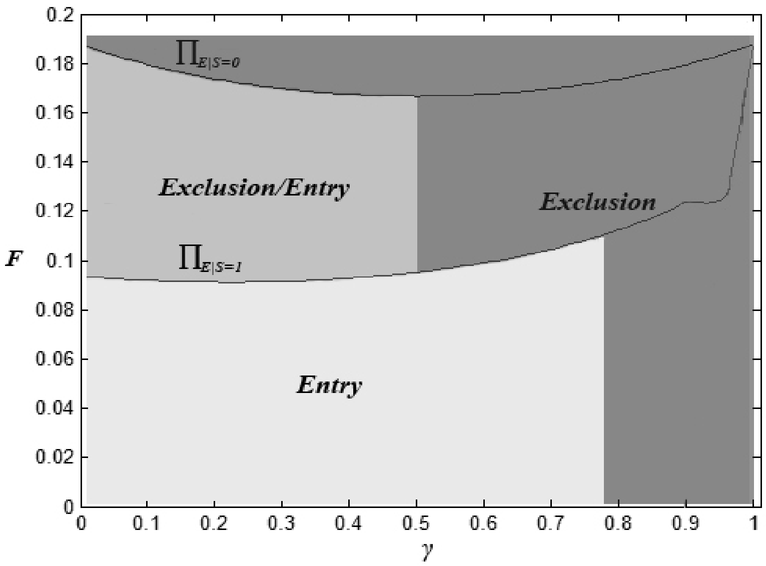

Figure 2b

1 References

Abito, J. M. and J. Wright (2006) "Exclusive dealing with imperfect downstream competition," mimeo. National University of Singapore.