Pricing Access: Forward-looking versus Backward-looking Cost Rules*

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Abstract

Regulators across many different jurisdictions and industries have recently adopted the practice of setting access prices based on the *current* costs of providing the relevant facilities. Though widely regarded as being efficient, the efficiency implications of using current costs instead of historical costs have not been formally analyzed. Our analysis shows that given stochastic costs, forward-looking access prices retard investment and are generally dominated by access prices determined by historical cost whenever investment is desired.

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1 Introduction

There is a worldwide trend towards opening some parts of network industries to competition as a way of enhancing the welfare derived from what were usually state-owned monopolies. The price at which entrants can obtain access to the networks (such as access to electricity distribution or origination and termination of calls in the case of telecommunications), is a key determinant of the welfare gains secured by such pro-competitive policies. As a result, considerable attention is devoted to the design and implementation of access pricing regimes.

There is general agreement that in most standard networks access prices should be set based on the costs of the facilities provided,[1] but the definition of the relevant costs are a matter of some dispute. It has been argued, for example, that access prices should reflect the opportunity cost to the incumbent, including the lost profit, on the grounds that doing so prevents inefficient entry. This approach, often referred to as the efficient component pricing rule, is discussed by Armstrong et al. (1996) and Laffont and Tirole (1994). This approach has merit from the viewpoint of ensuring productive efficiency. Allocative efficiency concerns motivate setting access prices based on the direct costs of the services provided. Recognizing both allocative and dynamic efficiency concerns, regulators typically set access prices at the long-run incremental cost of the service provided, where these cost measures allow for a reasonable return on capital outlays. Once a method for attributing common costs of infrastructure is employed, the resulting measures are typically referred to as the "total element (or service) long run incremental cost" (TELRIC or TSLRIC) of access.[2]

Our analysis attempts to shed some light on the effect of different asset valuation methods. Though there are many such methods, almost all are based on one answer to a fundamental question: should assets be valued for regulatory purposes at the cost of the initial investment, or at the cost of re-building the facility at the present time?

Initially, regulators adopted the former approach, basing asset values on a company's historical accounts. For instance, in 1985 OFTEL, the UK telecommunications regulator, set access prices to British Telecom's network based on its historical costs (Melody, 2000, p. 274). We call the use of historical measures of asset values in calculating access prices "backward-looking cost rules." This approach has been criticized by several authors, including Baumol and Sidak (1995).

[1]This assumes there are no network externalities that access prices need to internalize.

[2]It should be emphasized, however, that the TELRIC type label is usually also associated with a particular asset valuation methodology. This is by convention rather than necessity: in fact, any valuation method can be reflected in a TELRIC framework. Because our purpose is to study the effects of different valuation methods, we will minimize confusion by avoiding the use of TELRIC type labels.

The modern trend is towards the adoption of "forward-looking cost rules," whereby the costs used to determine access prices are based on the current cost of rebuilding facilities to provide the existing service, using the best available technology. In 1994 OFTEL switched away from fully allocated costs based on historical accounts to a system based on the computation of forward-looking incremental costs reflecting current replacement costs of capital assets (Melody, 2000, p. 274). In the US, the 1996 Telecommunications Act required regulators to set access prices for unbundled network elements equal to cost, but provided little guidance about how these prices should be set. The FCC proposed that the term "cost" should mean forward-looking economic cost (Salinger, 1998, p. 150). Hybrid solutions have also been adopted. For example, when the telecommunications regulator in the Netherlands implemented local loop unbundling, it used a scheme in which access prices were initially based on historical cost, and then through time converged to replacement cost (Rood and te Velde, 2003, p. 704). As the existing assets were highly depreciated, initial access prices were relatively low, but access prices will climb towards replacement cost, giving access seekers a greater incentive to invest in their own facilities.

The proponents of forward-looking access prices suggest that competitors should not have to pay for the high costs of an incumbent just because the incumbent invested at a time when costs were high.[3] In contestible markets, it is argued, a firm would be unable to recover historical costs that are more than the current stand-alone cost of re-building the network that it provides. Thus, to mirror contestible markets, the costs included in access prices should be based on current best practice. However, as Hausman (1999) has noted this misses an important point. The reason these markets are not contestable is that firms need to sink large amounts of money into irreversible investments. Given uncertainty over the future cost of such projects, it is critical that they face the right incentives to do so in the first place. This paper addresses this point by asking (i) if an incumbent, in a world of cost uncertainties, will invest earlier under a backward or forward-looking cost rule, and (ii) which rule leads to higher overall welfare.

We provide a model in which a firm has a single irreversible investment opportunity. The cost of carrying out the project varies stochastically through time. To focus on the effects of cost uncertainty, the (flow) return to the project is assumed constant for a given access price and there is no physical depreciation in the asset. We assume the incumbent's profit is increasing in the access price charged at any time. Given an access pricing rule, the firm must decide when to invest.

[3]The issue arises from the fact that, with costs falling over time, historical costs tend to be high and forward-looking costs low. This leads to an obvious conflict between access-seekers and access-providers over the appropriate methodology. Laffont and Tirole (2000, pp. 141–161) discuss the debate concerning backward versus forward-looking cost-based pricing of access. Temin (1997) notes that as early as the 1960s, AT&T argued in favor of forward-looking cost approaches to justify low rates. At that time the FCC was proposing to mandate access to microwave services, and AT&T responded by requesting that their own retail prices for comparable services be allowed to fall to the estimated cost of an entrant's service: a forward-looking cost standard.

Irrespective of the access price, the firm will delay investment until the present value of the completed project covers the cost of investing, including the cost of the delay option destroyed by investing. In doing so the firm waits too long from the regulator's point of view, since the firm ignores the surplus that would flow to competitors and consumers in the mean time. What is needed is a means of encouraging the incumbent to invest earlier. Higher access prices would provide such an incentive, by raising the profitability of the project and thereby raising the opportunity cost of delaying investment, but higher access prices reduce the flow of surplus to consumers through higher retail prices once the investment has been made. This suggests a trade-off: high access prices lead to a flow of surplus that is low but starts sooner, while low access prices lead to a flow of surplus that is high but starts later. The preferred access pricing scheme will match the marginal cost of bringing investment further forward in time (the lower total surplus resulting from raising access prices) and the marginal benefit (earlier investment raises the present value of any given cash flow).

This trade-off can be improved by using either backward or forward-looking access prices — backward-looking prices are determined by the cost of the project at the time the project is built, while forward-looking prices are set at each point in time based on the current cost of building the project. If the firm delays investment hoping for the required capital outlay to fall, then it will have to share some of the gains with competitors and customers — a lower construction cost implies lower access prices, and therefore a smaller profit flow — and this reduces the value of delaying investment. Because the access price under the forward-looking rule diverges from its initial value over time, it subjects the firm to additional risk. To achieve the same investment decision with forward-looking rules therefore requires that access prices be increased to compensate for this additional risk. When there is downward drift in the project's construction cost, implying a downward drift in forward-looking access prices (but none in backward-looking ones), forward-looking rules require even higher initial access prices if they are to induce the same investment behavior as backward-looking rules. The ability of a backward-looking rule to induce earlier investment for given access prices (or more generally, for the same market value of the incumbent), results in higher welfare.

There are two main policy implications. Firstly, policymakers should give the dynamic efficiency advantages of backward-looking rules more serious consideration. Secondly, if a forward-looking rule is used, the initial access price should be set at a level higher than would be the case if a backward-looking rule is adopted. A high rate is required to compensate the incumbent for the risk it bears when faced with forward-looking access prices. The incumbent will delay investment too long unless it receives such compensation.

Our work is related to some recent research which takes a forward-looking rule as given and considers how the rental rate on capital should be determined. Hausman (1999) considers a sunk telecommunications investment and shows that the allowed rental rate on capital for this investment project should be increased to account for the uncertainty that arises from allowing competitors access to the incumbent's facilities on a forward-looking (TELRIC) basis. Ergas and Small (2000) explore the relationship between economic depreciation and the value of the delay option in the context of regulated access. They establish conditions under which expected economic depreciation is identical to the value of the option to delay investment, and show that as a general matter the former (economic depreciation) is no less than the latter (the real option value). Salinger (1998) shows that the potential for competition, asset life uncertainty, and the installation of excess capacity for demand growth all raise the forward-looking access price. For instance, when firms build projects they typically invest in excess capacity to meet potential future growth in demand, thus avoiding having to come back and add small increments to the initial infrastructure (which would be inefficient). To the extent that forward-looking rules ignore these additional costs, competitors are getting a real option which they are not paying for — the option to use the excess capacity if and when needed. If the competitors do not pay for this option, then the incumbent will invest too little in such excess capacity. Offsetting these effects he finds that the potential for technological change that enhances the future value of an asset lowers forward-looking costs. Other papers which consider TELRIC prices include Jorde et al. (2000), Mandy (2002), Mandy and Sharkey (2003), Tardiff (2002), and Weisman (2003). However, none of these papers addresses how forward-looking access prices perform under stochastic costs, the feature which leads forward-looking costs to give fundamentally different incentives to backward-looking cost rules. Moreover, none of the papers attempts to evaluate the desirability of the forward-looking approach.

The practical importance of our analysis is highest in situations where additional infrastructure investment is desirable from the perspective of consumers, and it is efficient for downstream firms to share access to this infrastructure. For some networks, such as electricity lines, railways or ports, investment is needed to replace worn out assets or to serve new areas of demand. In other cases, such as telephony and the Internet backbone, technological developments may spur new investment needs. The scope for greenfields infrastructure investment is particular apparent in developing countries. Fay and Yepes (2003) estimate that the gap between demand for infrastructure services and available capacity is such that US$370 billion of new investment is required over the five year period to 2010, plus a further US$480 billion for maintenance expenditure. More than half of this US$850 billion total is estimated to be required in low and middle income countries.

The rest of the paper proceeds as follows. Section 2 sets up the firm's investment problem and describes the various regulatory regimes. In Section 3 the optimal investment policy is characterized for backward- and forward-looking rules, and the conditions under which the backward-looking rule leads to earlier investment than the forward-looking rule are explored. Section 4 contains a welfare comparison of the two rules, while Section 5 concludes with a summary of results and policy implications.

2 Setting up the model

A project, which can be launched at any time, involves a single, large, irreversible investment. If the project is launched at date t, it costs K_t.[4]

Assumption 1 *The cost of launching the project evolves according to the geometric Brownian motion $dK_t = \mu K_t dt + \sigma K_t d\xi_t$, where μ and σ are constants and ξ_t is a Wiener process. Fluctuations in K_t attract a (systematic) risk-premium of λ, for some constant λ satisfying $r + \lambda > \mu$ where r is the riskless interest rate.*

This assumption allows us to price contingent claims as though K_t evolves according to the 'risk-neutral' process[5]

$$dK_t = (\mu - \lambda)K_t dt + \sigma K_t d\xi_t.$$

The regulator wants a private firm, which we call the incumbent, to construct the project. It will impose a regulatory framework specifying the access price which the regulator can charge a competitor for use of the facility. We suppose that launching the project initiates an indefinite flow of profit $\pi_t = \pi(a_t)$ to the incumbent, the level of which depends on the access price a_t (which is assumed to be nonnegative). When the access price is zero, the competitor is able to use the incumbent's facility without charge. Higher access prices increase the incumbent's profit flow, but at a decreasing rate.

Assumption 2 *The incumbent's profit flow $\pi(a)$ is positive, bounded above, increasing and concave in a, and $\pi'(0)$ is finite.*

Once the project has been launched, the regulator observes a flow of total surplus $\theta_t = \theta(a_t)$ which also depends on the access price. We assume that once the investment has been made (and so is sunk), the flow of total surplus (consumer plus producer surplus) is increased by setting the access price as low as possible. This reflects the fact that the incremental cost of

[4]Any one of a wide variety of stochastic process could be used here. The nature of the chosen process will determine the trend in construction cost, as well as the volatility around this trend. We adopt geometric Brownian motion because the use of this relatively simple process is widespread in the real options literature and because it makes our model relatively tractable. (Nevertheless, the nonlinear dependence of the profit and total surplus flows on construction costs means that we still rely on a numerical analysis for our welfare analysis.) More complicated processes could incorporate mean reversion in construction costs and volatility which changes over time. However, geometric Brownian motion captures the essential aspects of trend and volatility that we argue must be considered when analyzing the impact of access price regulation on investment.

[5]We can interpret λ as the risk-premium of an asset with returns which are perfectly positively correlated with changes in K_t. Thus it captures the systematic risk of shocks to the project's cost. For further discussion of risk-neutral pricing see Dixit and Pindyck (1994, Chapter 4).

providing access is zero in the model, and so the access price that maximizes flow surplus is likely to be zero, or even negative.[6] We also note that, by construction, total surplus will be at least as large as the incumbent's profits.

Assumption 3 *The regulator's flow of total surplus $\theta(a)$ is decreasing in a, with $\theta(a) > \pi(a)$ for all a.*

Flow functions with these properties arise in various models. The following example, which shows one such situation in detail, will be used to motivate the functional forms used in the numerical analysis later in the paper.

Example 1 A vertically-integrated incumbent (firm 1) sells to consumers directly, while an entrant (firm 2) seeks access to the incumbent's facility in order to compete in the retail market. The incumbent's costs of providing the facility are entirely fixed (and sunk), since we set the marginal cost to zero without loss of generality. Retail competition is modelled as standard homogenous Bertrand competition. The firm which sets the lower retail price (say p) gets the total market $Q(p)$ demand at this price, for some decreasing function Q. The monopoly price p^M maximizes $(p-c)Q(p)$, where c is the per-unit cost of retailing the good, and the associated profits are denoted by π^M. The incumbent also sells some other independent services from the same capacity, generating a profit flow of π_0 and a flow of total surplus of θ_0.

The equilibrium is that if $a \leq p^M - c$, firm 1 sets $p_1 = c + a$ and firm 2 sets $p_2 = c + a$; if $a > p^M - c$, then $p_1 = p^M$ and firm 1 takes the whole market getting monopoly industry profits. The incumbent's profit flow is therefore

$$\pi(a) = \begin{cases} \pi_0 + aQ(a+c), & \text{if } a \leq p^M - c, \\ \pi_0 + (p^M - c)Q(p^M), & \text{if } a > p^M - c. \end{cases}$$

Firm 2's profit is 0. Consumers' surplus (from the retail market) equals $\int_{c+a}^{\infty} Q(p)\,dp$ for $a \leq p^M - c$ and $\int_{p^M}^{\infty} Q(p)\,dp$ for $a > p^M - c$, so that the flow of welfare is

$$\theta(a) = \begin{cases} \theta_0 + aQ(a+c) + \int_{c+a}^{\infty} Q(p)dp, & \text{if } a \leq p^M - c, \\ \theta_0 + (p^M - c)Q(p^M) + \int_{p^M}^{\infty} Q(p)dp, & \text{if } a > p^M - c. \end{cases}$$

Beyond some point, raising access prices does not benefit the firm (because no rival will want to produce, and so the firm will have a monopoly), while lowering access prices can only make it worse off. Thus, at least beyond a certain point, greater risk over access prices increases the downside loss, but not the upside gain. In fact, the incumbent's profit flow function satisfies all of the conditions in Assumption 2. Profit flow is positive for positive access prices, is increasing in a for $a \leq p^M - c$, and independent of a if $a > p^M - c$. Profit is concave in a under reasonable

[6] If there is downstream market power, then the access price that maximizes the flow of total surplus will typically be negative so as to offset the downstream markup.

properties on the demand function (for example, linear demand functions will suffice), since $\pi''(a) = 2Q'(a+c) + aQ''(a+c)$ if $a < p^M - c$. Finally, $\pi'(0) = Q(c)$ is indeed finite (for a well-defined demand function).

The situation for the regulator is the reverse, as the regulator's flow surplus first decreases in access prices and then, because the incumbent will operate as a monopoly once access prices reach the monopoly level, the flow surplus is constant. Thus, at least beyond a certain point, greater risk over access prices increases the upside gain, but not the downside loss. In fact, the flow of total surplus satisfies both of the conditions in Assumption 3.[7] Moreover flow surplus is always strictly greater than flow profits, given consumer surplus is always positive (even at the monopoly price). ■

Although we do not present the details of other motivating models in this paper, we have found that the profit and total surplus flow functions also exhibit the properties required of Assumptions 2 and 3 if the vertically-integrated incumbent and the entrant are Cournot competitors and demand is linear. This is also the case if the two firms compete in a Hotelling-type model including subscription and usage charges.[8]

The regulator chooses the charging regime, while the incumbent takes this charging regime as given and chooses its investment policy. The regulator's aim is to select the charging regime which, given the incumbent's response, leads to the greatest possible present value of future surpluses. In this paper we consider two different access pricing regimes.

Backward-looking (BL) access price. At time 0 the regulator sets the access price a_b that will prevail over the lifetime of the project if construction occurs immediately. If investment is delayed until date T, then the access price which prevails over the lifetime of the project is $a_b(K_T/K_0)$. That is, the access price is adjusted in line with changes in project cost *while the regulator waits for the incumbent to invest*, but as soon as the incumbent invests the access price is fixed in perpetuity.

Forward-looking (FL) access price. At time 0 the regulator sets the access price a_f. Regardless of when construction occurs, the access price which the incumbent is allowed to charge on date t equals $a_f(K_t/K_0)$. That is, the access price is adjusted in line with changes in project cost *both before and after investment*.

Thus, access prices reflect the actual cost of building the project under the BL regime, and the hypothetical cost of rebuilding it under the FL regime. We focus on these two access pricing regimes but other regimes could be evaluated using our model. For instance, regulators in the Netherlands have set access prices that are initially backward-looking but will converge to forward-looking ones over time. In this case, if investment is delayed until date T, then the

[7] Note that $\theta'(a) = a\,(dQ/da) < 0$ if $a < p^M - c$ and that $\theta(a)$ is independent of a if $a \geq p^M - c$.

[8] Details available from the authors upon request.

access price at date $T+t$ could be set equal to $a(e^{-\beta t}K_T + (1-e^{-\beta t})K_{T+t})/K_0$, where the constant β determines the speed with which access prices converge to the forward-looking level. We focus on the polar cases of BL and FL regimes because they are relatively easy to analyze and because many other possible schemes can be interpreted as combinations of these two regimes.

3 Investment behavior

This section explores the investment behavior of an incumbent faced with particular access charging regimes. We begin in Section 3.1 by calculating the investment payoffs under each regime, and then deriving the corresponding optimal investment policies. We compare the timing of the incumbent's investment under the regimes in Section 3.2.

3.1 Optimal investment policy

We denote the payoff to the incumbent at the time of investment by $P(K)$, where K is the cost of launching the project. It equals the present value of the profit flow initiated by investment, less the cost of launching the project. The precise form of $P(K)$ will depend on the access charging regime in place. For example, if the regulator sets an initial BL access price of a_b and the incumbent invests when the project costs K, then the access price is set equal to a_bK/K_0 for the lifetime of the project. The present value of the profit flow equals $\pi(a_bK/K_0)/r$, and the payoff to investment is

$$P_b(K) = \frac{\pi(a_bK/K_0)}{r} - K. \tag{1}$$

If the regulator sets an initial FL access price of a_f, then the access price at date t is a_fK_t/K_0. The resulting investment payoff is given in the following lemma.[9]

Lemma 1 *Under a FL access price, if the incumbent invests when the project costs K, then the present value of the incumbent's future profit flow equals $\Pi_f(a_fK/K_0)$, where*

$$\Pi_f(a) = \frac{1}{r}\cdot\frac{\gamma\delta}{\gamma+\delta}\left(\int_0^1 y^{\gamma-1}\pi(ay)dy + \int_0^1 y^{\delta-1}\pi(a/y)dy\right),$$

$$\delta = -\frac{\mu-\lambda}{\sigma^2} + \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu-\lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 1,$$

and

$$\gamma = \frac{\mu-\lambda}{\sigma^2} - \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu-\lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 0.$$

The payoff to investment equals

$$P_f(K) = \Pi_f(a_fK/K_0) - K. \tag{2}$$

[9] The proof of Lemma 1, together with the proofs of all other results, is contained in the appendix.

BL and FL access pricing regimes promote early investment in two distinct ways. Firstly, by allowing the incumbent to charge for access, they raise the project's profitability — this promotes early investment because it means that the firm forgoes greater profits whenever it delays investment. Secondly, by making the access price (and hence the profit flow) dependent on the cost of the project, these rules alter the trade-off between earlier investment and the investment payoff. When access is free (that is, set at marginal cost in our model) and the firm delays investment, each unit reduction in the cost of constructing the project translates into a unit increase in the incumbent's investment payoff. In contrast, under a BL or FL rule, any reduction in the project's construction cost also lowers the project's profit flow, partially offsetting the benefits of the reduced construction cost — some of the benefits of a lower construction cost are shared with competitors and consumers. The effect is to lower the volatility of the incumbent's investment payoff, and thereby lower the value of the real option to delay investment.

The incumbent chooses an investment policy which maximizes the value of the firm. In optimal stopping problems of the type facing the firm, it is optimal to invest as soon as the cost of doing so is less than some critical level $\hat{K}$. The following lemma gives the value of the firm for an arbitrary investment threshold.

Lemma 2 *If the project has not already been launched at time t, the value of the incumbent's entitlement to the project at that time is*

$$V(K_t;\hat{K}) = \begin{cases} P(\hat{K})\left(\dfrac{\hat{K}}{K_t}\right)^{\gamma} & \text{if } K_t \geq \hat{K}, \\ P(K_t) & \text{if } K_t < \hat{K}. \end{cases}$$

The incumbent's optimal investment policy is therefore to choose the investment threshold $\hat{K}$ which maximizes $P(\hat{K})\hat{K}^{\gamma}$. The following lemma describes the optimal investment threshold.

Lemma 3 *The optimal investment policy is to invest whenever the cost of doing so is less than the threshold $\hat{K}$ given implicitly by*

$$\frac{\hat{K}}{P(\hat{K})} \cdot \frac{dP(\hat{K})}{d\hat{K}} = -\gamma. \tag{3}$$

Notice that the firm's payoff function (1) under the BL rule can be written in terms of a_b/K_0 and $\hat{K}$. Similarly, a FL rule leads to an investment payoff (2) which can be written as a function of a_f/K_0 and $\hat{K}$. It follows from substituting these payoff functions into equation (3) that the incumbent's chosen investment threshold will be a function of a_b/K_0 under a BL rule, and of a_f/K_0 under a FL one. The precise optimal threshold for each regime can be found by substituting the appropriate payoff function into equation (3). The results are reported in the following proposition.

Proposition 1

1. *When faced with the BL regime with date $t = 0$ access price a_b, the incumbent chooses the investment threshold $\hat{K}_b = \hat{K}_b(a_b/K_0)$ given implicitly by $\hat{K}_b = R_b(a_b\hat{K}_b/K_0)$, where*

$$R_b(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{1}{\gamma+1} \cdot \frac{a\pi'(a)}{r}.$$

2. *When faced with the FL regime with date $t = 0$ access price a_f, the incumbent chooses the investment threshold $\hat{K}_f = \hat{K}_f(a_f/K_0)$ given implicitly by $\hat{K}_f = R_f(a_f\hat{K}_f/K_0)$, where*

$$R_f(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{\gamma}{\gamma+1} \cdot \frac{a}{r} \int_0^1 y^{\delta-2}\pi'(a/y)dy.$$

3.2 Comparing the different access pricing regimes

In this section, we focus attention on the timing of investment under the two regimes introduced in Section 2.

We begin our analysis by comparing BL and FL rules when they have the same initial access price: $a_b = a_f$. Proposition 2 shows that, provided the drift in cost is not too large, investment occurs sooner under a BL rule than under a FL rule with the same initial access price.

Proposition 2 *There exists a positive-valued function $N(\cdot, a_0/K_0)$ such that the BL rule with initial access price a_0 leads to earlier investment than the FL rule with the same initial access price if and only if*

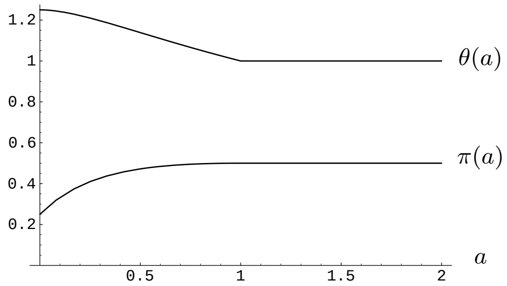
$$\mu \leq \lambda + \sigma^2 N\left(\frac{\sigma^2}{r}, \frac{a_0}{K_0}\right).$$

In particular, if (as will typically be the case in telecommunications) the project's cost has nonnegative systematic risk and the cost of launching the project is expected to fall over time, BL costs will lead to earlier investment.

To understand this result, first consider the case when $\mu < \lambda$. If the incumbent chose the same investment threshold for FL charges as for BL ones with the same initial access price, then the FL access price would trend downwards after investment, while the BL access price would stay at its investment-date level; the investment payoff under the FL regime would be lower than for the BL one. This encourages the incumbent to choose a lower threshold, spending less on launching the project, under the FL regime than under the BL regime. However, the trend in the project's replacement cost is just part of the story behind Proposition 2. Even when $\mu \geq \lambda$, the concavity of the incumbent's profit flow function means that the investment payoff will still be lower under the FL regime if FL access prices are sufficiently volatile. When this occurs, under FL costs the incumbent is motivated to wait until launching the project is cheaper.

We need a fairer way to compare BL and FL pricing rules when the project's cost has a nonzero trend. For instance, if costs are expected to fall, under the FL approach we should allow the firm a higher access price initially in order to compensate it for the lower access prices expected in the future. This raises the question of what quantity to set equal across rules.

Figure 1: Profit and total surplus flows



Notes. The bottom curve plots the profit flow $\pi(a)$, while the top curve plots the total surplus flow $\theta(a)$ for the situation described in Example 1 when the demand function is $Q(p) = p^{-2}$. Other parameters are $c = 1$ and $\pi_0 = \theta_0 = 0.25$.

One possibility is to set initial access prices in such a way that, whenever the firm chooses to invest, the present value of its profit flow is equal under BL and FL rules. This, after all, is the amount that competitors (and ultimately consumers) will pay the incumbent for building the facility. Alternatively, we could use the investment payoff as this is the quantity of interest to the incumbent's owners at the time they invest. However, if the firm invests at different dates under the resulting BL and FL rules, we would then be comparing payoffs received at two different dates. In order to avoid this awkward situation, we compare the different regimes in a way which equates the initial market value of the incumbent, where this is given by the function $V(K)$ in Lemma 2.

Due to the complexity of the problem, we must use numerical analysis for this comparison. We base our analysis on the situation described in Example 1, restricted to iso-elastic demand: $Q(p) = p^{-\varepsilon}$ for some constant $\varepsilon > 1$. The monopoly price is therefore $p^M = \varepsilon c/(\varepsilon - 1)$, while the flow functions are

$$\pi(a) = \begin{cases} \pi_0 + a(a+c)^{-\varepsilon}, & \text{if } a \leq \frac{c}{\varepsilon-1}, \\ \pi_0 + \varepsilon^{-\varepsilon}\left(\frac{c}{\varepsilon-1}\right)^{1-\varepsilon}, & \text{if } a > \frac{c}{\varepsilon-1}, \end{cases}$$

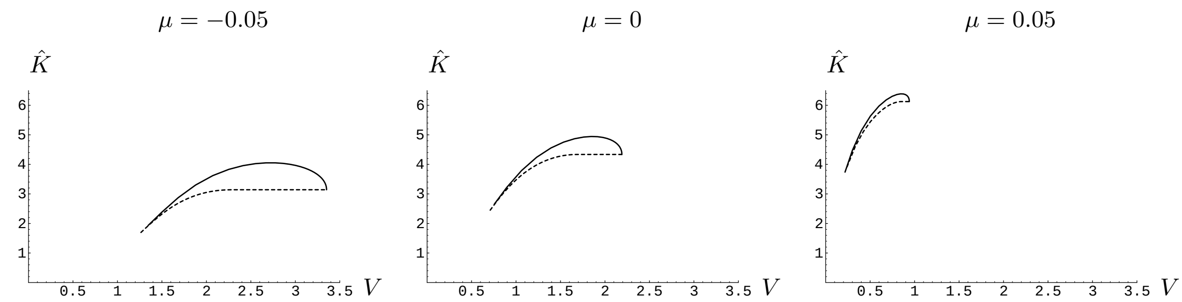
and

$$\theta(a) = \begin{cases} \theta_0 + \left(\frac{\varepsilon a + c}{\varepsilon-1}\right)(a+c)^{-\varepsilon}, & \text{if } a \leq \frac{c}{\varepsilon-1}, \\ \theta_0 + \frac{(2\varepsilon-1)c}{(\varepsilon-1)^2}\left(\frac{\varepsilon c}{\varepsilon-1}\right)^{-\varepsilon}, & \text{if } a > \frac{c}{\varepsilon-1}. \end{cases}$$

For the numerical analysis, we choose $r = 0.05$, $\lambda = 0.03$, $c = 1$, $\varepsilon = 2$, and $\pi_0 = \theta_0 = 0.25$. This implies that the monopoly price is $p^M = 2$ and that at this price the firm derives a profit flow of 0.25 from providing access and 0.25 from providing other services using the same capacity. The profit flow function is the lower function in Figure 1, while the upper curve plots the regulator's total surplus flow.

Figure 2 compares the incumbent's investment behavior under BL and FL rules where the regulator sets the initial access prices in such a way that the firm's initial market value is as given

Figure 2: Investment timing under BL and FL rules



Notes. The graphs plot the value of the investment thresholds ($\hat{K}$) as functions of the initial value of the firm (V), with the BL rule represented by the solid curves, and the FL rule by the dotted ones. The left-hand graph corresponds to a negative drift in replacement cost ($\mu = -0.05$), the middle graph to zero drift ($\mu = 0$), and the right-hand graph to a positive drift ($\mu = 0.05$). Other parameters are $\sigma = 0.2$, $r = 0.05$, and $\lambda = 0.03$.

on the horizontal axis. The graphs plot the value of the investment thresholds as functions of the common initial market value, reflecting the choice of the initial access prices, with the BL rule represented by the solid curves and the FL rule by the dotted ones. Each pair of curves intersects at a point representing the free-access outcome (the left hand intersection) and the monopoly outcome (the right hand intersection).[10] The left-hand graph corresponds to a negative drift in replacement cost ($\mu = -0.05$), the middle graph to zero drift ($\mu = 0$), and the right-hand graph to a positive drift ($\mu = 0.05$). Volatility is constant across the three graphs at $\sigma = 0.2$. For all the parameter values we consider, the BL rule induces earlier investment than the FL rule with the same initial market value. For example, when there is zero drift in the project's cost, under the BL regime the regulator can induce the incumbent to invest as soon as $K \leq 4.57$ if it allows the firm an initial market value of $V = 1.40$. In contrast, under the corresponding FL rule the incumbent will only invest once $K \leq 4.20$.

That BL rules are more successful than FL ones at promoting early investment can be explained by the volatility of access prices under the latter rules and the concavity of the incumbent's profit flow function. For simplicity, consider the situation when the project's construction cost has zero drift. If BL and FL rules are to give profit flows with the same present value, then the FL access price will have to start at a higher level than the (constant) BL access price. Suppose the project's cost falls before the incumbent invests. Then both initial access prices will fall, and so will the potential profit flows under both BL and FL rules. However, because

[10]When the initial access price is fixed at zero, access will always be free under the two rules, regardless of changes in the project's cost. When the initial access price is very high, access prices will usually be so high that the incumbent does not face competition and, again, the two rules will be identical. Similarly, regardless of the initial access price, if the drift is high enough, access prices will soon be at or above the monopoly level under both rules, explaining the similarity between the two rules evident in the right-hand graph.

the profit function is concave and the initial FL access price is higher than the initial BL one, the profit flow will suffer a bigger drop under the BL rule. Thus, delay is less attractive under a BL rule than under its FL counterpart.[11]

4 Welfare analysis

Up until this point, our attention has focused on the timing of the incumbent's investment decision under the two regimes. This section compares the two charging regimes from the regulator's point of view. In Section 4.1 we describe how the regulator assesses the charging regimes introduced in Section 2, and we compare the two rules' welfare performance using numerical analysis in Section 4.2.

4.1 Evaluating charging regimes

We let $S(K)$ denote the payoff to the regulator at the time of investment, where K is the cost of launching the project. We interpret $S(K)$ as the present value of the flow of surplus, less the cost of launching the project. The form this function takes depends on the access charging regime imposed by the regulator.

Under a BL regime the regulator observes a constant flow of surplus. This stream has present value $\theta(a)/r$, where a is the access price, implying that the regulator's payoff function is

$$S_b(K) = \frac{\theta(a_b K/K_0)}{r} - K. \tag{4}$$

The construction of the regulator's objective function is less straightforward under a FL regime.

Lemma 4 *Under a FL access price with initial access price a_f, if the firm invests when the project costs K, then the regulator's payoff function is*

$$S_f(K) = \Theta_f(a_f K/K_0) - K, \tag{5}$$

where

$$\Theta_f(a) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta} \left(\int_0^1 y^{\gamma-1}\theta(ay)dy + \int_0^1 y^{\delta-1}\theta(a/y)dy \right).$$

At any given time, the value of the regulator's future flow of surplus will depend on the current cost of launching the project, the investment threshold chosen by the incumbent, and the exact form of S.

[11]When drift is negative, an even bigger gap is initially required between BL and FL prices to equate the present values of the incumbent's profit flow. As a result, cost shocks result in an even bigger drop in the incumbent's profit flow under a BL rule, so that the difference in investment timing under the two rules is even greater.

Lemma 5 *If the project has not already been launched at time t, and the incumbent has chosen the investment threshold $\hat{K}$, then the net present value of the future surpluses at that time is*

$$W(K_t;\hat{K}) = \begin{cases} S(\hat{K})\left(\dfrac{\hat{K}}{K_t}\right)^{\gamma} & \text{if } K_t \geq \hat{K}, \\ S(K_t) & \text{if } K_t < \hat{K}, \end{cases}$$

where the regulator's payoff function S is given by one of equations (4) and (5) according to which access pricing regime is being used.

When evaluating different access pricing schemes (prior to the investment date), the regulator uses the present value of the future surpluses. Equivalently, it adopts the objective function $S(\hat{K})\hat{K}^{\gamma}$.

Like the incumbent, the regulator would wait for the cost to fall below some threshold before investing. However, the incumbent is generally too 'patient' for the regulator's liking. This is because the incumbent ignores the flow of surplus to customers when evaluating the investment payoff. To illustrate, consider the benchmark case in which a regulator gives a competitor free access to its facility. With the competitor provided with free access to the facility, the incumbent is too reluctant to invest. By introducing a positive (BL or FL) access price, the regulator is able to induce the incumbent to invest sooner. The cost of this strategy is that, because the regulator's surplus is a decreasing function of access prices, its payoff from investment will fall. At the optimal such access price, the marginal cost of raising the access price any higher would exactly match the marginal benefit resulting from the ensuing earlier investment.

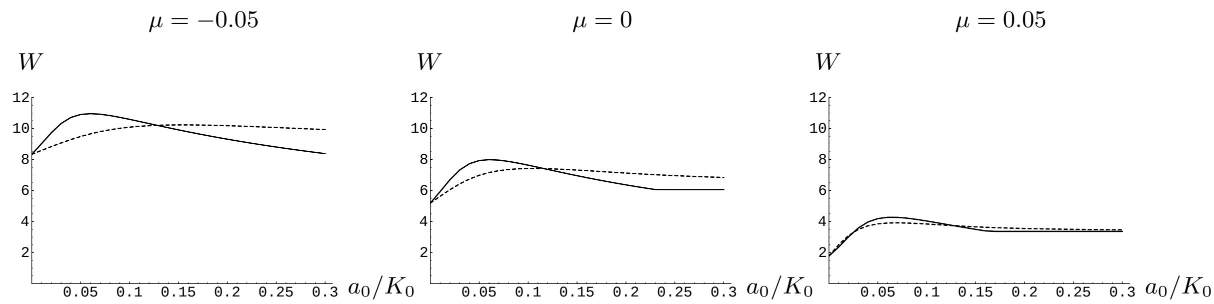
We now turn to the comparison of BL and FL rules from the regulator's perspective.

4.2 Welfare comparison of access rules

We adopt a similar approach to that in Section 3. The difference is that in Section 3 we were interested in the timing of investment by the incumbent, whereas now we compare the various charging regimes from the point of view of the regulator. We use the total surplus flow function shown in Figure 1.

The graphs in Figure 3 plot the regulator's objective function as a function of the initial access price (expressed as a proportion of K_0) for three different scenarios. For all three cases, and for both rules, overall welfare is low if a very low access price is set — while forcing the incumbent to offer free access raises the flow of surplus, it gives the incumbent little incentive to invest early, so that the present value of the surplus flow is actually relatively low. Allowing the incumbent to charge a very high access price is also suboptimal since although it leads to early investment the resulting flow of surplus is low. In all three cases, the best BL rule leads to higher welfare than the best FL rule. The welfare-maximizing BL access price appears to be insensitive to the drift in replacement cost. Furthermore, Figure 3 suggests that the welfare-maximizing

Figure 3: Overall welfare under BL and FL rules



Notes. The graphs plot the value of the regulator's objective function as functions of the initial access price (expressed as a proportion of K_0), with the BL rule represented by the solid curves and the FL rule by the dashed ones. The left-hand graph corresponds to a negative drift in replacement cost ($\mu = -0.05$), the middle graph to zero drift ($\mu = 0$), and the right-hand graph to a positive drift ($\mu = 0.05$). Other parameters are $\sigma = 0.2$, $r = 0.05$, and $\lambda = 0.03$.

FL access price exceeds its BL counterpart when replacement cost trends downwards, but that the two prices are similar when replacement cost trends upwards.

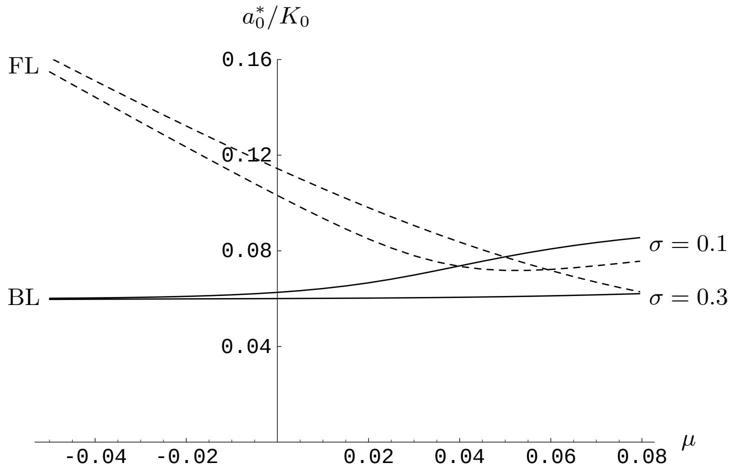
For the remainder of this section we restrict attention to the welfare-maximizing BL and FL rules, and use numerical analysis to consider a wider range of scenarios. Figure 4 illustrates the properties of the optimal access prices for BL and FL rules. For example, the optimal BL rule is described by the initial access price a_b^* which maximizes

$$S_b(\hat{K}_b(a_b/K_0))(\hat{K}_b(a_b/K_0))^\gamma.$$

The optimal FL rule is described by the analogous initial access price a_f^*. The dashed curves plot a_f^*/K_0 as a function of μ for two different levels of σ, while the solid curves plot a_b^*/K_0. Except when drift is high and volatility low, $a_f^* > a_b^*$; that is, the optimal initial FL access price is greater than the optimal initial BL one (and, except when drift is high, it is considerably higher).

When the project's construction cost is trending downwards, both the incumbent and the regulator have a strong incentive to delay investment, wait for the cost to fall, and then invest, thereby committing less capital to the project. Compared to the zero drift case, the regulator would need to offer profit flows with a higher present value (requiring higher access prices) in order to induce the incumbent to choose the same investment threshold. Figure 4 shows that the welfare-maximizing initial access price under the BL rule is almost constant with respect to drift, indicating that the regulator also finds it optimal to delay investment relative to the zero drift case. However, the welfare-maximizing initial access price under a FL rule is considerably higher when drift is negative. The reason is that access prices also trend downwards under this rule so that, when compared to the zero drift case, (1) a higher initial access price is needed just

Figure 4: Behavior of optimal BL and FL access prices



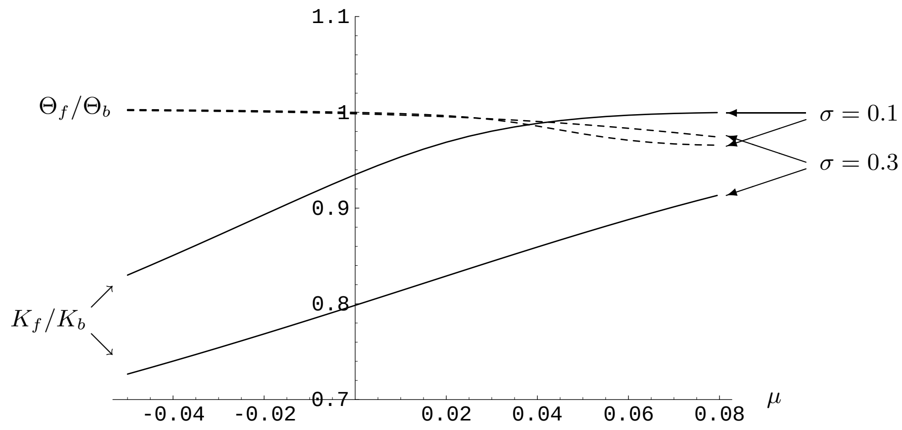
Notes. The curves plot the optimal initial access prices (as proportions of K_0) under BL costs (the solid curves) and FL costs (the dashed curves) as a function of the drift in costs. Each pair of curves corresponds to a different level of the volatility in the cost of launching the project.

to maintain the present value of the incumbent's profit flow and (2) a higher initial access price can be set without lowering the present value of the flow of surplus.

Figure 5 investigates two aspects of the welfare-maximizing BL and FL rules. The solid curves plot $\hat{K}_f/\hat{K}_b$ as a function of μ for two different values of σ. In all the combinations of drift and volatility we consider, $\hat{K}_f/\hat{K}_b < 1$, implying that investment occurs sooner under the welfare-maximizing BL rule than under its FL counterpart. The two rules induce markedly different investment behavior when replacement cost trends downwards. The dashed curves plot Θ_f/Θ_b as a function of μ, where Θ equals the present value of the flow of surplus, measured immediately after investment. The welfare-maximizing BL rule generates more surplus than the FL rule when drift is positive, and only very slightly less than the FL rule when drift is negative. Thus, we see that when drift is negative, or even slightly positive, the welfare-maximizing BL induces earlier investment and delivers only very slightly less surplus than the welfare-maximizing FL rule; that is, a BL rule delivers greater dynamic efficiency than a FL rule, while achieving similar allocative efficiency. When drift is more positive (and volatility is low), investment timing is similar under the two rules but the welfare-maximizing BL rule delivers a substantially more valuable flow of surplus; that is, a BL rule delivers greater allocative efficiency than a FL rule, without sacrificing dynamic efficiency.

These findings are summarized in Figure 6, which compares the performance of BL and FL rules when the regulator chooses the welfare-maximizing initial access price in each case. The graph plots the value of the regulator's objective function under FL costs as a proportion of its

Figure 5: Welfare assessment of rules with $a_b = a_b^*$ and $a_f = a_f^*$



Notes. The solid and dashed curves plot $\hat{K}_f/\hat{K}_b$ and Θ_f/Θ_b respectively, as functions of μ, where Θ equals the present value of the flow of surplus (measured immediately after investment). When $\hat{K}_f/\hat{K}_b < 1$ the welfare-maximizing BL rule leads to earlier investment than the welfare-maximizing FL rule; when $\Theta_f/\Theta_b < 1$ the welfare-maximizing BL rule delivers a more valuable flow of surplus than the welfare-maximizing FL rule.

value under BL costs as a function of the drift in costs — the height of each curve is

$$\frac{S_f(\hat{K}_f)(\hat{K}_f)^\gamma}{S_b(\hat{K}_b)(\hat{K}_b)^\gamma}.$$

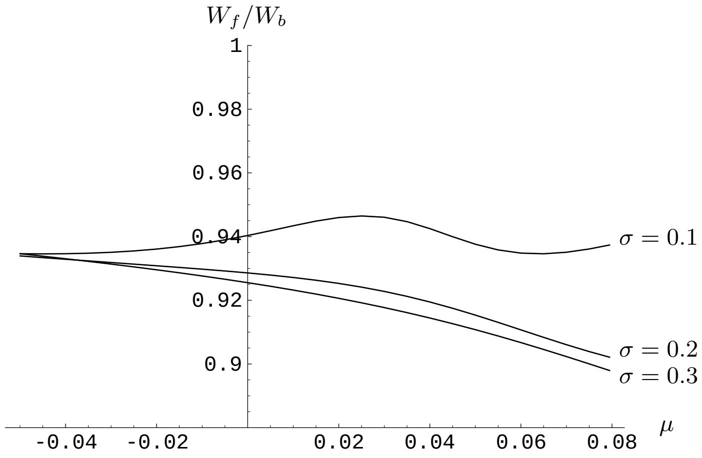
The three curves correspond to different levels of volatility in the cost of launching the project. The function is always less than 1 (even when drift is high and volatility low), so that the BL rule dominates the FL rule from a welfare perspective. In the usual case where the cost of the project is expected to fall over time, the BL rule is clearly preferred by the regulator.

5 Conclusion

Using a simple ex-ante model of an investment project, we found that when a regulator imposes a zero access price (or equivalently, prices access at marginal cost) a firm adopts an investment policy which is too conservative — it waits too long before investing. This inefficient delay arises because the firm ignores the surplus that would flow to competitors and consumers while it waits to invest. However, by imposing either backward-looking or forward-looking access prices, the regulator can induce the firm to invest sooner.

These rules work in two ways. Firstly, like any (positive) access pricing regime, by allowing the firm to charge for access they raise the profitability of the project, thereby raising the opportunity cost of delaying investment. Secondly, like any access regime in which the access price allows for recovery of the incumbent's costs of investment, if the firm delays investment hoping for the required capital outlay to fall, then it will have to share some of the gains

Figure 6: Welfare assessment of rules with $a_b = a_b^*$ and $a_f = a_f^*$



Notes. Each curve plots the value of the regulator's objective function under FL costs (W_f) as a proportion of its value under BL costs (W_b) as a function of the drift in costs.

with competitors and customers — a lower construction cost implies lower access prices, and therefore a smaller profit flow — and this reduces the value of delaying investment. We show that, for a given initial access price (and provided drift is not too positive), backward-looking rules promote earlier investment, because they reduce the risk the incumbent faces as a result of its investment. Numerical examples suggest that this result extends to any backward-looking and forward-looking rules that award the incumbent the same initial market value.

We found that when drift is negative or even slightly positive, the welfare-maximizing backward-looking rule induces earlier investment and delivers only a slightly lower flow of surplus than the welfare-maximizing forward-looking rule; that is, a backward-looking rule delivers greater dynamic efficiency than a forward-looking rule while achieving similar allocative efficiency. When drift is positive and volatility is low, investment timing is similar under the two rules but the welfare-maximizing backward-looking rule delivers a substantially more valuable flow of surplus; that is, a backward-looking rule delivers greater allocative efficiency than a forward-looking rule without sacrificing dynamic efficiency. In all the cases we considered, backward-looking rules dominated in terms of welfare.

The policy implications of this paper are twofold. Firstly, the dynamic efficiency advantages of backward-looking rules should be given more serious consideration by policymakers. Secondly, if a forward-looking rule is used, the initial access price should be set at a level higher than would be the case if a backward-looking rule is adopted. A high rate is required to compensate the incumbent for the risk it bears when faced with forward-looking access prices. Without such compensation, the incumbent will delay investment too long.

Although we only considered the cases of backward- and forward-looking access prices in this paper, our analysis offers some insights into how alternative regimes might perform. Consider, for example, access pricing schemes that combine elements of backward- and forward-looking prices. The greater the weight attached to forward-looking prices, the greater the risk facing the firm, and the higher the initial access price should be set if the regulator is to induce timely investment. One situation in which such a hybrid rule might perform well is where a new technology is involved. In this case, the project cost will be high initially, but trend downwards as the technology becomes established. The cost may be highly volatile at first, but volatility will fall over time. It is important that access prices are backward-looking in the initial high-volatility period, since the risk imposed on the firm by a forward-looking rule would cause it to delay investment. However, once volatility has fallen sufficiently, forward-looking prices would be more attractive because they would be trending down with project cost, thereby leading to higher surplus flows. Thus, a rule that is initially backward-looking but over time evolves to a forward-looking rule may perform well in this situation. Possibilities for future research therefore include expanding the set of access pricing regimes beyond the two polar cases considered in this paper, as well as considering alternative stochastic processes.

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Proofs

Proof of Lemma 1

We need to calculate the value $F(K)$ of an asset generating cash flow of $\pi(\rho K_t)$ for an arbitrary constant ρ. In keeping with the interpretation in footnote 5, suppose there is an asset (which does not pay a dividend) with price S_t evolving according to

$$dS_t = (r + \lambda)S_t dt + \sigma S_t d\xi_t.$$

Consider the portfolio made up of one unit of the asset being valued and $-K_tF'(K_t)/S_t$ units of the spanning asset. The value of this portfolio at time t equals

$$F(K_t) - K_tF'(K_t)$$

and, using Itô's Lemma, its value evolves according to

$$dF(K_t) - \frac{K_tF'(K_t)}{S_t}dS_t + \pi(\rho K_t)dt = \left(\frac{1}{2}\sigma^2K_t^2F''(K_t) + (\mu - r - \lambda)K_tF'(K_t) + \pi(\rho K_t)\right)dt.$$

This portfolio is therefore riskless, and must earn the riskless rate of return r. Thus

$$\frac{1}{2}\sigma^2K_t^2F''(K_t) + (\mu - r - \lambda)K_tF'(K_t) + \pi(\rho K_t) = r(F(K_t) - K_tF'(K_t)),$$

and $F(K_t)$ must satisfy the ordinary differential equation

$$0 = \frac{1}{2}\sigma^2 K^2 F''(K) + (\mu - \lambda) K F'(K) - rF(K) + \pi(\rho K).$$

Since $K = 0$ is an absorbing barrier for geometric Brownian motion, we must have

$$F(0) = \int_0^\infty e^{-rs}\pi(0)ds = \frac{\pi(0)}{r}.$$

It is straightforward to confirm that

$$F(K) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma + \delta}\left(K^{-\gamma}\int_0^K x^{\gamma-1}\pi(\rho x)dx + K^{\delta}\int_K^\infty x^{-\delta-1}\pi(\rho x)dx\right)$$

satisfies the ordinary differential equation above. Making the change of coordinate $x \mapsto yK$ in the first integral and $x \mapsto K/y$ in the second shows that

$$F(K) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma + \delta}\left(\int_0^1 y^{\gamma-1}\pi(\rho K y)dy + \int_0^1 y^{\delta-1}\pi(\rho K/y)dy\right),$$

as required.

Proof of Lemma 2

Clearly, if $K \leq \hat{K}$, $V(K;\hat{K}) = P(K)$, since investment occurs immediately. Suppose, instead, that $K > \hat{K}$, so that investment will be delayed for some unknown period. Following a similar process to that used in the proof of Lemma 1, V satisfies the ordinary differential equation

$$0 = \frac{1}{2}\sigma^2 K^2 V_{KK} + (\mu - \lambda) K V_K - rV.$$

The general solution to this equation is

$$V(K;\hat{K}) = C_1 K^{-\gamma} + C_2 K^{\delta},$$

where C_1 and C_2 are arbitrary constants and

$$\begin{aligned} \gamma &= \frac{\mu-\lambda}{\sigma^2} - \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu-\lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 0, \\ \delta &= -\frac{\mu-\lambda}{\sigma^2} + \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{\mu-\lambda}{\sigma^2} - \frac{1}{2}\right)^2} > 1. \end{aligned}$$

We require that $V(K;\hat{K}) \to 0$ as $K \to \infty$, so that the rights to the project are worthless when launching is prohibitively expensive. This forces $C_2 = 0$. Since investment in the project is triggered when K_t falls below $\hat{K}$, we must have $V(\hat{K};\hat{K}) = P(\hat{K})$. It follows that

$$V(K;\hat{K}) = P(\hat{K})\left(\frac{\hat{K}}{K}\right)^{\gamma}.$$

Proof of Lemma 3

Prior to investment, the value of the incumbent's entitlement to the project is proportional to $P(\hat{K})\hat{K}^{\gamma}$. The incumbent chooses $\hat{K}$ to maximize this expression, leading to the first order condition in (3).

Proof of Proposition 1

We need to consider the case of FL access prices. The first order condition for the incumbent's maximization problem can be written in the form $\hat{K}_f = R_f(a_f\hat{K}_f/K_0)$, where

$$R_f(a) = \frac{\gamma}{\gamma+1}\cdot\Pi_f(a) + \frac{1}{\gamma+1}a\Pi_f'(a).$$

Integration by parts confirms that

$$a\int_0^1 y^{\gamma}\pi'(ay)dy = \pi(a) - \gamma\int_0^1 y^{\gamma-1}\pi(ay)dy$$

and

$$a\int_0^1 y^{\delta-2}\pi'(a/y)dy = -\pi(a) + \delta\int_0^1 y^{\delta-1}\pi(a/y)dy.$$

Therefore

$$\begin{aligned} R_f(a) &= \frac{\gamma}{\gamma+1}\cdot\Pi_f(a) + \frac{1}{\gamma+1}a\Pi_f'(a) \\ &= \frac{1}{\gamma+1}\cdot\frac{\gamma\delta}{\gamma+\delta}\cdot\frac{1}{r}\left(\gamma\int_0^1 y^{\gamma-1}\pi(ay)dy + \gamma\int_0^1 y^{\delta-1}\pi(a/y)dy\right. \\ &\qquad\qquad \left. +a\int_0^1 y^{\gamma}\pi'(ay)dy + a\int_0^1 y^{\delta-2}\pi'(a/y)dy\right) \\ &= \frac{\gamma\delta}{\gamma+1}\cdot\frac{1}{r}\int_0^1 y^{\delta-1}\pi(a/y)dy. \end{aligned} \tag{A-1}$$

Integrating this expression by parts gives

$$\begin{aligned} R_f(a) &= \frac{\gamma\delta}{\gamma+1}\cdot\frac{1}{r}\int_0^1 y^{\delta-1}\pi(a/y)dy \\ &= \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\int_0^1 d(y^{\delta})\pi(a/y) \\ &= \frac{\gamma}{\gamma+1}\cdot\frac{\pi(a)}{r} + \frac{\gamma}{\gamma+1}\cdot\frac{a}{r}\int_0^1 y^{\delta-2}\pi'(a/y)dy. \end{aligned}$$

Proof of Proposition 2

The following interpretation of $R_f(a)$ will be useful.

Lemma A.1 *Let $\tilde{z}$ be the random variable with support $[1,\infty)$, density function $f(z) = \delta z^{-(\delta+1)}$ and distribution function $F(z) = 1 - z^{-\delta}$. Then*

$$R_f(a) = \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\cdot E[\pi(a\tilde{z})].$$

PROOF: The expected value equals

$$E[\pi(a\tilde{z})] = \delta \int_1^\infty z^{-(\delta+1)} \pi(az) dz.$$

The change of variable $z \mapsto 1/y$ leads to

$$E[\pi(a\tilde{z})] = \delta \int_0^1 y^{\delta-1} \pi(a/y) dy.$$

Inspection of equation (A-1) completes the proof. ■

The following result will also be useful. The concavity of π implies that the function $\pi(a) - a\pi'(a)$ is increasing in a for all nonnegative a, so that

$$\pi(a) - a\pi'(a) \geq \pi(0) > 0.$$

Therefore

$$a\pi'(a)/\pi(a) < 1 \text{ for all } a \geq 0. \tag{A-2}$$

We start by proving that there exists an increasing function $\delta^*(\gamma)$ such that the BL rule leads to earlier investment if and only if $\delta > \delta^*(\gamma)$.

The investment threshold under the BL rule is $\hat{K}_b = R_b(\hat{a}_b)$, where $\hat{a}_b$ (the access price on the investment date) is defined implicitly by $\hat{a}_b = (a_0/K_0)R_b(\hat{a}_b)$. Similarly, the investment threshold under the FL rule is $\hat{K}_f = R_f(\hat{a}_f)$, where $\hat{a}_f$ (the access price on the investment date) is defined implicitly by $\hat{a}_f = (a_0/K_0)R_f(\hat{a}_f)$. Therefore, the BL rule leads to earlier investment than the FL rule if and only if

$$\hat{a}_b = \frac{a_0 \hat{K}_b}{K_0} > \frac{a_0 \hat{K}_f}{K_0} = \hat{a}_f.$$

Note that $\hat{a}_b$ satisfies

$$(\gamma + 1)a_b = \frac{a_0}{rK_0} \left(\gamma\pi(\hat{a}_b) + \hat{a}_b\pi'(\hat{a}_b) \right). \tag{A-3}$$

Implicit differentiation with respect to γ, followed by some tedious manipulation, shows that

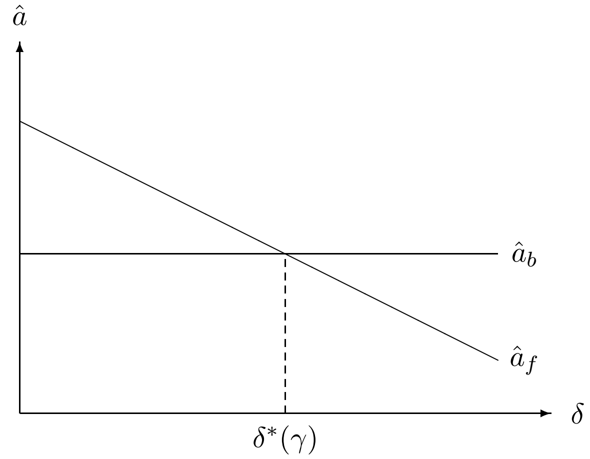
$$\frac{\partial \hat{a}_b}{\partial \gamma} = \frac{\hat{a}_b \left(1 - \frac{a_0}{rK_0}\pi'(\hat{a}_b) \right)}{\gamma \left((\gamma+1)\left(1 - \frac{a_0}{rK_0}\pi'(\hat{a}_b)\right) - \frac{a_0}{rK_0}\hat{a}_b\pi''(\hat{a}_b) \right)}.$$

Combining the inequality (A-2) with equation (A-3) gives

$$(\gamma + 1)\hat{a}_b = \frac{a_0}{rK_0} \left(\gamma\pi(\hat{a}_b) + \hat{a}_b\pi'(\hat{a}_b) \right) > \frac{a_0}{rK_0}(\gamma+1)\hat{a}_b\pi'(\hat{a}_b),$$

implying that $1 > (a_0/rK_0)\pi'(\hat{a}_b)$. The numerator of $\partial\hat{a}_b/\partial\gamma$ is therefore positive. Furthermore, the concavity of π ensures that the denominator of $\partial\hat{a}_b/\partial\gamma$ is also positive, proving that $\hat{a}_b$ is increasing in γ.

Figure 7: Proof of Proposition 2



We now turn to $\hat{a}_f$, which satisfies

$$(\gamma + 1)a_f = \gamma \frac{a_0}{rK_0} E[\pi(\hat{a}_f \tilde{z})], \tag{A-4}$$

where $\tilde{z}$ is the random variable introduced in Lemma A.1. Implicit differentiation with respect to γ, followed by some more tedious manipulation, shows that

$$\frac{\partial \hat{a}_f}{\partial \gamma} = \frac{\hat{a}_f}{\gamma \left(\gamma + 1 - \gamma \frac{a_0}{rK_0} E[\tilde{z}\pi'(\hat{a}_f \tilde{z})] \right)}.$$

Combining inequality (A-2) with equation (A-4) gives

$$\gamma \frac{a_0}{rK_0} E[\tilde{z}\pi'(\hat{a}_f \tilde{z})] < \gamma \frac{a_0}{rK_0} \frac{E[\pi(\hat{a}_f \tilde{z})]}{\hat{a}_f} = \gamma + 1,$$

so that the denominator of $\partial \hat{a}_f / \partial \gamma$ is positive. Therefore, $\hat{a}_f$ is an increasing function of γ.

It is easily seen that increasing δ shifts the distribution of $\tilde{z}$ to the left, in the sense of first order stochastic dominance. Since $\pi(az)$ is an increasing function of z, this change reduces $E[\pi(a\tilde{z})]$, for any given a. A similar calculation to that above proves that $\hat{a}_f$ is a decreasing function of δ.

Figure 7 plots $\hat{a}_b$ and $\hat{a}_f$ as functions of δ for an arbitrary value of γ. From above, $\hat{a}_b$ is constant and $\hat{a}_f$ is decreasing. The value of $\delta^*(\gamma)$ can be found from the point where the two curves intersect. Suppose that γ and δ are such that $\hat{a}_b$ and $\hat{a}_f$ take some common value $\hat{a}$. Noting that

$$\hat{a}\pi''(\hat{a}) < 0 < \gamma \left(1 - \frac{a_0}{rK_0} \pi'(\hat{a}) \right) E[\tilde{z}\pi'(\hat{a}\tilde{z})],$$

it is straightforward to show that

$$\frac{d\hat{a}_b}{d\gamma} < \frac{d\hat{a}_f}{d\gamma}.$$

Returning to Figure 7, where the two curves cross, $\hat{a}_f$ is more sensitive than $\hat{a}_b$ to small changes in γ. Since both curves move up when γ increases, the curve labelled $\hat{a}_f$ moves up further. The point of intersection must move to the right. That is, increasing γ raises δ^*.

Note that

$$\frac{\partial \gamma}{\partial(\mu - \lambda)/\sigma^2} > 0, \quad \frac{\partial \delta}{\partial(\mu - \lambda)/\sigma^2} < 0.$$

Therefore, holding σ^2/r fixed and increasing $(\mu - \lambda)/\sigma^2$ causes γ to rise, and δ^* to rise with it, while δ falls. Thus, when $(\mu - \lambda)/\sigma^2$ is sufficiently large, δ will be less than $\delta^*(\gamma)$, and $\hat{a}_f$ will be greater than $\hat{a}_b$. Thus, there exists a function $N(\cdot)$ such that $\hat{a}_b > \hat{a}_f$ if and only if $(\mu - \lambda)/\sigma^2 < N(\sigma^2/r)$.

Proof of Lemma 4

The proof follows that of Lemma 1, with Π_f replaced by Θ_f everywhere.

Proof of Lemma 5

The proof follows that of Lemma 2, with $P(\hat{K})$ replaced by $S(\hat{K})$ everywhere.