Platform investment and price parity clauses[*]

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Abstract

Platforms use price parity clauses to prevent sellers charging lower prices when selling through other channels. Platforms justify these restraints by noting they are needed to prevent showrooming, in which consumers search on the platform but then switch to buy in another channel, thereby undermining platforms' incentives to invest in providing search services for consumers. In this paper, we study the effect of price parity clauses on platforms' investment in providing search, and evaluate these restraints taking into account investment effects. We find, that wide price parity clauses tend to lead to excessive platform investment while narrow (or no) price parity clauses lead to insufficient platform investment. Even taking these investment effects into account, wide price parity clauses always lower consumer surplus, although their welfare implications are less clear cut.

Keywords: platforms, vertical restraints, showrooming, investment

1 Introduction

Price parity clauses (or platform MFNs as they are sometimes also known) have attracted considerable recent attention from policymakers and scholars. These clauses, imposed by platforms like Amazon or Expedia, require the price a firm offers on the platform is no higher than the prices the same firm offers when selling the

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same item through its own website (narrow price parity) or via any channel (wide price parity). In the latter case, it implies the firm offers its best price through the given platform.

Competition authorities and regulators have adopted different approaches to price parity clauses, depending on the jurisdiction. In some cases, platforms have removed contractual restrictions in the face of investigations or regulatory pressure (such as Amazon in Europe from 2013 and in the U.S. from 2019; also for narrow price parity clauses, Booking.com and Expedia in Europe from 2015, and Australia and New Zealand from 2016). In Europe, several national jurisdictions (Austria, Belgium, France, Italy) prohibit all parity clauses (even narrow ones) via online travel agencies (OTAs) outright. In Germany and Sweden regulation only applies to certain OTAs (HRS and Booking.com in Germany, and Booking.com in Sweden), while other OTAs continue to use price parity clauses within these markets. In other major markets, however, these OTAs continue to use wide price parity clauses. And in the UK, the Competition and Markets Authority found that wide price parity clauses restrict competition between price comparison websites for motor insurance, but it allowed narrow price clauses to remain. For a recent summary of these and other cases, and an argument for greater antitrust enforcement to be taken in the U.S., see Baker and Scott Morton (2018).

Platforms defend their price parity clauses by noting that they are necessary to prevent a free-riding problem, which is that consumers search on the platform for suppliers since it provides the lowest search costs, but then having found a good match, switch to buy through some other channel at a lower price if this is allowed (we call this "showrooming"). A key argument put forward by online search platforms such as Booking.com to defend their imposition of price parity clauses is that showrooming could undermine a platform's incentive to invest in improving the quality of the search and matching on its website. For instance, according to the OCED (2016) report (p.4), "When competition agencies around Europe started scrutinising these clauses and expressing concerns about their possible anticompetitive effects, Booking.com argued that these clauses were necessary to protect its investment in the facilities offered on its website" and (p.47) "There is less consensus when it comes to the anticompetitive effects of narrow agreements, which place restrictions only on the prices a hotel can set on its own site. This lack of consensus relates to the magnitude of the concerns about free-riding by hotels on OTAS' investments in their websites and search facilities, which in the extreme could have the effect of driving the OTAs out of business." We therefore explore how platforms' investment incentives in providing search benefits are affected by price parity clauses, and whether taking into account these effects changes the economic analysis of these restrictions.

To address these questions, this paper introduces a model of price parity clauses that includes a platform's investment in improving its search technology. We have in mind the platform investing in collecting and analyzing data to improve the search algorithm, providing more comprehensive information for the users, and offering a better user interface for search, all of which would increase the expected match value of using the platform.

Consider first the case of a monopoly platform. We find that it will not invest in search cost reduction if it cannot use a price parity clause but it will invest excessively in such search cost reduction when it can use a price parity clause. This is because without a price parity clause, its fees are constrained to be equal to only the convenience benefits it offers because of the showrooming problem, so it cannot recover any of its investment via higher fees. On the other hand, with a price parity clause in place, a platform can extract not only the additional social surplus generated by the lowered search cost but also the extra profit margin that firms lose in intensified competition on the platform. Taking into account these investment effects, a price parity clause can lead to higher or lower welfare. However, it always lowers consumer surplus. A price parity clause removes the restriction on the platform's fees implied by the direct market alternative since consumers always prefer to buy on the platform given prices are never higher. As a result, the monopoly platform fully extracts consumers' expected surplus from trade.

We next consider the use of price parity clauses in case there is an incumbent platform that faces potential competition from a differentiated entrant, and can first decide how much to invest in search cost reduction before the entrant does likewise. In this case, narrow price parity clauses do not solve the under-investment problem. Each platform does not want to invest in search cost reduction, given the other platform can free-ride on their lower search costs by not investing. Specifically, consumers who prefer to buy through the high-search-cost platform will search on the low-search-cost platform and then switch to complete their transaction on the high-search-cost platform. Moreover, the low-search-cost platform's fee is constrained by platform competition which prevents it from recouping its investment by charging higher fees to consumers who regard it as the preferred platform to finalize transactions. So our results support the idea that wide price parity clauses are required to incentivize investment in search cost reduction.

By removing platform competition and alleviating free-riding from the high-search-cost platform, wide price parity clauses help the low-search-cost platform build an advantage through charging higher fees and restore its incentive to invest. If this advantage is large enough, the incumbent will prefer to invest more than the rival in reducing search costs and be the platform consumers prefer to start their searches on. In this case, with the ability to use wide price parity clauses, the incumbent platform overinvests in search cost reduction to prevent the entrant from wanting to invest at all. As a result, the welfare effects are ambiguous. However, for the same reason as in the monopoly platform case, we find consumers are always worse off with wide price parity clauses. If the advantage of being the low-search-cost platform is not sufficiently large, the incumbent will not invest and completely free-rides on the entrant's investment. In this case, wide price parity clauses still harm consumers and lead to ambiguous effects on total welfare.

In the next section we review the related literature. In Section 3 we lay out the basic model and provide some preliminary analysis. We then use the model to analyze what happens for platform investment in the case of a monopoly platform (Section 4) and competing platforms (Section 5). Finally, in Section 6, we briefly conclude.

2 Literature review

A fast growing literature, which we have contributed to, has explored theoretically how price parity clauses can restrict competition, affect prices, and impact overall welfare. Platforms impose price parity clauses along with the *agency model* under which firms directly set on-platform prices to consumers and platforms charge a seller fee for each transaction. Price parity clauses can therefore be thought of as a type of vertical restraint imposed by platforms on participating sellers. Previous theoretical works include Edelman and Wright (2015), Boik and Corts (2016), Johnson (2017), Johansen and Vergè (2017), Carlton and Winter (2018), Ronayne and Taylor (2018), Wals and Schinkel (2018) and Gomes and Mantovani (2020). This stream of work emphasizes that price parity clauses lessen the normal substitution effect that arises when a platform raises its fee to sellers because the price of the direct channel or competing channels also have to increase in tandem with such a fee increase.[1] As a result, the literature generally finds price parity clauses result in

[1] An additional point made by Edelman and Wright (2015) is that high platform fees under price parity clauses can be used to fund platform benefits (including rewards) to consumers, which

higher fees and consumer prices, and lower consumer surplus. Some exceptions include Johansen and Vergè (2017) who show that given platform fees are constrained by the possible of firms delisting, the imposition of price parity clauses does not necessarily result in higher fees and lower consumer prices, and more recently (Mariotto and Verdier, 2020 and Liu et al., 2021) who also show possible positive effects of price parity clauses on consumers.

Our paper differs from this previous literature in two main ways. First, we consider a model of search platforms which naturally give rise to the showrooming problem we are interested in. Second, we study the impact of platform investment in reducing consumer search costs. By also focusing on search platforms, Wang and Wright (2020) looked into the interaction between showrooming and price parity clauses. They distinguish wide and narrow price parity clauses and show that, while wide price parity clauses almost always harm consumers, imposing narrow price parity clauses can benefit consumers if a severe showrooming problem threats the platforms' viability and the competition between platforms is sufficiently effective. In Wang and Wright (2020), a platform only makes a binary decision, whether to operate or not. However, arguably, the viability of certain platforms like Amazon and Expedia is not really in question, even in the absence of any price parity clauses. Rather, the concern is that showrooming could undermine a platform's incentive to invest in helping consumers find the best firm to buy from, which the firm or rival platforms might otherwise seek to free-ride on. Thus, we modify and extend the analytical framework in Wang and Wright (2020) to examine this new issue.

Two other papers have explored the implications of price parity clauses for investment, but they focus on very different types of investments. Specifically, Edelman and Wright (2015) look at investment in the convenience benefit offered by the platform in completing transactions. They show that price parity clauses leads to an over-investment in the provision of these platform benefits. Maruyama and Zennyo (2020) look at the implications of investment that boosts consumer demand. These are very different types of investment. We focus on investment that reduces search costs because improving search and matching is the core feature of these platforms that firms and/or rival platforms can free ride on, and which they have used to defend their use of price parity clauses. In addition to focusing on a different type of investment, we also, following Wang and Wright (2020), consider the effect of narrow price parity clauses on investment which is not considered in Edelman and Wright (2015) and Maruyama and Zennyo (2020).

can result in too many consumers joining and using the platform compared to the efficient level.

The ban of price parity clauses in several European countries has led to several recent empirical investigations of the effects of price parity clauses. Hunold et al. (2018) used data from Kayak (a price comparison website) to investigate the changes in the German hotel industry following the ban. They show that, along with providing more room availability and expanding the number of sales channels, the ban leads hotels to charge low price more frequently in Germany relative to countries without such a ban. Mantovani et al. (2020) provide quasi-experimental evidence on the full removal of price parity clauses in France for hotels listed on Booking.com. They show a significant decrease in hotel prices in the short run, but a more limited effect in the medium run. Ennis et al. (2020) evaluate the impact of EU's ban of wide price parity clauses (but not the narrow one) and show it is associated with a price drop on the direct selling channel. The main focus of the empirical literature so far has been on prices and product availability across different selling channels, with rather little discussion of the implications for platforms' investments.

3 The model and preliminaries

Our model setup follows that of our earlier paper Wang and Wright (2020), but with a couple of key differences. In that paper, in order to prevent showrooming from completely undermining the viability of the platform, we assumed there was a mass of "direct consumers" who only searched and purchased directly. This was to ensure that sellers faced an opportunity cost of undercutting on the direct price, thus limiting their incentive to engage in showrooming. In the current model, all consumers are ex-ante identical, and we ensure showrooming does not completely undermine the viability more directly, by assuming consumers enjoy some exogenous convenience benefits of using the platform versus transacting directly. This simplifies the model setting, allowing us to extend the model in another direction — modelling the platform's investment in search cost reduction. Despite these differences, much of the model setup and preliminary analysis closely follows Wang and Wright. Readers familiar with the setup there can skip to Section 4 after noting the description of b in the platform setup and of the investment setup below.

There is a continuum of consumers (or buyers) and risk-neutral firms (or sellers), of measure 1 in each case. Each firm produces a horizontally differentiated product and has its production cost normalized to zero. In the baseline setting, there is a single platform (M) which facilitates trades between the firms and consumers. In this section we present the model based on a single platform. When we extend the model to allow for platform competition we will explain how our assumptions need to be modified.

☐ **Preference**. Each consumer demands at most one unit of one of the products and has a match value for firm i's product, denoted v_i. This match value is realized independently across firms and consumers, and is distributed according to a common distribution function G over $[\underline{v}, \overline{v}]$ for any consumer and firm. We assume G is twice continuously differentiable with a weakly increasing hazard rate and a strictly positive density function g over $[\underline{v}, \overline{v}]$. An increasing hazard implies $1 - G(\cdot)$ is log-concave, which along with other assumptions, implies a firm's optimal pricing problem is characterized by the usual first-order condition. We define $\lambda(x) = \frac{1-G(x)}{g(x)}$ as the inverse hazard rate, which is decreasing.

☐ **Consumer search**. Firms can always be searched directly. When searching directly, each consumer incurs a search cost s_d per visit. By sampling firm i, a consumer l learns its price p_d^i and the match value v_l^i. The search cost can be understood as the non-trivial cost of investigating each firm's offerings, which is what enables a consumer to learn the relevant price p_d^i and their match value v_l^i. Consumers search sequentially with perfect recall.

The utility of a consumer l is given by

$$v_l^i - p_d^i - ks_d$$

if she buys from firm i at the price p_d^i after searching k times. The consumers' gross expected surplus of searching directly (including search costs, but not taking into account the price paid) is x_d,[2] which is implicitly given by

$$\int_{x_d}^{\overline{v}} (v - x_d) dG(v) = s_d. \tag{1}$$

We assume s_d is sufficiently small so that $\int_{\underline{v}}^{\overline{v}} (v - \underline{v}) dG(v) > s_d$. This, together with the fact the left-hand side of (1) is strictly decreasing in x_d and equals zero when $x_d = \overline{v}$ ensures a unique value of (1) exists satisfying $\underline{v} < x_d < \overline{v}$.

The standard results of sequential consumer search imply that, when searching directly, each consumer employs the following optimal cutoff strategy: (i) she starts searching if and only if $x_d \geq p_d$; (ii) she stops and buys from firm i if she finds a price p_d^i and match value v_l^i such that $v_l^i - p_d^i \geq x_d - p_d$; and (iii) she searches the

[2] The details of the derivation of x_d can be found in Wang and Wright (2020).

next firm otherwise. This implies that a consumers' actual net utility from firm i (i.e. $v_l^i - p_d^i$) must be at least equal to $x_d - p_d$, which is also the consumer's expected value of initiating a search.

□ **Platform**. A platform M provides search and transaction services to consumers. We assume M does not incur any marginal cost of handling transactions. If a firm i also sells over the platform, its price on the platform is denoted p_m^i. Searching on the platform costs $s_m > 0$ per visit, which is assumed to be less costly than searching directly, i.e. $s_m \in (0, s_d)$. By sampling firm i on M, a consumer l learns its price p_m^i and the match value v_l^i. Platforms thus are assumed to speed up the sequential comparison of different firms' offerings by standardizing the information provided and making it easily comparable. For example, a platform may make it easier to compare the suitability of hotels' locations, facilities, and room types, along with their relevant prices; or airlines' flight times, connections, aircraft types, cancellation policies and baggage policies, together with their relevant fares. In case all firms are available on M, the optimal stopping rule for a consumer searching on M is the same as when searching directly but with the consumers' (gross) expected surplus of searching via M being x_m, which is implicitly given by

$$\int_{x_m}^{\overline{v}} (v - x_m) dG(v) = s_m$$

and with the prices p_d^i and p_d replaced by p_m^i and p_m respectively, where p_m is the symmetric equilibrium price on M.

When consumers complete a transaction on the platform we assume they also obtain a convenience benefit of $b \geq 0$. This captures that the platform may make completing a transaction more convenient (e.g. with respect to payment and entering customer information) and may provide superior after-sale service (e.g. tracking delivery, manage bookings, etc). For instance, large platforms like Amazon, Booking.com and Expedia have created their own consumer Apps to provide such benefits.

Let us ignore consumers' option of searching directly for the moment. Then, consumers will start searching on M if and only if $x_m \geq p_m - b$, reflecting that b is an additional benefit they obtain when they make a purchase through M. Since $s_m < s_d$ and the left-hand side of (1) is decreasing in x_d, we have $x_m > x_d$. Consumers tend to search more when using M due to the low search cost; i.e. they hold out for a higher match value. We denote this difference in the gross surplus from searching through the platform and directly as

$$\triangle_s = x_m - x_d$$

and call it the *surplus differential* of the platform. It reflects the additional surplus consumers enjoy from being able to search at a lower cost on the platform, ignoring any difference in prices.

□ **Showrooming**. In the case of a single platform, showrooming means that consumers can buy directly for a lower price after finding a good match through searching on the platform. With competing platforms, consumers can also search through one platform to find a good match and then buy through another platform. Showrooming is possible only if consumers can observe a firm's identity when they search on the platform. We further assume that consumers can costlessly switch in either direction (and multiple times). Such costless switching ensures consumers can switch back to buy on the platform in case they find that the price on the other channel is higher than expected.

□ **Instruments**. We allow the platform to set a non-negative per-transaction fee charged to firms when they make a transaction through M, which we denote f. An obvious way to avoid the showrooming problem is to charge firms a lump-sum fee instead. However, lump-sum fees charged to firms have not been used even when price parity clauses (both narrow and wide) have been banned. This suggests it is not a choice that platforms find practical to implement in practice. One reason for this, outside of our model, could be that with heterogenous consumers and firms, lump-sum fees would cause some firms to no longer join, which through cross-group network effects, could lead to a downward spiral of reduced consumer and firm participation. With competing platforms, this additionally means firms will prefer to avoid multihoming, so the platform which introduces the lump-sum fees may lose sellers and consumers to the one that does not. Wang and Wright (2020) provide further justification for the focus on transaction fees, and also discuss the role of per-click fees and referral fees. [3]

□ **Investment**. Platforms can invest in reducing their search cost s_m.[4] Specifically,

[3]Some platforms use percentage fees. In our model, the effect of percentage fees is similar to transaction fees given that all firms are ex-ante homogenous. Percentage fees will only make a substantive difference if firms are heterogenous and they pay different amounts to the platform as a result of percentage fees.

[4]Note we have assumed initially M has lower search costs ($s_m < s_d$) since otherwise consumers would never have a reason to use it.

suppose that M can reduce its search cost from s_m to s'_m and thereby increase the consumers' expected (gross) match value from some initial level x_m^0 to its new level $x_m = x_m^0 + y$. Since y is monotonically increasing in $s_m - s'_m$ according to

$$\int_{x_m^0+y}^{\overline{v}} [v - (x_m^0 + y)]dG(v) = s_m - (s_m - s'_m),$$

we can equally focus on the choice of y instead of the choice of $s_m - s'_m$. This formulation captures anything that M might invest in to improve the utility consumers get from using the platform to find firms, e.g., it would include investing in collecting and analyzing data to improve the search algorithm, providing more comprehensive information for the users, and a better user interface for search. This might also include a platform investing in improving its recommendations so as to filter out matches that are less likely to be relevant, thereby shifting consumers match values from random search to a distribution $\hat{G}$ that stochastically dominates G.

M's investment cost is $C(y)$ with $C(\cdot)$ strictly increasing, continuously twice differentiable, convex, $C(0) = 0$ and $C'(0) < 1$. Later we will show the standard result, that the firms' mark-up is just equal to the inverse hazard function. To ensure existence and second-order conditions for the platform's maximization problem holds, we assume $\lambda'(x_m^0 + y) + C'(y) > 1$ for some sufficiently large y, and that $\lambda(x_m^0 + y) + C(y)$ is convex in y. The latter property is true provided C is sufficiently convex in its argument. Indeed, for many reasonable distributions of G, such as generalized Pareto (which includes the Uniform, Exponential, Constant Elasticity, etc), Normal, Logistic, Type I Extreme Value, and Weibull, λ is linear or convex in its argument, so the required second-order condition follows whenever C is convex.

A social planner maximizes the incremental total welfare of investment by solving

$$\max_y \{y - C(y)\},$$

which yields the efficient investment level y^e implied by the first-order condition

$$C'(y^e) = 1. \tag{2}$$

Our assumptions ensure the existence and uniqueness of y^e, and that it corresponds to the welfare maximum.

□ **Timing and equilibrium concept**. The timing of the game is as follows:

1. M chooses the investment level y. The investment level y is observed by all parties.

2. M sets the fee f to maximize its profits. Firms and consumers observe the fee.

3. Firms decide whether to join M and set prices.

4. Without observing firms' decisions, consumers decide whether to search on M or search directly (possibly switching search channels along the way) if they search at all. And if they search, they carry out sequential search until they stop search or complete a purchase.

The assumption that the platform makes its investment and fee decisions sequentially is innocuous. We adopt this timing to be consistent with our timing assumptions when there are competing platforms. In the case of a single platform M, we focus on a symmetric perfect Bayesian equilibrium where all firms make the same joining decisions and set the same prices. We adopt the usual assumption that consumers hold passive beliefs about the distribution of future prices upon observing any sequence of prices. In any user subgame where firms and consumers make decisions, we select an equilibrium in which all firms join the platform and set the same prices if such a symmetric equilibrium exists. Thus, we rule out a trivial equilibrium in which consumers do not search through the platform because they expect no firms to join, and firms do not join because they do not expect any consumers to search through the platform. Finally, in case firms' direct prices (or prices on the platform) are not pinned down as part of the equilibrium of a user subgame, we determine equilibrium prices $p(n)$ by assuming there is an exogenous positive mass n of consumers that only search and buy directly (or on the platform) and let equilibrium prices p_d in the direct market (or p_m on the platform) be the limit of $p(n)$ as n goes to zero.

4 Monopoly platform investment

In this section, we explore the incentive of a monopoly platform to invest in reducing consumer search costs under various specifications of showrooming and price parity clauses (PPCs). We first establish the benchmark case in Section 4.1 where switching between channels is forbidden and without PPCs. In Section 4.2, we show the negative impact of showrooming on platform investment. Section 4.3 shows how the imposition of PPCs restores the platform's incentive to invest.

4.1 Benchmark without showrooming

We can separately analyze firms' pricing behavior on and off the platform if switching between channels is forbidden. A consumer who searches directly will buy from firm i if and only if $v^i - p_d^i \geq x_d - p_d$. Therefore, firm i's profit is proportional to $p_d^i(1 - F(x_d - p_d + p_d^i))$. The equilibrium prices in the direct market are given by

$$p_d = \lambda(x_d). \tag{3}$$

We assume the search cost s_d is sufficiently low so that

$$x_d > \lambda(x_d). \tag{4}$$

This ensures that $x_d > p_d$, so consumers expect a positive surplus from searching in the first place.

A consumer who searches on M will buy from firm i if and only if $v^i - p_m^i \geq x_m - p_m$. Therefore, firm i's profit is proportional to $(p_m^i - f)(1 - F(x_m - p_m + p_m^i))$. The equilibrium prices on M are given by

$$p_m(f) = f + \lambda(x_m). \tag{5}$$

We assume

$$x_m + b > \lambda(x_m) \tag{6}$$

so that consumers expect a positive surplus from searching and buying through M when it doesn't charge anything.

We denote the difference in the equilibrium markups across the two channels as

$$\triangle_m = \lambda(x_d) - \lambda(x_m)$$

and call it the *markup differential* of the direct market. Since $s_m < s_d$, we have $x_m > x_d$, which implies $\lambda(x_m) \leq \lambda(x_d)$ or $\triangle_m \geq 0$ given our assumption that the hazard rate is weakly increasing. That is, the equilibrium price markups of firms are lower on M, reflecting that consumers search more on M.

The absence of a showrooming possibility implies that consumers can only complete a purchase from a particular firm on the channel that they found the firm on.

With all firms available for searching on M, a consumer prefers to start their search through M provided the expected utility exceeds the one of searching directly, or

$$x_d - p_d \leq x_m + b - p_m. \tag{7}$$

Substituting (3) and (5) into (7), consumers will use M to search if and only if

$$f \leq \triangle_s + \triangle_m + b. \tag{8}$$

Consumers benefit from M due to lower search costs (the surplus differential), intensified competition (the markup differential), and transaction convenience (b). Equation (8) says that in order to attract consumers, platform fees cannot exceed the sum of the three benefits that M provides consumers. The platform can only make a positive profit if consumers choose to use it. For given s_m, the equilibrium involves M setting the fee that leaves consumers indifferent between searching on the platform and searching directly. That is

$$f^* = \triangle_s + \triangle_m + b. \tag{9}$$

So the benefits coming from the lower search cost, the fiercer on-platform competition and the transaction convenience are fully offset by the monopoly transaction fee. The resulting equilibrium price on M is obtained by substituting (9) into (5), implying

$$p_m(x_m) = \triangle_s + \lambda(x_d) + b. \tag{10}$$

The equilibrium outcome involves all consumers searching and purchasing on M. The platform's profit is $\Pi^* = \triangle_s + \triangle_m + b$.

We are now ready to analyze the effects of platform investment. After choosing an investment level y, by the same logic as above, consumers' expected match value from searching on M will be $x_m^0 + y$ and M can increase its fee to

$$f(y) = x_m^0 + y - x_d + \lambda(x_d) - \lambda(x_m^0 + y) + b.$$

M's optimal investment level y^* is given by maximizing $f(y) - C(y)$ with respect to y, and given our assumptions, is characterized by the first order condition:

$$C'(y^*) = 1 - \lambda'(x_m^0 + y^*). \tag{11}$$

Comparing (2) and (11), it is clear from the increasing hazard rate property ($\lambda'(\cdot) \leq 0$) that M invests weakly more than the efficient level. Throughout the paper, by "increasing/decreasing" or "over-/under-investment" we mean in a weak sense unless specified otherwise.

Proposition 1. (Platform investment in the benchmark model)
When both showrooming and PPCs are absent, the monopoly platform over-invests relative to the efficient investment level.

When the costs of searching on the platform decrease, consumers tend to search more on the platform and obtain higher surplus (other things equal). This happens both because the expected match value becomes higher as the platform's search cost is lower, and also since competition between firms is intensified so firms will price lower for any given platform fee. The increase in expected match value is a social benefit, which the platform correctly internalizes when deciding how much to invest in reducing search costs. However, the reduction in firms' markups from intensified competition is not a social benefit. It allows M to set a higher fee while keeping consumers still willing to use the platform, and so ultimately results in a transfer from firms to M. Thus, it is the intensified firm competition induced by a reduction in search costs which explains why M over invests in search cost reduction in the absence of showrooming.

4.2 Showrooming

Suppose now consumers are able to switch to buying directly having found a good match through the platform, potentially at a lower price. The equilibrium in the previous section in which prices are higher on the platform than off the platform by the amount $\triangle_s + b$ would indeed lead consumers to switch in this way. As a result, M would want to lower the fee f it charges firms. However, depending on the fee charged, firms may also want to raise their prices on M and/or lower their direct prices to induce consumers to switch, given that firms avoid paying the fee f on consumers that purchase directly.

We first characterize consumers' optimal search strategy. Consumers always prefer searching on M to searching directly as $x_m > x_d$ and switching incurs no cost. But consumers will search on M only if they expect non-negative net surplus, i.e.,

$$x_m - \min\{p_m - b, p_d\} \geq 0. \tag{12}$$

If $p_m - b > p_d$, consumers will switch and make their purchase directly. If instead $p_m - b \leq p_d$ consumers who search on M expect to make their purchase on M. The value of stopping in this setting is the highest value between purchasing immediately on M and switching to purchase in the direct search market, which is therefore given by

$$v^i - \min\{p_m^i - b, p_d^i\}.$$

The value of continuing to search on the platform is given in (12). Using the standard argument for optimal sequential search, consumers' optimal stopping strategy is therefore:

- if $v^i - \min\{p_m^i - b, p_d^i\} < x_m - \min\{p_m - b, p_d\}$, continue to search on M.
- if $v^i - \min\{p_m^i - b, p_d^i\} \geq x_m - \min\{p_m - b, p_d\}$,
  - stop and buy on M immediately if $p_m^i - b \leq p_d^i$.
  - stop and switch to purchase from direct search market if $p_m^i - b > p_d^i$.

With consumers' optimal strategy specified above, we can now specify the equilibrium when showrooming is possible, for any given level of platform investment that is already determined in stage 1. The equilibrium outcome involves all consumers searching and purchasing through M.

Proposition 2. (Showrooming equilibrium)
Stage 2: M sets the fee $f^* = b$.
Stage 3: If $f \leq b$, firms' prices are given by (3) and (5); if instead $f > b$, firms' prices are given by $p_d = \lambda(x_m)$ and (5).

When the platform's fee is set above b, a firm can do better inducing consumers to switch to buy directly. In the equilibrium that would result, all consumers would search on M but switch to purchase directly with direct prices determined as if firms competed on M but without facing any fees. To rule this switching equilibrium out, M has to lower its fees to b. In this case, the fee is no more than the convenience benefits of using the platform and therefore firms cannot profitably induce consumers to switch. Obviously, the equilibrium fee under showrooming is independent of consumers' expected match value of using M, and is therefore independent of M's investment in increasing the value. M then has no incentive to make investment at all as all the benefits arising from the investment go to consumers but M needs to bear the investment cost. As a result $y^* = 0$ and $x_m = x_m^0$ in this case with showrooming.

Proposition 3. (Platform investment under showrooming)
When consumers are free to switch between channels, the monopoly platform underinvests relative to the efficient investment level.

Showrooming activities impose a binding constraint on M's fee. Consumers make switch/purchase decisions after they have enjoyed the additional benefit brought about by the additional platform investment (i.e. they can more easily find a good match given the lower search cost). This asynchrony between the timings of consumers obtaining search benefits and making purchases causes M to be unable to capture profits from the investment it makes in reducing the search costs for consumers. In contrast, consumers obtain the convenience benefit b only after they purchase on M, which does allow M to sustain a strictly positive fee up to the value of convenience benefit, and any investments in convenience benefits would be able to be captured by M in its monopoly pricing (as was the case in Edelman and Wright, 2015).

4.3 Price parity clauses

One way a monopoly platform can eliminate showrooming and the constraint it implies for the platform's fee is to use PPCs, thereby requiring the price firms set on the platform be no higher than the price they set for the direct channel. If a firm joins M and thereby accepts a PPC, its direct price must be at least as high as its price on the platform. Thus, the showrooming constraint is ruled out. However, PPCs allow M to do even better, raising its fee beyond the level it sets in the benchmark case without showrooming. We first characterize the platform's and firms' pricing equilibrium under PPCs in stages 2 and 3.

Proposition 4. (Equilibrium with PPCs)
Stage 2: M sets the fee

$$f^* = x_m - \lambda(x_m) + b. \tag{13}$$

Stage 3: Suppose $f \leq f^*$. Firms' prices on M are given by (5) and the firms' direct prices are given by the maximum of (3) and (5). If $f > f^*$, firms do not join M and their direct prices are given by (3).

Facing the same or lower price on the platform, but lower search costs, consumers will all search on M and will not switch to buy directly. In equilibrium, firms will all participate since if they do not, they will not attract any business given all consumers are searching on M. The optimal fee charged by M implies firms will set their prices (both through the platform and directly) equal to $x_m + b$, so consumers expect zero surplus from search in equilibrium and are just willing to search. Despite the high on-platform price induced by the high fee, consumers do not want to search directly, since this would imply a negative surplus given in equilibrium prices are set equally high regardless of which channel they come through but search costs are higher when they search directly and they would lose b. Given this, the binding constraint for M under PPCs is that consumers still would like to participate in search on the platform. M therefore sets a fee which leads to zero consumer surplus.

Under PPCs, M invests in search so as to maximize its fee f, which it collects on all consumers. This is equivalent to maximizing:

$$\max_{y} \left\{ x_m^0 + y - \lambda(x_m^0 + y) + b - C(y) \right\}.$$

This leads to the same solution as in (11). It implies that, just as in the case without showrooming, M over-invests in search cost reduction. When PPCs are used, M can extract not only the additional value generated by the lowered search cost but also the extra profit margin that firms lose from intensified competition on M. As a result, M is strictly better off imposing PPC, and so would always choose to impose such a contract.

With showrooming but in the absence of PPCs, the total welfare is $x_m^0 + b$, reflecting that there will be no investment in search cost reduction. If PPCs are imposed, the total welfare is $x_m^0 + y^* + b - C(y^*)$ where y^* is determined by (11). Clearly, PPCs increase total welfare if and only if $y^* > C(y^*)$. Since there is insufficient investment without PPCs and excessive investment with PPCs, the effect of PPCs on welfare is in general ambiguous.

We next consider how PPCs impact consumer surplus. Without PPCs, since there will be no investment, consumer surplus is $x_m^0 - \lambda(x_m^0)$, reflecting that the fee firms pay will just equal b. Recall from (4) that $x_d > \lambda(x_d)$. Since $x_m^0 \geq x_d$ and $\lambda(x_d) \geq \lambda(x_m^0)$, we have that consumer surplus is strictly positive without PPCs. Under PPCs, however, consumer surplus is zero as M sets its fee at the level at which consumer surplus is fully extracted.

We summarize our findings in the following proposition.

Proposition 5. (Platform investment under PPCs)

- *With PPCs, M over-invests in search relative to the efficient investment level.*
- *Taking into account how the platform's investment depends on PPCs, the effect of PPCs on total welfare is ambiguous. However, PPCs unambiguously lower consumer surplus.*

Taking into account the effects of investing in search cost reduction therefore does not change the common view that when a monopolist platform imposes PPCs, consumers are harmed. Moreover, the result that M invests too much under PPCs reinforces the result in Edelman and Wright (2015) who reach the same conclusion with respect to the platform's investment in increasing transaction benefits, although for quite different reasons.

We can illustrate our general findings on the welfare effects of PPCs more explicitly by supposing the distribution of v is the generalized Pareto distribution

$$G(v) = 1 - \left(1 - \frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)^{\frac{1}{\varepsilon}}$$

on the interval $[\underline{v}, \overline{v}]$ with $\varepsilon > 0$, and the cost function is the power function $C(\Delta x_m) = \frac{c}{\eta}(y)^{\eta}$, with $c > 0$ and $\eta > 1$. Then (11) implies

$$y^* = \left(\frac{1+\varepsilon}{c}\right)^{\frac{1}{\eta-1}}$$

and so $y^* > C(y^*)$ if and only if

$$\eta > 1 + \varepsilon.$$

To interpret this condition, note that $\eta = 2$ implies C is quadratic and $\varepsilon = 1$ implies G is linear. In this special case, the welfare under a PPC exactly equals that arising without a PPC. If the cost function is less convex than quadratic or the distribution function is more convex than linear, then welfare will be lower under a PPC. Conversely, welfare is higher under a PPC if the cost function is more convex than quadratic or the distribution function is more concave than linear. More generally, welfare is lower with a PPC if the difference in the shape parameters $\eta - \varepsilon$ is less than unity, and is higher in the opposite case.

A straightforward extension of this model is to allow an exogenous fraction of consumers who cannot switch channels. In this case, even without a price parity clause, the platform will want to invest in reducing its search costs. The logic is that the platform can set a fee up to the point where non-switching consumers are indifferent between searching on the platform and searching directly. The higher the benefit from searching on the platform (via the platform's investment in reducing search costs), the higher the fee the platform can charge. All other consumers, who remain free to switch, will search on the platform and buy directly (i.e., showroom). As the fraction of these non-switching consumers increases, the platform's incentive to invest in reducing search costs will increase since it gets a larger increase in revenue from doing so.

5 Competing platform investment

In this section we modify the previous monopoly model to allow for platform competition. We start by supposing there is an incumbent platform, M^I, that is in the identical situation to that modeled above for the monopoly platform. It already provides a basic search service with search costs s_m. By investing the amount y^I at the cost $C(y^I)$ it can raise consumers' gross surplus from searching for a match on the platform from x_m^0 to $x_m^0 + y^I$. Following the incumbent's decision, an entrant platform, M^E, also decides how much to invest in reducing its search cost. M^E offers a basic search service with search costs s_m, and it can reduce this search cost with the same technology available to M^I. Define $x_m^j = x_m^0 + y^j$ to be the gross surplus available for consumers searching and purchasing on platform M^j $(j = I, E)$ after the respective platforms have made their investment decisions. We assume that there exists y such that $x_m^0 + y + b - \lambda(x_m^0 + y) < C(y)$ which ensures that in our setting platforms' profits eventually become negative if they invest too much.

The platforms are horizontally differentiated. In an earlier version of this paper (Wang and Wright, 2016), we derived the equilibrium for competing homogenous platforms. The equilibrium outcome in the homogenous case corresponds to the limit case of the equilibrium outcome we derive here as the degree of differentiation goes to zero provided we focus on the equilibrium in which the low search cost platform captures all profits. Consumers have heterogenous preferences regarding which platform provides higher convenience benefits. More specifically, half of the consumers obtain convenience benefit b from buying on M^I and $b-a$ from buying on M^E, while the other half of the consumers obtain convenience benefit b from buying on M^E and $b - a$ from buying on M^I. We will refer to the platform on which a consumer obtains b rather than $b-a$ as the consumer's "preferred platform". Before choosing which platform to use, each consumer observes a random shock a, which is drawn from a continuously differentiable distribution $H(a)$ on $[0, b]$, which has a strictly positive density on $[0, b]$. Suppose t is a non-negative constant. We assume the maximization problems $\max_x\{x(1+H(t-x))\}$ and $\max_x\{x(1-H(x-t))\}$ each have a unique solution which can be characterized by the respective first order conditions. Note this requirement is satisfied when the corresponding density function h is log-concave.[5] We assume $\triangle_s + \triangle_m + b \geq \frac{1}{H'(0)}$, so that without PPCs, platform competition is effective in that it leads to fees lower than the fees that would be set by a monopoly platform. Finally, we assume

$$\max_y \left\{ \frac{1}{2}(x_m^0 + y - \lambda(x_m^0 + y) + b) - C(y) \right\} > \max \left\{ \frac{b}{2}, \frac{1}{2H'(0)} \right\}. \tag{14}$$

As we will show later, this means that a platform prefers the maximum profit it can earn by serving half of the consumers and fully extracting their surplus to the equilibrium profit it would get if it also served half of the consumers but its fees were constrained by showrooming or platform competition (in which case it does not invest at all in reducing search costs). In case search costs and expected prices are the same on both platforms, the tie-breaking rule is such that consumers search and buy on their preferred platform.

In this model, M^I first decides its investment level (stage 1), followed by M^E (stage 2), after which the two platforms choose their fees simultaneously (stage 3). The assumption that platforms choose their fixed investments in search cost reduction sequentially is natural and allows us to focus on pure strategy equilibria.[6]

As before, without any restriction on the selection of equilibria in the user subgame, many equilibria of the full game are possible. We select the equilibrium in which firms join both platforms provided this exists. For expositional purposes, we

[5]To show this, suppose $h(\cdot)$ is log-concave. Then, $x(1 - H(x - t))$ is log-concave and therefore quasi-concave. So $\max_x x(1-H(x-t))$ has a unique maximizer. Now consider $\max_x x(1+H(t-x))$. Taking the log of the objective function, we have $\ln x + \ln(1 + H(t-x))$. The first term is concave. Taking the second derivative of the second term, we have $\frac{h'(t-x)(1+H(t-x))-(h(t-x))^2}{(1+H(t-x))^2}$. Since $h(\cdot)$ is log-concave and therefore $H(\cdot)$ is log-concave, the numerator of this expression is negative, which implies $\ln(1 + H(t - x))$ is concave. We can conclude $x(1 + H(t - x))$ is quasi-concave and the maximization problem has a unique maximizer.

[6]For the case without price parity restrictions, or with narrow price parity only, the results we obtain would be unchanged if investment decisions were instead made simultaneously. Each platform would still invest nothing since it can free ride on the investment of the other. However, with wide price parity, if one platform chooses a relatively low investment level, the other platform would best respond by investing slightly more, while if one platform chooses a relatively high investment level, the other platform will want to not invest at all, thus ruling out a pure strategy equilibrium.

only state equilibrium outcomes in the propositions in this section instead of characterizing the firms' full equilibrium strategies at stage 3. Our explanations still consider all possible unilateral deviations and the equilibrium continuation strategies that then apply, such as when a platform undercuts in its fee enough so as to rule out an equilibrium where all firms join both platforms in the user subgame.

Consider the symmetric equilibrium fee f^* in the absence of showrooming and PPCs. In equilibrium, consumers use their preferred platform. If M^j deviates by setting $f_S^j < f_S^*$, consumers whose preferred platform is M^k $(k \neq j)$ and draw $a < f^* - f^j$ will use M^j. M^j maximizes the deviation profit $f^j \left(\frac{1}{2} + \frac{1}{2}H(f^* - f^j)\right)$. Alternatively, M^j can increase its fee above f^*. In this case, consumers whose preferred platform is M^j and draw $a < f^j - f^*$ will use M^k. M^j maximizes the deviation profit $f^j \left(\frac{1}{2} - \frac{1}{2}F(f^j - f^*)\right)$. In either case, imposing symmetry on the first order conditions implies

$$f^* = \frac{1}{H'(0)}. \tag{15}$$

Now suppose that having searched a firm on a particular platform, consumers can switch and buy from the firm directly or through the other platform. In order to prevent consumers from switching, both platforms' fees cannot exceed b, meaning the equilibrium fee will be the minimum of b and $\frac{1}{H'(0)}$.

Recall that narrow PPCs require that the price a firm sets on the platform be no higher than the price the firm sets when it sells directly, ruling out the direct purchase option. If narrow PPCs are adopted by both platforms, the showrooming constraint is removed and only the competition constraint is effective. Thus, each platform will always (at least weakly) want to impose narrow PPCs given doing so removes the possibility of showrooming. The following proposition characterizes the equilibrium outcome for platforms and firms taking into account the constraint implied by showrooming and narrow PPCs.

Proposition 6. (Competing platforms' fee-setting absent wide PPCs)

Stage 3: Both platforms set the symmetric fee $f^* = \min\left\{b, \frac{1}{H'(0)}\right\}$ *if they do not impose narrow PPCs, and the symmetric fee* $f^* = \frac{1}{H'(0)}$ *if they do impose narrow PPCs.*

Stage 4: In the absence of wide PPCs, firms join both platforms and set the common on-platform price $f^* + \lambda(x_m)$ *and the direct price* $\lambda(x_d)$.

Since the equilibrium fees in both scenarios without wide PPCs (i.e. without any PPCs or with narrow PPCs) do not depend on the platform's search costs, platforms have no incentive to invest in reducing search costs. A platform that invests in reducing search costs will attract all consumers to search on that platform, but consumers will continue to complete their transactions on their preferred platform.

Proposition 7. (Competing platforms' investment absent wide PPCs)
Absent wide PPCs, platforms under-invest relative to the efficient investment level.

Things are different with wide PPCs, which require that the price a firm sets on the platform be no higher than the price the same firm sets through any other channel. Platform fees will no longer be constrained by showrooming or direct competition. Instead, in the presence of wide PPCs, consumer participation pins down platform fees. In addition, the platform with the lower search cost can potentially lower its fee to attract firms to list exclusively given consumers will all be searching on this platform. This explains why differences in the platforms' investment in search cost reduction matter for the equilibrium analysis.

Suppose M^j, $j = I, E$, is the platform with a strictly lower search cost after the investments have taken place (the case with symmetric search costs is considered in the proof of Proposition 8). We first characterize the equilibrium fees set by platforms following the given levels of search cost on platforms. Under these fees, the platform with lower search cost sets a higher fee and consumer surplus is fully extracted.

Proposition 8. (Competing platforms' fee-setting under wide PPCs)
Suppose M^j has a strictly lower search cost and both platforms impose wide PPCs.
Stage 3: Equilibrium fees are

$$f^j = \alpha(x_m^j - \lambda(x_m^j) + b) \quad \textit{and} \quad f^k = (2 - \alpha)(x_m^j - \lambda(x_m^j) + b),$$

where α satisfies $1 \leq \alpha \leq 2$.
Stage 4: All firms join both platforms and set

$$p_m(f^j, f^k) = \frac{f^j}{2} + \frac{f^k}{2} + \lambda(x_m^j). \tag{16}$$

> *Consumers search on M^j and complete transactions on their preferred platform (i.e. the platform that gives them higher convenience benefit).*

The different values of α map out a continuum of equilibria in the fee setting subgame. As long as the sum of the fees is such that the price in (16) exactly extracts consumers' surplus, then the equilibrium conditions do not pin down the exact fee set by each firm. While each platform serves half of the consumers, each different $\alpha > 1$ corresponds to a specific equilibrium revenue division between the two platforms, reflecting the different fees set by each platform. Among these different divisions of revenue, we know that $\alpha = 1$ selects the symmetric equilibrium in fees (provided it exists) regardless of the differences in search costs. On the other hand, $\alpha = 2$ selects the equilibrium which is best for the platform with lower search costs (since it gets all the revenue and the platform with higher search costs gets no revenue although the two platforms still equally split the market). In the limit case as the degree of differentiation between platforms goes to zero because b goes to zero, the equilibrium outcome in this case with $\alpha = 2$ corresponds to the equilibrium outcome that would arise for the case in which competing platforms are homogenous.[7] We restrict α to be no smaller than 1 to reflect that the platform with lower search cost should get a larger share of industry revenue, given the platforms are otherwise symmetric.

We next consider the investment decisions made by each platform. Recall that M^j is the platform with lower search cost in the market. Define

$$\pi(x_m^j)\equiv\pi(x_m^0 + y^j) = x_m^0 + y^j - \lambda(x_m^0 + y^j) + b,$$

and let

$$z = \arg\max_{y^j} \left\{ \frac{1}{2}\alpha\pi(x_m^0 + y^j) - C(y^j) \right\}.$$

Under wide PPCs, there are two types of equilibria, depending on whether

$$\frac{1}{2}\alpha\pi(x_m^0 + z) - C(z) \geq \frac{1}{2}(2-\alpha)\pi(x_m^0 + z) \tag{17}$$

holds or not. The next proposition characterizes the equilibrium platform investment levels, provided platforms adopt wide PPCs and set fees $f^j = \alpha\pi(x_m^j)$ and $f^k = (2-\alpha)\pi(x_m^j)$ as in Proposition 8.

Proposition 9. *Consider the case in which both platforms are allowed to adopt wide PPCs. Assume $\alpha \in \left[\frac{4}{3}, 2\right]$. Both platforms adopt wide PPCs. Furthermore:*

- *M^I preempts M^E: When (17) holds, M^I chooses $y^I = y^* \geq z$ which satisfies*

$$\frac{1}{2}\alpha\pi(x_m^0 + y^*) - C(y^*) = \frac{1}{2}(2-\alpha)\pi(x_m^0 + y^*), \tag{18}$$

and M^E chooses $y^E = 0$.

[7] The details of the convergence result can be found in Wang and Wright (2016).

- *M^I free-rides on M^E: When (17) does not hold, M^I chooses $y^I = 0$ and M^E chooses $y^E = z$.*

The condition $\alpha \in \left[\frac{4}{3}, 2\right]$ ensures that both platforms have an incentive to adopt wide price parity clauses, and that neither platform has an incentive to deviate from the equilibrium and significantly lower its fees to attract firms to list exclusively.[8] When (17) holds, M^I makes significant investment in order to preempt M^E. The L.H.S. of (18) is the profit of the platform with lower search cost when its investment level equals y^*, while the R.H.S. of (18) is the profit of the platform with higher search cost when its rival's investment level is y^*. The investment level y^* removes M^E's incentive to become the better platform (i.e. to have lower search costs). If M^E invests just slightly more than y^*, the profit it can get (i.e. just less than the L.H.S. of (18)) is slightly less than its profit from free-riding on the rival's investment (i.e. the R.H.S. of (18)). When (17) is violated, however, M^I chooses not to invest at all and free-rides on M^E's investment. In either case, neither platform can do better by removing wide PPCs. Intuitively, wide PPCs remove showrooming possibilities (either from the firms' direct channel or from the other platform), which is something each platform prefers (at least weakly) to do.

The results in Proposition 9 can be further strengthened when v is distributed according to a generalized Pareto distribution and its cumulative distribution function is not too concave. The following example shows that there exists a unique $\overline{\alpha} \in (1, 2)$ such that the first equilibrium prevails when $\alpha \geq \overline{\alpha}$ and the second equilibrium prevails when $\alpha < \overline{\alpha}$.

Example: Use the same specification of $G(v)$ and $C(y)$ as in Section 4.3. Then,

$$z = \arg\max_y \left\{ \frac{\alpha}{2} \left((1+\varepsilon) \left(x_m^0 + y \right) + b - \varepsilon \overline{v} \right) - \frac{c}{\eta} (y)^\eta \right\}$$

implies

$$z = \left(\frac{\alpha (1+\varepsilon)}{2c} \right)^{\frac{1}{\eta - 1}}.$$

The L.H.S. of (17) is greater (smaller) than the R.H.S. of (17) when $\alpha = 2$ (when $\alpha = 1$). Then the L.H.S. of (17) minus R.H.S. of (17) is

$$D(\alpha) = (\alpha - 1) \left[(1+\varepsilon) x_m^0 + b - \varepsilon \overline{v} \right] + (\alpha - 1) \left(\frac{\alpha}{2c} \right)^{\frac{1}{\eta-1}} (1+\varepsilon)^{\frac{\eta}{\eta-1}} - \frac{c}{\eta} \left[\frac{\alpha (1+\varepsilon)}{2c} \right]^{\frac{\eta}{\eta-1}}.$$

[8] When $\alpha \in \left[1, \frac{4}{3}\right]$, it is possible that one platform will want to drop its wide price parity clause while the other platform still imposes it.

So

$$D'(\alpha) = (1+\varepsilon)\, x_m^0 + b - \varepsilon\overline{v} + (1+\varepsilon)^{\frac{\eta}{\eta-1}} \left(\frac{1}{2c}\right)^{\frac{1}{\eta-1}} \left[\frac{(\alpha-1)\,\alpha^{\frac{1}{\eta-1}-1}}{\eta-1} + \alpha^{\frac{1}{\eta-1}} - \frac{\alpha^{\frac{1}{\eta-1}}}{2(\eta-1)}\right].$$

The first term in brackets in $D'(\alpha)$ is $(1+\varepsilon)\, x_m^0 + b - \varepsilon\overline{v}$ which is positive since we require $\pi(x_m^0) > 0$. The term in large square brackets is non-negative if and only if $2\alpha\eta - \alpha - 2 \geq 0$, or $\eta \geq \frac{1}{2} + \frac{1}{\alpha}$, which is satisfied by $G(v)$'s that are not too concave. So we can conclude that the L.H.S. of (17) cross with the R.H.S. of (17) at a unique $\alpha = \overline{\alpha}$.

Whether the investment is higher or lower than the efficient level is ambiguous. Take the equilibrium with $y^I > 0$ as an example. There are three effects impacting on M^I's investment at the same time when M^I invests to preempt M^E in equilibrium. First, if M^I invests less than y^*, it cannot prevent M^E from investing more and becoming the platform with lower search cost. Thus, a higher investment level is needed to preempt M^E's investment. Second, an increase in y^I decreases $\lambda(x_m^I)$ given firms compete more aggressively when search costs are lower. This loss of firms' mark-up represents a transfer to M^I under wide PPCs. However, this pure transfer is not taken into account when determining the efficient level of investment. These two effects suggest there will be over-investment when platforms can use wide PPCs. Finally, unless $\alpha = 2$, M^E free-rides on M^I's investment. That is, M^I's return on investment is multiplied by $\frac{1}{2}\alpha$, which is less than one if $\alpha < 2$. This effect pushes towards under-investment. Thus, it is ambiguous whether wide PPCs cause excessive investment. The investment level in the equilibrium with $y^E > 0$ can also be either insufficient or excessive. The latter two effects mentioned above, with one positive and one negative on investment level, are still present, while the preemption effect does not exist for M^E as it is the second mover.

We summarize our findings in the following proposition.

Proposition 10. (The effect of wide PPCs)

Suppose there are competing differentiated platforms.

- *With wide PPCs, platforms overinvest in search if α is close enough or equal to 2, but may underinvest in search for lower α.*
- *Wide PPCs lower consumer surplus.*
- *Wide PPCs lower total welfare if α is close enough or equal to 2, and may increase total welfare for lower α.*

Consumers are unambiguously worse off under wide PPCs as they get zero surplus compared to the positive surplus they obtain without it. Without wide PPCs, total welfare is $x_m + b$ as platforms have no incentive to invest. Under wide PPCs, total welfare is $x_m + y^* + b - C(y^*)$ or $x_m + z + b - C(z)$. Wide PPCs improve efficiency if $y^* > C(y^*)$ or $z > C(z)$. The effect is ambiguous in general. However, since overinvestment takes place when α is close to 2, wide price parity clauses are more likely to reduce welfare when α is close to 2.

6 Conclusions

Previous research has shown that price parity clauses can be harmful to consumers since they remove competitive pressures on platform fees. However, previous research has ignored the positive role that their clauses can have in protecting the incentives of platforms to invest in search benefits, which is the most obvious defence platforms have for using these clauses. This paper analyzes the implications of price parity clauses for platform investment in providing search benefits, and how such platform investment may change the effects of price parity clauses.

We find that for investments that improve search, there is insufficient investment without wide price parity clauses and excessive investment with wide price parity clauses. Thus, our paper would seem to provide some support for platforms' arguments that wide price parity clauses are actually beneficial since they promote investment. However, the welfare effects of price parity clauses are at best ambiguous. Moreover, wide price parity clauses unambiguously lower consumer surplus. Thus, while investment incentives could potentially provide a welfare justification for allowing for wide price parity clauses, there should be no presumption that this is indeed the case. Any justification for wide price parity clauses based on investment would also need to trade off the loss in consumer surplus with any possible efficiency benefits arising from higher platform investment.

In terms of narrow price parity clauses, we find that they don't help incentivize platforms to invest in improving search because each platform can still free-ride on the other platform's superior search by slightly undercutting on fees. This suggests narrow price parity clauses cannot be justified on the grounds of supporting investment in search. On the other hand, in the absence of any price parity clauses, platform search and matching is like a public good. As such, if price parity clauses are banned, there could be grounds to subsidize investment in providing improved platform search, for instance via favorable tax treatment for such investment.

In reality, the free-riding might be milder than assumed in our model, which leaves a role for narrow price parity clauses to promote platform investment. This could be because some consumers won't bother to switch back to their preferred platform or to the direct channel after searching on another platform in order to get a better price. Also, a firm might offer different versions of products on different selling channels. For example, hotels offer certain types of rooms only on selective OTAs and consumers might not be able to switch to a different platform to find the same room. In these circumstances, we expect a platform can at least recover part of the surplus generated from their investments under narrow price parity clauses. With these considerations in mind, and provided platform competition under narrow price parity clauses is effective, the end result under narrow price parity clauses could be an acceptable compromise between sustaining investment and maintaining price competition. On the other hand, if platform competition itself is not very effective because it is restrained by things like network effects, consumer stickiness, and best price guarantees, then it may be better to ban narrow price parity clauses as well, so that fees can be pinned down by the showrooming constraint from the direct channel.

While we allowed for platform competition in considering investment in search cost reduction, we have not yet done so in the context of advertising investments, another important form of investments that is likely to be impacted by price parity clauses. In exploring the implications on advertising investments, one would need to take into account that platforms like Expedia and the firms they host (e.g. hotels) compete in advertising to attract consumers (e.g. through a general search engine such as Google). By heavily spending in advertising, platforms can lower firms' direct exposure to consumers and therefore weaken their outside option of withdrawing from the platforms and selling independently. This might weaken firms' own advertising propensity, making them increasingly reliant on platforms. In future work it would be interesting to explore how this channel may work, with and without the use of price parity clauses.

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Appendix.

Proof of Proposition 2. We first establish the equilibrium pricing rules in stage 2. There are two user subgames we need to distinguish.

■ The user subgame following $f \leq b$: First, note, $f \leq b$ implies $f - b + \lambda(x_m) \leq \lambda(x_d)$ as $\lambda(x_m) \leq \lambda(x_d)$. This implies $p_m - b \leq p_d$ given the proposed equilibrium pricing strategies. That is, in the proposed equilibrium, consumers will make purchases on M rather than switching.

Consider a unilateral deviation by firm i designed to induce switching. Note any deviation that does not induce switching can be ruled out for the same reason as in the benchmark case. This deviation requires $p_m^i - b > p_d^i$. This is always possible as firm i can manipulate p_m^i and p_d^i simultaneously. In this case, consumers who want to buy from i will switch to buy from firm i directly. Consumers who visit firm i through M $(1/(1 - G(x_m))$ of them) will choose to continue to search through M if they do not buy from firm i. Only consumers with $v^i - p_d^i \geq x_m - (p_m + f_B - b)$

will buy from firm i. Therefore, firm i's maximization problem is given by

$$\max_{p_d^i} p_d^i \left[\frac{1 - G(x_m - (p_m - b) + p_d^i)}{1 - G(x_m)} \right]. \tag{19}$$

Then note

$$\begin{aligned}
&\max_{p_d^i} p_d^i \left[\frac{1 - G(x_m - (p_m - b) + p_d^i)}{1 - G(x_m)} \right] \\
=&\max_{p_d^i} p_d^i \left[\frac{1 - G(x_m - (f - b + \lambda(x_m)) + p_d^i)}{1 - G(x_m)} \right] \\
\leq&\max_{p_d^i} p_d^i \left[\frac{1 - G(x_m - \lambda(x_m) + p_d^i)}{1 - G(x_m)} \right] \\
=&\lambda(x_m).
\end{aligned}$$

The first equality follows from the definition of p_m in the equilibrium pricing strategy. The first inequality follows from our assumption that $f \leq b$. The second equality follows since $p_d^i = \lambda(x_m)$ is the argument maximizing the expression. Since $\lambda(x_m)$ is firm i's profit in the proposed equilibrium, the inequality above shows that firm i cannot make a profitable deviation from the proposed equilibrium when $f \leq b$. Note that if $f > b$ then the inequality is reversed, and there is a profitable deviation that induces consumers to switch.

■ The user subgame following $f > b$: First, note, $f > b$ implies $f - b + \lambda(x_m) > \lambda(x_m)$. This implies $p_m - b > p_d$ given the proposed equilibrium pricing strategies. That is, in the proposed equilibrium, consumers will always switch to buy directly after searching on M.

Consider a unilateral deviation by firm i. If firm i deviates such that $p_d^i < p_m^i - b$, firm i's sales are still all through direct purchases. In this case, firm i cannot be better off by choosing a price different from $p_d^i = \lambda(x_m)$, given all other firms are choosing this direct price. This is because when all other firms are charging $p_d = \lambda(x_m)$ and $p_d < p_m - b$, consumers expect to use the platform as a showroom and make purchases directly. If the deviation is such that $p_d^i < p_m^i - b$ and firm i expects consumers to buy from it directly, a consumer who visits firm i will buy from firm i directly only if $v^i - p_d^i \geq x_m - p_d$. Firm i chooses p_d^i to maximize

$$p_d^i \left[\frac{1 - G(x_m - \lambda(x_m) - p_d^i)}{1 - G(x_m)} \right].$$

So firm i's best response is indeed exactly $p_d^i = \lambda(x_m)$.

Now consider a unilateral deviation by firm i such that $p_d^i \geq p_m^i - b$ so as to induce consumers not to switch. Consumers buy from firm i through M only if $v_i - (p_m^i - b) \geq x_m - p_d$. Firm i's maximization is

$$
\begin{aligned}
&\max_{p_m^i}(p_m^i - f)\left[\frac{1 - G(x_m - p_d + p_m^i - b)}{1 - G(x_m)}\right] \\
=&\max_{p_m^i}(p_m^i - f)\left[\frac{1 - G(x_m - \lambda(x_m) + p_m^i - b)}{1 - G(x_m)}\right] \\
<&\max_{p_m^i}(p_m^i - f)\left[\frac{1 - G(x_m - \lambda(x_m) + p_m^i - f)}{1 - G(x_m)}\right] \\
=&\lambda(x_m).
\end{aligned}
$$

The first equality follows from the definition of p_d in the equilibrium pricing strategy. The first inequality follows from our assumption that $f > b$. The second equality follows since $p_m^i = \lambda(x_m)$ is the argument maximizing the expression. Since $\lambda(x_m)$ is firm i's profit in the proposed equilibrium, the inequality above shows that firm i cannot make a profitable deviation from the proposed equilibrium when $f > b$.

■ M's strategy in stage 1: Given the firms' pricing equilibrium in stage 2 (and consumers' corresponding optimal search behavior as described in the text), we can now work out M's optimal fees. In stage 1, the platform therefore chooses the highest possible f subject to $f \leq b$ (since otherwise all consumers will switch to buying directly) and also subject to consumers choosing to search on M in the first place. The latter condition requires $x_m + b - p_m \geq 0$ or $f \leq x_m + b - \lambda(x_m)$. The two constraints imply $f^* = \min\{b, x_m + b - \lambda(x_m)\} = b$ as $x_m - \lambda(x_m) > x_d - \lambda(x_d) > 0$ from (4). Thus, M sets $f^* = b$. □

Proof of Proposition 4. We solve the game backwards.

■ The user subgame in stage 2: Provided the direct price is weakly higher, consumers will never want to switch to buying directly. Given consumers search only through M, prices on M are determined by (5) following the same argument as in the benchmark case. Direct prices have to be at least as high as these. A firm cannot do better by not joining since then it will get zero profit given all consumers are searching on M. Because of the PPC, firm i is also unable to deviate by raising p_m^i and lowering p_d^i to induce consumers to switch.

■ M's strategy in stage 1: Consumers will prefer to search through M, provided M offers a strictly lower search cost and a weakly lower price. In addition, consumers must expect a non-negative surplus from searching and buying on M, which requires

$$x_m + b - p_c \geq 0. \tag{20}$$

Substituting (5) into (20), we have that

$$f \leq x_m + b - \lambda(x_m). \tag{21}$$

The platform maximizes its profit by setting f to make (21) hold with equality which gives (13). $\square$

Proof of Proposition 8. We show the fees, prices and user choices described in Proposition 8 characterize an equilibrium in the subgames following the investment choices. Given that all consumers use M^j to search and other firms join both platforms and price according to (16), it is optimal for an individual firm i to price according to (16) if it also joins both platforms. Firm i does not have any incentive to exclusively join M^k, $k \neq j$, even if M^k reduces its fee as all consumers search on M^j and they will not find firm i if firm i is only listed on M^k.

Let us first consider what happen if the platform with lower search cost, M^j, reduces its fee to f^j in order to attract exclusive listings. To identify how low M^j's fee needs to be in order to attract exclusive listings in the user subgames, we need to consider when an individual firm would like to deviate from a candidate equilibrium in which all firms join both platforms. Suppose firm i exclusively joins M^j and sets price p'_m on M^j. Note that in this case firm i's price is no longer constrained by a PPC. Consumers whose preferred search platform is M^j will buy from firm i if $v^i - p'_m \geq x^j_m - p_m(f^j, f^k)$, or equivalently, $v^i \geq x^j_m - p_m(f^j, f^k) + p'_m$. A consumer whose preferred platform is M^k will buy from firm i if $v^i - p'_m + b - a \geq x^j_m - p_m(f^j, f^k) + b$, or equivalently, $v^i \geq x^j_m - p_m(f^j, f^k) + a + p'_m$. Firm i's deviating profit, denoted $\pi^d(f^j, f^k; x_m^j)$, is

$$\begin{aligned}
\pi^d(f^j, f^k; x_m^j) = & \max_{p'_m} \frac{(p'_m - f^j)}{1 - G(x_m^j)} \Bigg[\frac{1}{2}\Big(1 - G(x_m^j - p_m(f^j, f^k) + p'_m)\Big) \\
& + \frac{1}{2}\int_0^b \Big(1 - G(x_m^j - p_m(f^j, f^k) + a + p'_m)\Big) dH(a) \Bigg] \\
= & \max_{z} \frac{(z + \frac{1}{2}(f^k - f^j))}{1 - G(x_m^j)} \Bigg[\frac{1}{2}\Big(1 - G(x_m^j - \lambda(x_m^j) + z)\Big) \\
& + \frac{1}{2}\int_0^b \Big(1 - G(x_m^j - \lambda(x_m^j) + a + z)\Big) dH(a) \Bigg]
\end{aligned}$$

The equality comes from the change of variables $z = p'_m - (\frac{1}{2}f^j + \frac{1}{2}f^k)$. Since firm i's equilibrium profit is $\lambda(x_m^j)$, it will not deviate in this way if

$$\pi^d(f^j, f^k; x_m^j) \leq \lambda(x_m^j).$$

Now consider whether platforms would like to charge a fee different from the equilibrium fee given their rival sets the equilibrium fee. Both platforms will not raise their fee above the equilibrium level as, given (16), this leads to a negative expected payoff to consumers and consumers will stop using platforms to search and buy. M^k will not reduce its fee below the equilibrium level as it can neither reduce price on M^k to attract consumers due to M^j's wide PPC nor induce firms to join M^k exclusively due to the fact that all consumers search using M^j. Given the result above, M^j has to reduce its fee to at least $\hat{f}$ to attract exclusive selling, where $\hat{f}$ is given by

$$\pi^d(\hat{f}, f^k; x_m^j) = \lambda(x_m^j).$$

Then, M^j will not reduce its fee if the profit from attracting firms' exclusive selling, i.e. $\hat{f}$, is no higher than its equilibrium profit, i.e. $f^j/2$,

$$\frac{f^j}{2} \geq \hat{f}.$$

Since $\pi^d(f^j, f^k; x_m^j)$ is decreasing in f^j, the condition for M^j not to want to reduce its fee becomes

$$\pi^d\left(\frac{1}{2}f^j, f^k; x_m^j\right) \leq \lambda(x_m^j).$$

Plugging in $f^j = \alpha(x_m^j - \lambda(x_m^j) + b)$ and $f^k = (2 - \alpha)(x_m^j - \lambda(x_m^j) + b)$, we can rewrite this condition as

$$\max_{z} \frac{\big(z+(1-\frac{3}{4}\alpha)(x_m^j-\lambda(x_m^j)+b)\big)}{1-G(x_m^j)}\Bigg[\frac{1}{2}\Bigg(1-G(x_m^j-\lambda(x_m^j)+z)\Bigg) + \frac{1}{2}\int_0^b\Bigg(1-G(x_m^j-\lambda(x_m^j)+a+z)\Bigg)dH(a)\Bigg] \leq \lambda(x_m^j). \tag{22}$$

Since

$$\lambda(x_m^j)=\max_{z}\frac{z}{1-G(x_m^j)}\Bigg(1-G(x_m^j-\lambda(x_m^j)+z)\Bigg),$$

condition (22) will hold when α is relatively large, e.g., $\alpha \geq \frac{4}{3}$. Note that the exact maximum lower bound of α which supports inequality (22) can be either higher or lower than 1, but definitely no higher than $\frac{4}{3}$.

In the off-equilibrium subgames in which both platforms choose the same level of investment, it is natural to consider symmetric equilibrium fees. This corresponds to the previously analyzed case of $\alpha = 1$ but with $y^I = y^E$. From the discussion above, we know that, when $\alpha = 1$, full surplus extraction by platforms might not be feasible as platforms may find it profitable to attract exclusive listing. So M^E's profit is at most $\frac{1}{2}(x_m^j - \lambda(x_m^j) + b)$ if M^E' investment level is the same as M^I's. This result will be later used to support our arguments concerning M^E's incentive to set $y^E = y^I$. □

Proof of Proposition 9. First, consider the case $\frac{1}{2}\alpha\pi(x_m^0+z)-C(z) \geq \frac{1}{2}(2-\alpha)\pi(x_m^0+z)$. Note that the L.H.S. of (18) strictly decreases in y when $y \geq z$ as z is the maximizer of the L.H.S. of (18), while the R.H.S. of (18) strictly increases in y. By our assumption, $\pi(x_m^0+y) < C(y)$ for some y, then $\alpha\pi(x_m^0+y) < C(y)$ for some y. This implies the L.H.S. of (18) eventually becomes negative for large enough y. At the same time, the R.H.S. of (18) is always positive. Then, the R.H.S of (18) and the L.H.S. of (18) must cross each other at some $y \geq z$.

We next check whether M^I and M^E have an incentive to deviate from the equilibrium strategies described in Proposition 9 when (17) holds. Obviously, M^E has no incentive to invest any positive amount below y^* as its revenue would not change but its costs would increase. M^I has to make sure that M^E does not want to invest more than y^*. When $y^* > z$, M^E will only want to invest an infinitesimal amount more than y^* if it tries to be the platform with the lowest search cost. This is because M^E is maximizing $\frac{1}{2}\alpha\pi(x_m^E) - C(y^E)$ subject to $y^E > y^*$. Since the objective function without the constraint is maximized at z and $y^* > z$, a further increase above y^* will decrease M^E's profit. M^E will not choose such a $y^E > y^*$ if this profit is lower than its equilibrium profit $\frac{1}{2}(2-\alpha)\pi(x_m^0 + y^*)$. Given (18), M^E does not want to do so. M^E will not choose $y^E = y^*$ either, since as we argued in the proof of Proposition 8, its profit by choosing $y^E = y^I$ is at most $\frac{1}{2}\pi(x_m^0 + y^*) - C(y^*)$. This is lower than the L.H.S. of (18) and therefore the R.H.S. of (18). Alternatively, M^I may invest nothing in search cost reduction in the first stage so as to free ride on M^E's investment.[9] Then M^E will invest $y^E = z$. M^I's profit is $\frac{1}{2}(2-\alpha)\pi(x_m^0 + z)$. M^I will prefer choosing y^* to choosing 0 if

$$\frac{1}{2}\alpha\pi(x_m^0 + y^*) - C(y^*) = \frac{1}{2}(2-\alpha)\pi(x_m^0 + y^*) > \frac{1}{2}(2-\alpha)\pi(x_m^0 + z).$$

But since $\pi(x_m^0 + y^*)$ increases in y^* and $y^* > z$, this condition always holds.

Second, consider the case $\frac{1}{2}\alpha\pi(x_m^0 + z) - C(z) < \frac{1}{2}(2-\alpha)\pi(x_m^0 + z)$. We check whether $y^I = 0$ and $M^E = z$ constitute an equilibrium in this sequential-move game. Given that M^E is the second mover and $y^I = 0$, M^E solves $\max_y \frac{1}{2}\alpha\pi(x_m^0+y) - C(y)$. By the definition of z, M^E chooses $y^E = z$. We next check whether M^I can profitably deviate. For any observed y^I, M^E will only choose either $y^E = 0$ so as to free ride or some $y^E \geq y^I$ to attract all consumers to search using M^E. By choosing $y^E \in (0, y^I)$, M^E cannot change the equilibrium outcome but incurs a positive cost.

Suppose that M^I chooses y^I such that $y^E = 0$. M^I's profit is $\frac{1}{2}\alpha\pi(x_m^0 + y^I) - C(y^I) \leq \frac{1}{2}\alpha\pi(x_m^0+z) - C(z) < \frac{1}{2}(2-\alpha)\pi(x_m^0+z)$. The first inequality comes from the definition of z and the second inequality comes from the presumption of this case. So M^I does not want to do so. Next, suppose M^I chooses y^I such that $y^E > y^I$. M^I's profit is $\frac{1}{2}(2-\alpha)\pi(x_m^0 + y^I) - C(y^I) \leq \frac{1}{2}\alpha\pi(x_m^0 + z) - C(z) \leq \frac{1}{2}(2-\alpha)\pi(x_m^0 + z)$. The first inequality comes from the fact that z is the maximizer of an objective function with higher coefficient (i.e. $\alpha \geq 2 - \alpha$) and the second inequality comes from the presumption. So M^I will not deviate in this way. Finally, suppose M^I chooses y^I such that $y^E = y^I$. M^I's profit is at most $\frac{1}{4}\pi(x_m^0 + y^I) - C(y^I)$ if full surplus extraction is still feasible when both platforms have the same search cost. However, $\frac{1}{4}\pi(x_m^0 + y^I) - C(y^I) \leq \frac{1}{2}\alpha\pi(x_m^0 + z) - C(z) \leq \frac{1}{2}(2-\alpha)\pi(x_m^0 + z)$ given that $\alpha \geq 1$ and a similar logic as above applies. So M^I will not deviate in this way.

We finally show both platforms have an incentive to adopt wide PPCs. Suppose (17) holds. If M^I does not adopt any form of PPC or adopts only a narrow PPC, its fee will be constrained by showrooming (related to either the direct channel or

[9] We can also rule out M^I investing some amount between 0 and y^*. Since $\frac{1}{2}\alpha\pi(x_m^0 + y) - C(y)$ is assumed to be single-peaked, if M^I chooses some y^I between 0 and y^*, M^E can always invest an amount which is slightly higher than y^I and make a profit higher than $\frac{1}{2}(2-\alpha)\pi(x_m^I)$.

M^E) and therefore its revenue will be independent of its investment. So M^I will choose $y^I = 0$ if it decides not to adopt a wide PPC. Given that $y^I = 0$, M^E sets $y^E = z$ and adopts a wide-PPC. The resulting profit satisfies $\frac{1}{2}\alpha\pi(x_m^0 + z) - C(z) > \max\{\frac{b}{2}, \frac{1}{2H'(0)}\}$ by our assumption in (14), implying M^E prefers adopting a wide PPC to imposing no restraints at all or adopting only a narrow PPC. Given M^E's response, M^I's profit is therefore at most $\frac{1}{2}(2-\alpha)\pi(x_m^0 + z)$ if firms do not choose to exclusively list on M^I. Without attracting exclusive listing, this profit is lower than M^I's profit when adopting a wide PPC and choosing $y^I = y^*$, due to (17). Another possibility is that, without a wide PPC, M^I's constrained fee is so low such that firms choose to exclusively list on M^I. But we showed in the proof of Proposition 8 that the platform with lower search cost is worse off by inducing exclusive listing provided $\alpha \geq 4/3$. Given that M^I adopts a wide PPC, M^E cannot be worse off by also adopting a wide PPC. If M^E does not invest in search, the effect of adopting a wide PPC is to remove the showrooming constraint otherwise faced by M^E. If instead M^E invests in search, it is better to adopt a wide PPC to protect its investment from showrooming activities related to the direct channel (if no restriction is adopted) or through M^I (if M^E adopts a narrow PPC only).

When (17) does not hold, M^I free-rides on M^E's investment in equilibrium. If M^I no longer adopts a wide PPC and sets $y^I = 0$, M^E will continue to impose a wide PPC and choose $y^E = z$. This makes M^I the platform with higher search cost which cannot attract any exclusive listing. So M^I's profit is at most $\max\{\frac{b}{2}, \frac{1}{2H'(0)}\}$, which is lower than its equilibrium profit $\frac{1}{2}(2-\alpha)\pi(x_m^0+z)$ since $\frac{1}{2}(2-\alpha)\pi(x_m^0+z) > \frac{1}{2}\alpha\pi(x_m^0+z) - C(z) \geq \frac{b}{2}$, where the first inequality comes from the presumption that (17) does not hold and the second inequality comes from (14). Given M^I adopts a wide PPC, M^E will do the same.

□

Proof of Proposition 10. We first consider the extreme case of $\alpha = 2$. The equilibrium of the full game involves that M^I invests a positive amount $y^* > z$ where y^* satisfies

$$\pi(x_m^0 + y^*) - C(y^*) = 0, \tag{23}$$

and M^E does not invest. The equilibrium investment level y^* is unambiguously higher than the efficient level. In the case where a monopoly platform imposes a PPC, the monopoly platform maximizes the L.H.S. of (23) and the resulting investment level is higher than the efficient level y^e. Now y^* is so high such that the L.H.S. of (23) equals zero. We thus can conclude $y^* > y^e$.

Consumers are unambiguously worse off under wide PPCs as they get zero surplus compared to the positive surplus they obtain without them. Without wide PPCs, total welfare is $x_m + b$ as platforms have no incentive to invest. Under wide PPCs, total welfare is $x_m^0+y^*+b-C(y^*)$. Wide PPCs reduce efficiency if $y^* < C(y^*)$. Recall y^* is determined by (23) and therefore $C(y^*) - y^* = x_m^0 - \lambda(x_m^0 + y^*) + b > 0$ since $x_m^0 > \lambda(x_m^0) \geq \lambda(x_m^0 + y^*)$. It is clear that wide PPCs decrease total welfare.

We next consider any α that supports the equilibrium characterized in Propositions 8 and 9. We first determine whether the equilibrium investment is excessive or insufficient. The equilibrium investment level y^* made by M^I is determined by (18) such that

$$y^* = -b - (x_m^0 - \lambda(x_m^0 + y^*)) - \frac{C(y^*)}{1-\alpha}. \tag{24}$$

It is clear from (24) that y^* continuously increases in α. We also know that $y^* > y^e$ when $\alpha = 2$. So, when α is close to 2, wide PPCs leads to over-investment. If we substitute the efficient investment level y^e into (24) and get a corresponding $\hat{\alpha} \in [1, 2]$, we can plug $\hat{\alpha}$ into the L.H.S. of (22). If the inequality in (22) still holds strictly, we know that the type of equilibrium we focus on exists when $\alpha = \hat{\alpha}$ under which the equilibrium investment level is efficient. Since both y^* and the L.H.S. of (22) are continuous in α, we can conclude that the platform investment is insufficient if $\alpha < \hat{\alpha}$ and is excessive if $\alpha > \hat{\alpha}$, provided (22) holds strictly at $\alpha = \hat{\alpha}$.

We next consider the impact of wide PPCs on total surplus. The comparison depends on whether $C(y^*) - y^* \geq 0$. From (24), we have

$$C(y^*) - y^* = x_m^0 - \lambda(x_m^0 + y^*) + b - \frac{(2-\alpha)C(y^*)}{\alpha - 1}. \tag{25}$$

From the discussion above, we know that wide PPCs decrease total surplus when $\alpha = 2$. Since the R.H.S. of (25) is continuous in α, we can conclude that wide PPCs decreases total surplus when α is close to 2.

Note that, if we set $C(y^*) = y^*$ where y^* is determined by (24), we can get an α that is welfare-neutral, i.e. $\tilde{\alpha} = \frac{x_m^0 - \lambda(x_m^0+y^*)+b+2C(y^*)}{x_m^0-\lambda(x_m^0+y^*)+b+C(y^*)} > 1$. If the inequality in (22) holds strictly at $\alpha = \tilde{\alpha}$, we can conclude that wide PPCs increase total surplus if $\alpha < \tilde{\alpha}$ and decrease total surplus if $\alpha > \tilde{\alpha}$.

Suppose condition (17) is violated. M^E invests z in equilibrium. By definition $\frac{1}{2}\alpha\pi'(x_m^0 + z) = C'(z)$ and $1 = C'(y^e)$. Note that $\pi'(x_m^0 + z) = 1 - \lambda'(x_m^0 + z)$. Then,

$z \geq y^e$ if and only if $\frac{1}{2}\alpha(1 - \lambda'(x_m^0 + z)) > 1$. Wide PPCs improve welfare if and only if $z \geq C(z)$. We can conclude that the impact of wide PPCs on welfare is, in general, ambiguous.

$\square$