Marketplace leakage*

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Abstract

A key issue for the design of online marketplaces is addressing leakage. Buyers may use the marketplace to discover a seller or to obtain certain conveniences, but the seller may then want to take transactions off the marketplace to avoid transaction fees. Assuming buyers are heterogenous in their switching cost or inconvenience cost of purchasing directly, we provide a model in which there is partial leakage in equilibrium. We use the model to analyze different strategies the marketplace can use to attenuate the effects of leakage: investing in transaction benefits, charging referral fees, hiding information, introducing seller competition on the marketplace and hiding sellers that try to induce too much leakage.

Keywords: platforms, disintermediation, showrooming, steering, two-sided markets.

1 Introduction

Leakage, the phenomenon of platform participants meeting via a platform but taking their transactions off the platform, is a common problem facing platforms (namely, marketplaces) that attempt to charge transaction fees.[1]

Marketplaces use a variety of instruments and measures to combat this issue: investing in additional benefits of completing transactions on the marketplace (e.g. escrow, insurance, and payment facilities on Airbnb, Amazon, eBay, Preply), charging only for referrals and not transactions (e.g. Capterra, Thumbtack), obscuring some information about transaction parties in order to make it more difficult for them to find each other outside the marketplace before completing the transaction (e.g. Airbnb, AngelList, eBay, Preply, Upwork), penalizing sellers who attempt to take buyers off the platform by demoting them in search results (e.g.

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[1] Leakage is also sometimes referred to as disintermediation or showrooming.

Booking.com, Expedia, CoachUp), and imposing price parity clauses to prohibit sellers from attracting buyers with lower prices to their own websites (e.g. Booking.com and Expedia) or bans if providers are caught taking clients off the platform (Preply, Upwork).

In this paper we provide a simple model to study how a marketplace should optimally respond to a leakage problem. In our benchmark setting there is a single monopoly seller that wants to sell to buyers who have unit demand. The marketplace charges a transaction fee when buyers purchase from the seller on the marketplace. Buyers discover the seller (and its prices) on the marketplace, and face additional costs to complete the transaction directly, i.e. outside the marketplace. The buyers' reluctance to transact directly could reflect a disutility due to the marketplace's superior transaction infrastructure (e.g. payments, security, shipping logistics, etc.) or a cost of switching to purchase directly. The seller sets two prices: one for buyers who purchase from it via the marketplace and one for buyers who purchase directly. Reflecting the heterogeneity in buyers' reluctance to purchase directly, the greater the discount the seller offers for buyers to transact directly, the more such buyers it can attract directly and thereby avoid paying the marketplace's transaction fee on.

In this setting, we show that the marketplace always sets a fee that induces some (but not complete) leakage to arise in equilibrium. The extent of leakage that arises in equilibrium depends on how much the seller is willing to lower its direct price below the one it offers on the marketplace—a higher marketplace fee leads the seller to offer a bigger discount for direct transactions. The marketplace takes this relationship into account in setting its profit maximizing fee. As the underlying leakage problem becomes less severe (i.e. buyers are more reluctant to switch to purchasing directly), the marketplace's fee increases (up to some maximum level) and it makes strictly more profit in equilibrium.

We explore several different strategies the marketplace can use to avoid or limit leakage: (1) investing in transaction benefits, (2) charging the seller for referrals rather than for transactions, (3) hiding information, (4) using a price-parity clause to rule out leakage, (5) allowing a competing seller to also sell on the marketplace and steering in favor of one or the other seller, depending on how much leakage each seller induces. For each of these strategies, we analyze the tradeoff that it creates for the marketplace, and we analyze how the tradeoff depends on consumers' switching costs (which capture the propensity for leakage) and other relevant parameters.

2 Literature review

By now there is an extensive literature studying the implications of showrooming (consumers visiting an offline seller to search for the right product and then switching to buy the product online at a cheaper price) and webrooming (consumers doing their initial search online and then purchasing their chosen product offline). This literature has examined how competition between different combinations of online and/or offline sellers works when switching between competing sellers and/or channels is possible (Wu et al., 2004, Shin, 2007, Loginova, 2009, Balakrishnan et al., 2014, Jing, 2018, and Bar-Isaac and Shelegia, 2022). The key difference in our setting is that the underlying leakage problem arises from consumers choosing between buying from the same seller (or sellers) either on a marketplace or directly.

Our focus is also different in that we evaluate different strategies that the marketplace can use to address the "showrooming" problem. Related strategies have been looked at in other environments. An earlier economics literature explored restrictions (price floors and restricted territories) imposed by manufacturers in order to limit free riding across the retailers that sell their products, which in turn ensures each retailer retains an incentive to promote the manufacturer's products (Telser 1960 and Mathewson and Winter 1984). Mehra et al. (2018) consider strategies (price matching and exclusive products) that offline stores use to counter traditional showrooming. Jing (2018) studies how the showrooming problem faced by an offline seller competing with an online seller is affected by the ability of the offline seller to open its own online store. In our marketplace setting, the strategies considered and the incentives to choose them are very different, given the purpose is not to protect a seller from competition from other sellers, but rather to stop a seller from inducing its consumers to buy from it directly so as to avoid the marketplace's transaction fees.

The setting in our paper is closest to Wang and Wright (2020) who look at a platform's use of a price-parity clause that prevents third-party sellers from undercutting in their direct channel (or other cheaper channels) so as to eliminate showrooming. Theirs is a search framework with many sellers, and consumers search on the platform to discover their ideal match. In the absence of a price-parity clause, consumers are able to switch costlessly to buy directly from the seller of their choice, and taking this into account, the platform will always set a fee that prevents any leakage from happening in equilibrium. Our framework differs in that switching costs are heterogeneous, so switching does end up happening in equilibrium, even with a monopoly seller. Moreover, we look at the profitability of many different strategies a platform can use to address the leakage problem, including the price-parity clause they consider.

There is also an emerging empirical literature that studies marketplace leakage and the factors that give rise to it. Hunold et al. (2020) provide evidence of steering in response to leakage on hotel booking platforms — they show Booking and Expedia give less prominent placement to hotels that have lower prices on the hotel's website or on a competing platform's site. Gu and Zhu (2021) show that more online trust between consumers and third-party freelancers increases leakage on an online freelancer marketplace. For a similar type of online labor marketplace, Zhou et al. (2021) show higher customer-agent interaction frequency, higher transaction prices, service repetitiveness and proximal customers are some of the factors that also increase leakage.

Finally, our paper fits within a broader literature that analyzes various non-price design choices faced by platforms. Broad frameworks for looking at these choices include Bhargava (2022), Choi and Jeon (2022), and Teh (2022). Some of the strategies we consider to address leakage such as hiding communications and steering buyers have been studied by others in different contexts. However, most papers on platform design do not consider the possibility of leakage as a factor explaining design choices. An exception is Peitz and Sobolev (2022) who study conditions under which a platform will make inflated recommendations to buyers on whether to purchase from a seller whose product the platform has superior information about. In an extension, they show how the possibility of leakage makes the platform more inclined to induce the inflated recommendations outcome.

3 Baseline model

There is a single platform (marketplace) M, a single seller S and measure one of buyers that each wish to purchase one unit of S's product. S's product is valued at v by all buyers and it incurs a marginal cost equal to c, where $v > c$. The buyers have an outside option valued at zero.

S can sell to buyers through two channels: via M, in which case it must pay M a fee per transaction f, or via its direct channel (with no transaction fee). S can set different prices across these two channels: p_m when selling via M and p_d in its direct channel. Facing these prices, buyers make their purchase and channel decisions.

Buyers have a preference for using M. They face no cost of going to M, and they face a disutility s of buying directly from S, which is drawn from a distribution G on the interval $[0, \overline{s}]$, with everywhere positive density on $(0, \overline{s})$. This bias in favor of M can be interpreted in two different ways. One is that buyers are uninformed about the existence of S, so they rely on $M$to discover S (and its prices in both channels), after which they face a cost s to switch and buy directly from it. The other is that they already know about S's existence, but they perceive a disutility s if they buy directly instead of through M. For instance, this could reflect that direct payments to S are less convenient or seem less secure, or logistics are inferior in the case of shipping a product directly rather than via M. Either interpretation of buyers is allowed in our baseline model, and indeed, there may be some buyers of each type.

These interpretations allow us to cover different types of applications. In case M is a marketplace and S is an individual (e.g. Airbnb, CoachUp, eBay, Etsy, Fiverr, OpenSea, Preply, Rover, TaskRabbit, Upwork etc.), it is natural to think of all buyers discovering the seller via a marketplace. On the other hand, if M is a marketplace for brands that have their own websites (e.g. Amazon.com, Apple's App Store, Booking.com, Tmall), some buyers may be aware that a seller they are interested in has its own website, but they might still feel that going through the marketplace is more convenient or more secure.

The timing is completely natural. There are three stages: (i) M sets its fee f; (ii) observing the fee, S sets its price p_d, and if it decides to list on M, sets its price p_m as well; and (iii) buyers decide whether to buy and which channel to use.

Before proceeding, it is useful to briefly discuss our key assumptions. A simple way to avoid leakage is to charge S a fixed fee for listing instead of a fee per transaction. There are several reasons why we think an analysis with per-transaction fees is still interesting. Obviously in practice many marketplaces, such as those discussed above, do charge their sellers for transactions. There are several possible reasons for this. One is that sellers typically face budget constraints, so are unable to pay the marketplace upfront for the value they expect to get from all future transactions. Another is that in reality, there is considerable heterogeneity and uncertainty in the demand and/or revenue that sellers expect to face on the marketplace, which means a fixed fee that tries to fully extract a seller's surplus may result in the seller not participating at all. In Online Appendix A, we show how our baseline model analysis and results extend to allow for a fixed fee that extracts an exogenous fraction of S's surplus. And we do consider the possibility of fixed referral fees in Section 5.2, where we introduce uncertain demand, which is necessary to show that referral fees don't always dominate a fee per transaction. Finally, note that with unit demand and positive marginal costs, an ad-valorem (i.e. proportional) fee is equivalent to a per-unit transaction fee, something we prove for our baseline model in Online Appendix B. For this reason, we stick with a per-unit fee for expositional purposes.

Although we will analyze the case with competing sellers in Section 5.5, for most of our analysis we focus on the simpler case with a single seller. The way we interpret the single seller in our model is that M is actually a marketplace for many different independent sellers, and our analysis applies to each one of them. Alternatively, if different sellers are ex-ante identical other than the measure of consumers that are interested in each, our analysis would still apply even if M could only set a single fee across all the different sellers since each seller's and M's profit would just scale with this measure, and the choice of M's optimal fee would remain.

A final key assumption is that buyers are homogenous in their willingness-to-pay for S's product. Allowing for heterogeneity in this willingness-to-pay would create downward sloping demand for S's product, but would destroy the tractability of our framework because it would mean we have two dimensions of heterogeneity: willingness-to-pay for the product and willingness-to-pay across the two channels. S's optimal prices no longer have closed-form solutions, which makes it hard to say much about M's optimal fee or other strategies to avoid leakage.

4 Baseline analysis

Each buyer draws a switching cost (or disutility) s and purchases directly iff

$$v - p_d - s \geq \max\{v - p_m, 0\}.$$

The buyer compares the direct channel with the best alternative: buying on M if $p_m \leq v$, or the outside option, which gives utility of zero.

We can simplify the analysis by eliminating weakly dominated strategies to obtain the following preliminary result (proof is in the appendix).

Lemma 1. *M's optimal fee and S's optimal prices satisfy* $0 < f \leq v - c$ *and* $p_d < p_m = v$.

These properties are intuitive. M must choose a positive fee f, but low enough so that S can make non-negative profits selling through M. Meanwhile, S will set its price to extract the entire buyer surplus on M and will undercut in the direct channel, in order to attract buyers with the lowest switching costs and avoid paying M's fee when selling to them.

Given Lemma 1, S's pricing problem reduces to setting $p_d \leq v$ to maximize its profit

$$\pi = (p_d - c)\, G(v - p_d) + (v - c - f)(1 - G(v - p_d)). \tag{1}$$

Since S can attract all buyers to purchase directly by setting $p_d \leq v - \overline{s}$ (where $\overline{s}$ is the upper bound of the support of $G(.)$), then S will never want to price less than $v - \overline{s}$. Thus, S's optimal p_d is such that $v - \overline{s} \leq p_d \leq v$.

Denote by $p_d(f)$ the unique solution in p_d to the first-order condition (FOC)

$$G(v - p_d) - g(v - p_d)(p_d - (v - f)) = 0,$$

so that

$$p_d(f) = v - f + \frac{G(v - p_d(f))}{g(v - p_d(f))}. \tag{2}$$

Provided G is well behaved[2], so there is a unique solution to this FOC that characterizes the profit maximizing price, $p_d(f)$ is decreasing in f. The higher the transaction fee charged by M, the more aggressively S will price in the direct channel in order to induce leakage.

S's profit maximizing price $p_d^*(f)$ is then given by

$$p_d^*(f) = \begin{cases} v - \overline{s} & \text{if} \quad p_d(f) \leq v - \overline{s} \\ p_d(f) & \text{if} \quad v - \overline{s} \leq p_d(f) \leq v \\ v & \text{if} \quad p_d(f) \geq v \end{cases} .$$

The corresponding profit for M is

$$\Pi(f) = f(1 - G(v - p_d^*(f)))$$

so

$$\Pi^* = \max_{f \leq v-c} \{f(1 - G(v - p_d^*(f)))\} . \tag{3}$$

The expression of M's profits reflects the leakage tradeoff. If $f = 0$, S will set $p_d = v$ since it has no reason to try to shift transactions to the direct channel. However, once $f > 0$, because there are some buyers with s arbitrarily close to 0, S will always want to lower p_d to induce some leakage. As M increases f further, it leads S to price more aggressively in the direct channel (i.e. lower p_d^*), which means more leakage and fewer transactions on M. This implies $\Pi^* < v - c$ since at $f = v - c$, some buyers will no longer purchase on M.

Initially, we are interested in the effect on the equilibrium fee, the direct price, leakage and M's profit of changing the distribution of switching costs so that overall, buyers have higher switching costs. The immediate effect of higher switching costs is that the direct channel is relatively less appealing to buyers, which should increase M's profits as fewer buyers switch (all other things equal). On the other hand, precisely because the direct channel is less appealing, S has to price more aggressively in the direct channel to attract buyers, which leads more buyers to switch. Finally, M can respond to leakage being less appealing by raising its fee, or it can respond to S pricing more aggressively in the direct channel by lowering its fee. Either way, M's optimal adjustment of its fee can help mitigate the effect on leakage of a change in switching costs.

To obtain more specific results, we proceed by assuming G is the uniform distribution; i.e.

[2] A sufficient condition is that g is continuous over its support, and is weakly decreasing in its argument, so that G/g is strictly decreasing in $p_d(f)$.

$$G(s) = \frac{s}{\mu},$$

for $s \in [0, \mu]$, where $\mu > 0$, with $G(s) = 0$ for $s < 0$ and $G(s) = 1$ for $s > \mu$. This ensures we can get a closed form solution.[3] We are interested in how things change when switching costs increase. Switching costs can be interpreted as capturing the underlying propensity for leakage: higher switching costs correspond to a lower propensity for leakage (holding everything else constant). A change in switching could arise from an exogenous shock in preferences or technology, or it could be the result of strategic design decisions by M. We capture an increase in switching costs by increasing μ. If $\mu_2 > \mu_1 > 0$, then $G_2(s)$ stochastically dominates $G_1(s)$, so an increase in μ corresponds to an overall increase in switching costs in the sense of first-order stochastic dominance.[4]

Assuming $f \leq v - c$, we have

$$p_d(f) = v - \frac{f}{2}$$

and therefore S's profit maximizing price $p_d^*(f)$ is given by

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad f \geq 2\mu \\ v - \frac{f}{2} & \text{if} \quad f \leq 2\mu \end{cases}. \tag{4}$$

The corresponding profit for M is

$$\Pi(f) = \begin{cases} 0 & \text{if} \quad f \geq 2\mu \\ f\left(1 - \frac{f}{2\mu}\right) & \text{if} \quad f \leq 2\mu \end{cases}. \tag{5}$$

Recalling that M will always set $f \leq v - c$, and relegating the details to the appendix, we obtain the following proposition.

Proposition 1. *Suppose* $G(s) = \frac{s}{\mu}$ *and* $\overline{s} = \mu$. *There is always positive but partial leakage equal to* $\frac{1}{\mu}(v - p_d^*(\mu))$, *where the equilibrium transaction fee, direct price and marketplace*

[3] We can still get a closed form solution provided G is a power function. In Online Appendix C we repeat the analysis of the model in this section with $G(s) = \frac{1}{\mu}s^{\alpha}$ for $s \in \left[0, \mu^{\frac{1}{\alpha}}\right]$ and show we get the same qualitative results. We stick to the simpler linear function for exposition purposes.

[4] Formally, G_2 first-order stochastically dominates G_1 if $G_2(s) \leq G_1(s)$ for all s, with strict inequality over an interval with positive measure.

profit are as follows:

$$f^*(\mu) = \min\{\mu, v-c\}$$
$$p_d^*(\mu) = v - \min\left\{\frac{\mu}{2}, \frac{v-c}{2}\right\}$$
$$\Pi^*(\mu) = \begin{cases} \frac{\mu}{2} & \text{if } \mu \leq v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if } \mu \geq v-c \end{cases}. \tag{6}$$

In response to an increase in switching costs (an increase in μ), the marketplace's fee weakly increases, the seller's direct price weakly decreases, the extent of leakage weakly decreases, and the marketplace's profit increases.

Given there are some buyers with arbitrarily small switching costs, any positive fee will lead to some leakage. However, it never makes sense for $M$to set a fee that leads all buyers to switch to purchase directly. At such a high fee, it will get no demand. This is why there is always positive but partial leakage in equilibrium.

Proposition 1 says that M's fee increases and it does better when switching costs of buying directly increase (in the first-order stochastic sense), while S's direct price and leakage decrease as a result. To unpack the effects of an increase in switching costs, consider what happens as μ increases starting from a low level (i.e. where switching costs are relatively low). First note that in the range of fees at which M still has some demand, (4) implies that S's direct price only changes in response to a change in M's fee and does not depend on μ directly. If fewer buyers purchase directly as a result of higher switching costs, M will want to increase its fee. S will respond by decreasing its direct price to induce more buyers to switch in order to avoid M's higher fee. This decrease in S's direct price mitigates the initial decrease in leakage, and indeed turns out to leave the net amount of leakage unchanged. But with a higher fee, M is strictly better off.

As switching costs μ continue to increase above $\mu = v - c$, M's fee is pushed to the maximum level $v - c$ beyond which S would drop out. At this point, further increases in switching costs will not lead to any further increase in M's fee, and so S no longer decreases its direct price. With S's direct price held constant, we are only left with the immediate effect of higher switching costs, which is to decrease leakage and increase M's profit.

5 Strategies to mitigate leakage

In this section we explore six alternative strategies that can help mitigate leakage: (1) investing in transaction benefits, (2) charging the seller for referrals rather than for transactions, (3) hiding information, (4) using a price-parity clauses to rule out leakage, (5) adding a rival seller onto the marketplace; and (6) in the case there is an alternative seller that the marketplace can show, hiding the seller if it tries to induce too much leakage. In each case, we explain the tradeoffs that arise from using the alternative strategy, modifying the baseline model as necessary to capture the relevant tradeoff. And we explore how the tradeoffs change as we increase switching costs.

5.1 Investing in transaction benefits

A direct way the marketplace could try to reduce or even eliminate leakage is by offering buyers some benefits from completing transactions on the marketplace. This means that buyers and sellers have an incentive to use the marketplace even for repeat interactions. For example, StyleSeat, a marketplace that helps consumers discover and book appointments with beauty salons and stylists has added the ability to pay through the platform, schedule bookings 24/7, receive personalized reminders, handle last minute cancellations, manage receipts and records of past transactions, and handle no-shows and rebookings.

We model this by assuming that M must invest $K(b)$ to provide buyers with a benefit b from completing their transaction on M, where $K(b)$ is an increasing and convex function of b.[5] The baseline model is otherwise unchanged.

Suppose M has chosen b and consider the resulting game. A buyer with switching cost s purchases directly iff

$$v - p_d - s \geq \max\{v + b - p_m, 0\}.$$

Following the same logic as in the baseline, M always sets $f \leq v + b - c$ and S sets $p_m = v + b$ and $p_d \leq v$. Thus, the decision S now faces is to set $p_d \leq v$ to maximize its profit

$$\pi = (p_d - c)\, G(v - p_d) + (v + b - c - f)(1 - G(v - p_d)).$$

S's profit-maximizing price $p_d^*(f)$ is given by

$$p_d^*(f) = \begin{cases} v - \overline{s} & \text{if} \quad p_d(f) \leq v - \overline{s} \\ p_d(f) & \text{if} \quad v - \overline{s} \leq p_d(f) \leq v \\ v & \text{if} \quad p_d(f) \geq v \end{cases},$$

[5] One could conduct an equivalent analysis if the benefit was relevant to S, or to both buyers and S. It also does not matter whether Minvests in b before or after (or at the same time) it sets its fee f, provided it is chosen before S sets its prices.

where

$$p_d(f) = v + b - f + \frac{G(v - p_d(f))}{g(v - p_d(f))}.$$

Provided G continues to be well behaved (as in the baseline), so there is a unique solution to this FOC that characterizes the profit maximizing price, $p_d(f)$ is decreasing in f and increasing in b.

A key difference relative to the baseline is that S no longer has an incentive to induce leakage (i.e. set $p_d^*(f) < v$) as soon as $f > 0$. Rather, it only has an incentive to induce leakage when $f > b$. This is because, as long as $f \leq b$, S's margin on M $(v + b - c - f)$ is higher than the maximum margin it can obtain in its direct channel $(v - c)$. Taking this into account, M will always set $f \geq b$. Provided M has chosen $b > 0$, it has the option to set $f = b$ and obtain a profit of $b - K(b)$ given there is no leakage. Alternatively, it can set a higher fee $(f > b)$ and face some leakage.

Stepping back to the first-stage problem that also involves setting b, the corresponding profit for M is

$$\Pi^* = \max_{\substack{b \geq 0 \\ b \leq f \leq v - c + b}} \left\{ f \left(1 - G\left(v - p_d^*(f)\right)\right) - K(b) \right\}.$$

To obtain closed form solutions with full characterization of the optimum, we adopt the same uniform distribution as in the baseline model and assume K is quadratic. Relegating the calculations to the appendix, we obtain the following proposition.

Proposition 2. *Suppose $G(s) = \frac{s}{\mu}$ and $\overline{s} = \mu$, and $K(b) = \frac{k}{2}b^2$, with $k > 0$. Then the marketplace's optimal choice of transaction benefits is*

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \mu \leq \frac{1}{2k} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ \frac{1}{k} - \frac{v-c}{2k\mu} & \text{if} \quad \mu \geq \overline{\mu} \end{cases},$$

where

$$\overline{\mu} = \frac{v - c}{2} + \frac{1}{4k} + \sqrt{\left(\frac{v - c}{2}\right)^2 + \frac{1}{16k^2}}.$$

The marketplace's optimal level of investment in transaction benefits b^* is decreasing in the investment cost k, and initially constant in switching costs μ, then decreasing in μ for $\mu \in \left[\frac{1}{2k}, \overline{\mu}\right]$, and finally increasing in μ for $\mu \geq \overline{\mu}$. Meanwhile, the equilibrium amount of leakage is initially constant at zero, then increasing in μ for $\mu \in \left[\frac{1}{2k}, \overline{\mu}\right]$, and finally decreasing in μ for $\mu \geq \overline{\mu}$.

The result in Proposition 2 that M's investment in transaction benefits b^* is everywhere decreasing in k is not at all surprising. Higher costs of investment naturally lead to less investment in transaction benefits.

More interesting is the comparative static of b^* in μ. Proposition 2 shows b^* attains its minimum when M faces moderate switching costs ($\mu = \overline{\mu}$), and the highest level occurs at extreme levels of switching costs, i.e. $\mu \to 0$ and $\mu \to \infty$ (note indeed b^* tends to $\frac{1}{k}$ again as $\mu \to \infty$).

To explain this result, it is useful to trace through what happens as switching costs (measured by μ) increase. When they are sufficiently low, the competitive pressure of the direct channel is highest, so the most effective way for M to generate revenue is to set $f = b$, so S does not want to induce any leakage. With all buyers purchasing on M, it extracts the full marginal value of its investment in b, thereby setting b to maximize $b - K(b)$, and setting $b^* = \frac{1}{k}$. As μ increases from this low base (i.e. $\mu > \frac{1}{2k}$), value extraction becomes more attractive, and it becomes optimal for M to set $f > b$. This results in increasing leakage. With fewer buyers purchasing on M, M no longer extracts the full marginal value of an investment in b, so b^* decreases. Essentially, M finds it optimal to save on investing in transaction benefits and instead rely on higher switching costs to extract more from S. In this intermediate range of switching costs, the equilibrium level of leakage is increasing in μ, reflecting higher fees and a lower level of investment in transaction benefits. Past a certain point however (i.e. for $\mu > \overline{\mu}$), the switching costs are high enough that the equilibrium amount of leakage starts to decrease as switching costs increase. This is the point where M sets its fee to extract the entire margin from S for transactions conducted on M (i.e. $f^* = v + b^* - c$). Since the fee cannot increase any further, any increase in switching costs from here onwards results in more buyers choosing to purchase via M instead of via the direct channel. This means M extracts a higher share of the total value created by transaction benefits, and so its investment incentive (its choice of b^*) is now increasing.

5.2 Referral fees

Another obvious way to fight leakage is for M to prevent the problem from arising in the first place: rather than charging the seller a transaction fee, M can charge the seller for referring buyers to it, i.e. informing them of its existence. For example, Thumbtack, a leading marketplace for home services, has adopted this approach: it charges service providers for new leads, without any transaction fees whatsoever.

To keep things as simple as possible, we focus on the interpretation of our model in which all buyers need to visit M to discover S, but then some switch to buy directly. In this scenario, M can sidestep the leakage issue by charging S only for referring buyers to it, and not based on whether or not the buyer completes the transaction on M.[6] By just charging for each buyer that comes via M (regardless of whether they end up buying on M or directly), M eliminates S's incentive to induce buyers to switch to purchase directly. This is because now S has the same marginal cost (c) regardless of whether it serves a buyer through M or directly. It is then easy to see that S will set $p_d = p_m = v$and make $v - c - r$ for each buyer referred by M, where r is the referral fee. Indeed, referred buyers purchase on M since they have no reason to incur s given the prices are identical. The alternative for S is not to sign with M and get nothing, given that all buyers rely on M to discover S. This implies M can charge $r = v - c$, so S is just willing to accept referrals from M, and M obtains a profit of $v - c$. This is strictly higher than M's profit in (3) with a transaction fee, which as noted there, is strictly less than $v - c$. Thus, in this simple framework, M always prefers to use a referral fee over a transaction fee.

Given this, one may wonder why we don't observe a majority of marketplaces that face leakage use referral fees instead of transaction fees. A key reason is that referral fees are quite risky from the point of view of sellers: they do not a priori know for sure they will get anything from the referral. The buyer may be a genuine buyer or one that turns out not to be interested. As a result, the referral fee must be lowered to compensate sellers for this risk. More generally, a referral fee (or any kind of fixed fee) eliminates the marketplace's ability to price discriminate across buyers that differ in the size, value or number of their transactions. This is why a fee charged at the transaction level may work better than a fee that is a fixed amount per buyer. To show this formally, we introduce uncertain buyer demand and show that it generates a realistic tradeoff for M between using referral fees and using transaction fees.

We model uncertain buyer demand by assuming M does not know ex-ante (when it sets its fee) the demand facing S. Specifically, we assume all buyers have the same probability q of actually valuing the product at v and therefore conducting a transaction, and probability $1 - q$ of valuing the product at zero, but M does not know q. This is compatible with two interpretations: all buyers have the same draw (v or zero) or each buyer draws their valuation (v or zero) independently, with the same probability q.[7] When M sets its fee, it only knows the prior distribution of q. For simplicity, we describe this by a continuous distribution function H on the interval $\left[\underline{q}, \overline{q}\right]$, where $0 \leq \underline{q} < \overline{q}$ and $\overline{q}$ is not necessarily finite. This distribution H is assumed to be independent of the distribution G. Other than the

[6] This type of referral fees has also been studied by Condorelli et al. (2018), who look at whether an intermediary should charge for referrals rather than buying and reselling.

[7] If there was uncertainty over the number of buyers, then this would create a tradeoff between transaction fees and fixed fees. To generate a tradeoff between transaction fees and fees charged per buyer that is referred we need uncertainty over the number of transactions per buyer.

introduction of random q, everything else is assumed to be the same as before.[8]

Consider first the case M charges a transaction fee f, as in the baseline. The timing is now as follows:

1. M sets its transaction fee f
2. The value of q is drawn and, having observed it, S decides whether to accept the contract or not
3. S sets its prices (both direct and, if it accepts the contract, on M)
4. Buyers visit M and are informed of S if it is listed on M. Buyers then realize the value of the product, before deciding whether to purchase, and if so, in which channel. S pays f to M for each transaction on M.

The choices of M's optimal fee f and S's optimal prices p_m and p_d are identical to before because all payoffs are multiplied by q, but q does not affect the choices between different options, including whether S should accept the contract. This is because in case a buyer values the product at zero, which happens randomly, such a buyer is irrelevant to S's and M's decisions and profit. Thus, M's corresponding expected profit in this setting is just $\Pi^* \int_{\underline{q}}^{\overline{q}} q dH(q)$, where Π^* is the profit from the baseline model with no uncertainty given by (3).

Now consider the case where M instead charges S a referral fee for each referred buyer. The timing is as follows:

1. M sets its referral fee r
2. The value of q is drawn and, having observed it, S decides whether to accept the contract or not
3. S sets its prices (both direct and, if it accepts the contract, on M)
4. Buyers visit M and are informed of S if it is listed on M. S pays r to M for each such buyer referred. Buyers then realize the value of the product, before deciding whether to purchase, and if so, in which channel.

When M charges a referral fee, buyers can still decide whether to complete the transaction on M or buy directly. The key difference in this case is that, even if a buyer decides to purchase directly, S still has to pay for the referral of that buyer (since the buyer needed M

[8] Thus, the baseline setting corresponds to the case when H collapses to the mass point at $\underline{q} = \overline{q} = 1$.

to discover S).[9] Since there is no transaction fee, S has no incentive to lower its price in its direct channel, so all buyers will just purchase via M since they have no reason to incur the cost s to switch and buy directly.

If M knew the probability q that each buyer is actually interested in S's product, it would optimally charge a referral fee $r = (v-c)\,q$ and extract all the expected surplus. But since M does not observe q, it has to take into account that a higher referral fee makes it less likely that S (which knows the value of q) will accept the offer. Specifically, S accepts the offer if and only if $(v-c)\,q \geq r$. If instead $(v-c)\,q < r$, then S rejects the offer and M makes no profit. Thus, M's expected profit is $\max_r \left\{r\left(1-H\left(\frac{r}{v-c}\right)\right)\right\}$, or, defining $y = \frac{r}{v-c}$, the profit can equivalently be written as $(v-c)\max_y \{y\,(1-H\,(y))\}$.

Comparing expected profits under the two types of fees, M's profit is higher with a transaction fee if and only if

$$\Pi^* \int_{\underline{q}}^{\overline{q}} q dH\,(q) > (v-c) \max_y \{y\,(1-H\,(y))\}\,. \tag{7}$$

From (3), we know that $\Pi^* < v-c$, as noted above. On the other hand, we also know that $\int_{\underline{q}}^{\overline{q}} q dH\,(q) > \max_y \{y\,(1-H\,(y))\}$.[10]

Thus, M faces a tradeoff between using the two types of fees. Referral fees sidestep the leakage problem, thereby allowing it to extract more surplus from buyers who are in fact interested in S's product. On the other hand, transaction fees allow M to extract more surplus when there is a lot of uncertainty around buyer demand because they align the revenue extracted with buyer demand.

To make this tradeoff more precise, we adopt the same uniform distribution as in the baseline model for $G\,(.)$, and assume that H follows the generalized Pareto distribution

$$H\,(q) = \begin{cases} 1-\left(1+\frac{\varepsilon\left(q-\underline{q}\right)}{\sigma}\right)^{-\frac{1}{\varepsilon}} & \text{if} \quad \varepsilon \neq 0 \\ 1-e^{-\frac{q-\underline{q}}{\sigma}} & \text{if} \quad \varepsilon = 0 \end{cases},$$

where $\varepsilon < 1$, and the support is $q \geq \underline{q}$ if $0 \leq \varepsilon < 1$, and $\underline{q} \leq q \leq \underline{q} - \frac{\sigma}{\varepsilon}$ if $\varepsilon < 0$. This distribution covers the uniform distribution ($\varepsilon = -1$), the exponential distribution ($\varepsilon = 0$), and the Pareto distribution ($\varepsilon > 0$ and $\underline{q} = \frac{\sigma}{\varepsilon}$). Here, ε measures the shape of the distribution and σ measures the scale (or dispersion) of the distribution. Note in case $\varepsilon > 0$, we must

[9] In practice, this can take the form of a per-click referral fee, i.e. a fee that the seller pays for every buyer that clicks on the seller's profile on the marketplace or on the link to the seller's direct website.

[10] To see this, note if we define $q^* = \arg\max_q \{q\,(1-H\,(q))\}$, then $\int_{\underline{q}}^{\overline{q}} q dH\,(q) - q^*\,(1-H\,(q^*)) = \int_{\underline{q}}^{q^*} q dH\,(q) + \int_{q^*}^{\overline{q}} (q-q^*)\,dH\,(q) > 0$.

have $\underline{q} < \frac{\sigma}{\varepsilon}$ since otherwise the maximization problem on the right-hand side of (7) will not be well defined. With these distribution functions we show the following proposition.

Proposition 3. *Suppose $G(s) = \frac{s}{\mu}$ and $\overline{s} = \mu$. If $\sigma \geq \underline{q}$, then M prefers a transaction fee over a referral fee iff*

$$\frac{\Pi^*(\mu)}{v-c} > \frac{\sigma^{\frac{1}{\varepsilon}}\left(\frac{\sigma-\varepsilon\underline{q}}{1-\varepsilon}\right)^{1-\frac{1}{\varepsilon}}}{\underline{q}+\frac{\sigma}{1-\varepsilon}}, \tag{8}$$

where $\Pi^*(\mu)$ is given by (6). If $\sigma < \underline{q}$, then M prefers a transaction fee over a referral fee iff

$$\frac{\Pi^*(\mu)}{v-c} > \frac{\underline{q}}{\underline{q}+\frac{\sigma}{1-\varepsilon}}. \tag{9}$$

If switching costs as measured by μ are sufficiently high (low), M will prefer to use a transaction fee (referral fee). If the dispersion in buyer demand as measured by σ is sufficiently low and provided $\underline{q} > 0$, then M will prefer to use a referral fee. Moreover, an increase in μ, or σ, or ε always shifts M's tradeoff towards using a transaction fee, while an increase in $\underline{q}$ always shifts M's tradeoff towards using a referral fee.

To understand the effect of μ (switching costs) on the choice of fee type, note that with a referral fee, M does not have to worry about switching costs, which are irrelevant for its profit. With a transaction fee, higher switching costs increase M's profit (indeed, recall $\Pi^*(\mu)$ is increasing in μ), and so using a transaction fee naturally becomes more attractive with high switching costs.

To understand the effects of σ, $\underline{q}$ and ε on the choice of fee type, it is useful to first point out that they each have a monotonic effect on the normalized variation of q, i.e. the standard deviation of q divided by its mean. The normalized variation matters because more uncertainty over buyer demand relative to the expected level intuitively shifts the tradeoff in favor of using a transaction fee given it allows M to capture the expected value of q without any discount for uncertainty. To confirm this intuition, the normalized standard deviation of the generalized Pareto distribution H can be defined when $\varepsilon < \frac{1}{2}$, and equals

$$\frac{\sigma}{\left(\sigma+\underline{q}(1-\varepsilon)\right)\sqrt{1-2\varepsilon}}.$$

Clearly, this expression is increasing in σ and ε, and decreasing in $\underline{q}$. Thus, when σ or ε increase, or $\underline{q}$ decreases, the normalized variation increases, shifting the tradeoff in favor of using a transaction fee.

The above intuition is based on what happens when $\sigma \geq \underline{q}$. In case $\sigma < \underline{q}$, the logic is a bit different. With relatively low dispersion in the distribution, M prefers to set a low fee

($r^* = \underline{q}$) so that S always joins, i.e. for any realized q. There is no longer any distortion under the referral fee, but M's profit is fixed at $(v - c)\,\underline{q}$. This can be compared to M's profit under a transaction fee, which depends on the expected value of q, i.e. $\Pi^*(\mu)\left(\underline{q} + \frac{\sigma}{1-\varepsilon}\right)$. In this case, an increase in σ and ε shift the tradeoff in favor of using a transaction fee simply because they increase the expected value of q but leave the amount M can extract under a referral fee unchanged. On the other hand, an increase in $\underline{q}$ increases M's profit by the full amount $v - c$ with a referral fee, but by less than this amount with a transaction fee, thereby shifting the tradeoff in favor of a referral fee.

5.3 Limiting communication

In an effort to prevent leakage, marketplaces can make communication between buyers and sellers more difficult. This can involve hiding the identity of the parties and banning or restricting communications between them until after they have already committed to transact, thereby making it difficult for them to deal directly. Often, communications can only be done via the marketplace itself, and the marketplace blocks sharing of identifying information.

While these types of restrictions help reduce leakage, they also have a downside. Limiting communication on the marketplace can make it more difficult for buyers to ascertain the quality of the seller's product. In other words, buyers are left with more uncertainty about the value of the seller's product. For instance, AngelList and its syndicate leads connect investors with startup founders who are seeking to raise capital. Most syndicate leads on AngelList have very strict policies prohibiting investors to directly contact founders (e.g. via email). This makes it hard for investors to ask founders questions and to thoroughly evaluate their competence and the prospects of their startups.

In this section we capture the tradeoff associated with placing limits on communication in a simple way. Obviously, the strategy of limiting communication is only relevant to the extent buyers rely on M to discover S. To keep things as simple as possible, we will stick to that interpretation. As in Section 5.2, we assume that a priori, buyers have a probability $0 < q < 1$ of actually valuing S's product at v and probability $1 - q$ of valuing the product at zero.[11] Unlike Section 5.2, we treat q as a fixed parameter known to all parties.

If buyers are allowed to freely communicate with S, they can find out with certainty whether they value S's product or not, as well as its price in the direct market, just as in the baseline model. Alternatively, M has the option to limit communication between the buyer and S (e.g. by hiding the seller's identity, requiring that all communication is intermediated

[11] Once again, this covers the case all buyers have the same draw (v or zero), as well as the case each buyer draws their valuation independently with the same probability q.

and vetted by the marketplace, etc.). We assume that this effectively prevents buyers from being able to switch and buy from S directly. However, it also lowers the probability that any given buyer will find out their true valuation for S's product from one to $\eta \leq 1$.[12] This means that with probability $1-\eta$, a given buyer will never be sure whether their value for S's product is v or zero, so they must make their purchase decision based on the expected value qv. When $\eta = 1$, there is no information loss from limiting communication, so M will always want to do so, as we will see. When $\eta = 0$, there is complete information loss, so all buyers make their purchase decisions based on expected value qv. The lower η, the higher the information loss.

If M does not place any limits on communication, then things are straightforward. M's and S's pricing behaviors are the same as in the baseline, except all payoffs must be multiplied by q. Thus, M's expected profit is $q\Pi^*$, where Π^* is given by (3).

Suppose now M bans communication. Assuming S lists on M, leakage is impossible so S's only relevant price is the on-platform price p_m. It has two pricing options. The first one is to set $p_m = v$ and only sell to the fraction ηq of buyers who learn that their valuation of its product is v. Its resulting profit is $\eta q\,(v-c-f)$. The second option for S is to set the lower price $p_m = qv$, so that it also sells to the $1-\eta$ buyers who never learn their true valuation for its product. Its resulting profit in this case is $(1-\eta+\eta q)\,(qv-c-f)$. Thus, S prefers the first (high-price) option if and only if

$$f > \frac{(1-(2-q)\,\eta)\,qv}{1-\eta} - c.$$

Otherwise, S chooses the second (low-price) option.

In turn, this implies that there are two possible optimal options for M when choosing f:

1. Set $f = v - c$, which induces S to choose the high-price option, resulting in M making a profit of $\Pi_H(\eta) = \eta q\,(v-c)$.
2. Set $f = \frac{(1-(2-q)\eta)qv}{1-\eta} - c$, which induces S to choose the low-price option, resulting in M making a profit of $\Pi_L(\eta) = (1-\eta+\eta q)\left(\frac{(1-(2-q)\eta)qv}{1-\eta} - c\right)$.

Thus, M's profit after banning communications is $\max\{\Pi_H(\eta), \Pi_L(\eta)\}$, which needs to be compared to the profit $q\Pi^*$ without any limits on communication. Relegating the rest of the analysis to the appendix, we obtain the following proposition.

[12] The idea that limiting the exchange of information reduces transaction value on a platform is also present in Piolatto (2020). However, Piolatto does not consider the platform's choice of how much information exchange to allow.

Proposition 4. *M prefers to ban communications for all* $\eta > \frac{\Pi^*}{v-c}$. *If* $q > \frac{c}{v-\Pi^*}$, *then* M *also prefers to ban communications when* $\eta < \eta_L$, *where* $\eta_L > 0$ *is uniquely defined by* $\Pi_L(\eta_L) = q\Pi^*$. *If* q *is close enough to one,* M *always prefers to ban communications. Finally, if* $G(s) = \frac{s}{\mu}$ *and* $\overline{s} = \mu$, *then* M *will prefer to ban (allow) communication when* μ *is sufficiently close to zero (one).*

Recall $0 < \Pi^* < v - c$, so the first result in the proposition means there is always some cutoff between 0 and 1 such that for all higher η, the marketplace prefers to ban communications. This reflects that when η is close enough to one, the information loss from banning communication is small, while retaining the upside of eliminating leakage. Surprisingly, provided q is not too small (i.e. provided $q > \frac{c}{v-\Pi^*}$), then M also prefers to ban communications for η sufficiently close to zero.

To explain these properties, it is useful to note that $\Pi_H(\eta)$ is increasing in η, whereas $\Pi_L(\eta)$ is decreasing in η (whenever $\Pi_L(\eta) > 0$). To understand the difference, recall that option 1 involves only selling to buyers who learn that they value S's product at v. When η is higher, the number of such buyers is larger (i.e. there is less information loss due to the communication ban), so M's profit is also higher under the communication ban. Option 2 involves setting a lower fee such that S also sells to buyers who learn nothing. In this case, buyer demand is decreasing in η: fewer informed buyers means more buyers are willing to buy at $p_m = qv$, which makes this option more profitable for M, especially if q is high. Furthermore, the maximum fee f that M can charge under this option is higher when there are fewer informed buyers since S prefers option 2 when η is low and q is high. Both of these effects mean that, provided q is not too small and η is close enough to zero, M will also want to ban communications. Indeed, if q is high enough, then this means M will prefer to ban communications for the full range of η.

5.4 Price-parity clauses

Suppose the marketplace can require that the participating seller must set its direct price no lower than its price on the marketplace. If the seller sets a lower direct price, the marketplace commits to delist the seller. This type of clause in a contract is known as a price-parity clause. While this is not in the marketplace's interest ex-post in case the seller does undercut given that it would deprive the marketplace of all revenue, being able to commit to such a clause can make the marketplace better off ex-ante if it stops the seller from undercutting in the first place. Such price-parity clauses have been used by hotel booking platforms, price comparison platforms, as well as Amazon.

How a price-parity clause works depends on whether buyers depend on M to discover S. If they do, then with this restriction in place, S will always be willing to stay on M provided $f \leq v - c$, since it cannot get any profit if it delists. As a result, S sets $p_d = p_m = v$ and M sets $f = v - c$, so M extracts the maximum profit $\Pi = v - c$, which is always strictly more than what it would get without the price-parity clause. Thus, M will always impose a price-parity clause, which results in higher direct prices for buyers.

Things are less obvious when buyers don't depend on M to discover S since then S has the option of rejecting M's offer and still making positive profit by just selling directly. If S lists, it sets $p_d = p_m = v$ to comply with the price-parity clause and obtain $v - c - f$. If S delists, it obtains

$$\max_{c \leq p_d \leq v} \{(p_d - c) G(v - p_d)\}.$$

Thus, M will set f as high as possible so that S is just willing to list, i.e.

$$f = v - c - \max_{c \leq p_d \leq v} \{(p_d - c) G(v - p_d)\}. \tag{10}$$

This is also equal to M's profit given there is no leakage, so all buyers buy through M.

As a result, M prefers to impose a price-parity clause if its profit in (10) exceeds its baseline profit given in (3), i.e. if

$$v - c - \max_{c \leq p_d \leq v} \{(p_d - c) G(v - p_d)\} \geq \max_{f \leq v - c} \{f (1 - G(v - p_d^*(f)))\}. \tag{11}$$

In the appendix, we show that this inequality always holds, which leads to the following proposition.

Proposition 5. *The marketplace always prefers to adopt a price-parity clause.*

The reason a price-parity clause is always better for M in our setting is that it ensures M's fee does not lead to any distortion (via leakage), so the transaction fee acts like a fixed fee.[13] Given S does not want to induce leakage, it sets both its prices at v, and M's fee is just pinned down by making S indifferent between listing on M and not listing. This means M extracts the maximal possible surplus subject to S being willing to participate.

Thus, to some extent, the fact that imposing price-parity is always desirable for M here is not too surprising. Assuming ex-ante commitment to price parity and ex-post enforcement are possible, this is a way to eliminate the constraint arising from leakage. This may lead one to wonder why price parity is not more widely used by real-world marketplaces. One

[13] Liu et al. (2021) show that a platform facing a monopoly seller may be worse off by introducing a price-parity clause provided that it faces elastic seller demand with the right curvature properties.

reason is that both commitment and enforcement are difficult in practice. Monitoring sellers for transgressions of the price parity clause is costly because there are many ways in which sellers can offer buyers lower prices in their direct channels, which are not easily observable by the marketplaces they participate on. And commitment may lack credibility for less mature marketplaces. Lacking a large number of buyers and sellers, a marketplace may decide not to kick out a seller just because it has induced some buyers to purchase directly. Another reason is that for large and dominant marketplaces, imposing price parity runs the risk of being found anticompetitive by the relevant authorities.[14] The latter reason explains why marketplaces may prefer to discipline sellers less explicitly, which is something the next section explores.

5.5 Competing sellers and steering

Thus far we have assumed the marketplace attracts a single seller. As discussed in the baseline setting, this could capture the idea that the marketplace has many different sellers, each of which is in a different product or service category, so they can be treated separately. In this section we explore whether a marketplace would want to host a second competing seller and the impact seller competition has on leakage. In doing so, we will also consider the possibility that the marketplace can steer buyers towards the seller that induces less leakage. For example, CoachUp, a marketplace connecting fitness enthusiasts with private coaches for many different sports, provides preferential treatment in its listings to coaches that conduct more repeat transactions through the marketplace.

To analyze seller competition and steering in a tractable way, we suppose there are only two sellers, one of which is the same as in the benchmark case (the high-quality seller), and the other whose product is valued at u, where $c \leq u \leq v$ (the low-quality seller). The reason to allow for vertically differentiated sellers is to make the steering decision less trivial. If $u = v$, so the two sellers are identical, then M never loses anything by steering towards the seller that induces less leakage, thereby inducing the sellers to not undercut in the direct channel at all. As a result, M can charge its maximum fee (i.e. $f^* = v - c$) and obtain maximum profits. However, when $u < v$, if M charges $f^* = v - c$, then it may lose the ability to credibly prevent the high-quality seller from inducing leakage if the low-quality seller find it unprofitable to sell on M at such a high fee. The model is otherwise the same as the baseline setting.

We first treat the case when M cannot steer buyers, before exploring the implications of M being able to steer buyers to one of the sellers by hiding the other. We will use the usual

[14] As discussed in Wang and Wright (2020), there have been competition and regulatory cases involving several large marketplaces that have used price-parity clauses.

asymmetric Bertrand tie-breaking rule throughout: when buyers are indifferent between buying from the high-quality and low-quality seller, they choose the high-quality seller.

5.5.1 Competition without steering

Even without steering, allowing for two competing but vertically differentiated sellers makes the analysis of leakage considerably more complicated. Fundamentally, this is because competition between sellers in the direct market can lead to more leakage than a single seller (the high-quality one) would optimally want to induce. Despite the added complications, we are able to prove the following results.[15]

Proposition 6. *Suppose there are two competing sellers, a high-quality seller with value v and a low-quality seller with value $u \leq v$, and switching costs are distributed according to $G(s) = \frac{s}{\mu}$ with $\overline{s} = \mu$. In equilibrium, the high-quality seller always makes all sales on both channels. The marketplace's profit with two competing sellers is always weakly lower than the marketplace's profit with the high-quality seller only (i.e. the baseline), and indeed is weakly decreasing in u.*

The proposition shows that in the absence of steering, M is always weakly worse off with competing sellers. Indeed, M's profits are the same as in the case with just the high-quality seller only when $u - c \leq \frac{v-c}{2}$ and $\mu \geq 2(u-c)$, i.e. when the low-quality seller is a sufficiently weak competitor and the switching costs are high enough so that leakage is not too serious of an issue. Otherwise, introducing competition always hurts M—and so does making the low-quality seller more competitive, i.e. increasing u. This runs counter to the usual intuition that marketplaces benefit from introducing more competition among their sellers in the absence of any innovation incentives.

To understand this result, note that for the range of M's fee where the low-quality seller constrains the pricing of the high-quality seller (i.e. when $f \leq u - c$), competition pins down the high-quality seller's prices ($p_m^h = c + f + v - u$ and $p_d^h = c + v - u$), thus increasing the difference between its price on M and its direct price relative to the case when the high-quality seller is a monopolist (recall from the baseline that was only $p_m - p_d = \min\left\{\frac{f}{2}, \mu\right\}$). This "involuntary" leakage makes M strictly worse off. Otherwise, the low-quality seller doesn't constrain the high-quality seller's price, which is the case when the low-quality seller is a sufficiently weak competitor and the switching costs are high enough.

[15] The proof in the appendix contains the full equilibrium characterization, including M's optimal fee and profits. Using those expressions, it is easily seen that M's optimal fee is weakly increasing and its profit is increasing in the switching cost parameter μ, consistent with Proposition 1 for the baseline setting.

By the same logic, when u decreases, the competitive pressure exerted by the low-quality seller is lower, so the high-quality seller has more market power, implying it can adjust prices in a way that induces less leakage, which benefits M. When u is sufficiently low and the switching costs are high enough, the presence of the low-quality seller becomes irrelevant and the outcome is the same as that with the high-quality seller only.

5.5.2 Vertically differentiated sellers and steering

We now extend the above analysis to the case M can steer buyers by only showing one of the two sellers depending on their respective prices. In order to explore steering, we naturally focus on the interpretation of our model in which buyers only know about the sellers if they discover them first on M (otherwise the analysis is the same as in the previous subsection). Once a seller is shown on M, buyers become aware of it, and so also know the seller's price in the direct channel. The reliance on M for discoverability means M can leverage competition between sellers to discipline them from undercutting too much in the direct channel.[16]

Specifically, given f and a set of seller prices, M's objective is simply to maximize the number of transactions on M. This means it will show the seller with the lowest $p_m - p_d$, i.e. which induces the least amount of leakage, subject to the constraint that buyers obtain non-negative surplus purchasing from the seller shown by M. This is the key difference relative to the previous setting with no steering: here, the two sellers are competing to minimize the difference between their direct price and their price on M, as opposed to competing in price on each channel.

To be precise, we assume that M makes its steering decision in the following way, given prices $\left(p_m^l, p_d^l\right)$ set by the low-quality seller and $\left(p_m^h, p_d^h\right)$ set by the high-quality seller:

- If both sellers are offering non-negative surplus to buyers on M (i.e. if $p_m^l \leq u$ and $p_m^h \leq v$), then show the seller with the lowest difference between price on M and direct price. If both sellers have the same difference (i.e. if $p_m^l - p_d^l = p_m^h - p_d^h$), then show the high-quality seller.

- If only one seller is offering non-negative surplus to buyers on M, then show that seller.

- If neither seller is offering non-negative surplus to buyers on M (i.e. if $p_m^l > u$ and $p_m^h > v$), then show the high-quality seller.

[16] Unlike the case of the price-parity clause analyzed in Section 5.4, here M cannot commit to hide a seller if it induces some leakage. Rather, M decides which seller(s) to show based on the prices they set. As a result, there may be leakage in equilibrium as we show below.

In the first case above, M is indifferent between showing either seller. Breaking the tie in favor of the high-quality seller does not have any impact on M's profit in the equilibrium of the full game.

In the third case above, M is indifferent between showing either seller or showing neither, because in all cases it makes zero profits—no buyers are willing to purchase on M from either seller. This tie-breaking assumption does have an impact on the overall equilibrium, by ensuring that M will never set $f > v - c$ when f is already high enough that the low-quality seller cannot make non-negative total profits and so is irrelevant. This treatment is consistent with the baseline setting with a monopoly seller in which M never sets $f > v - c$.[17]

The need to rely on M for being discovered and M's ability to steer also imply that here sellers may be willing to incur losses on M, which are then recouped in the direct channel, provided the total profit is non-negative. For example, if $f > u - c$, then the low-quality seller loses money on sales via M, but is willing to compete to be recommended as long as there exists $p_d^l \leq u$ such that

$$\left(p_d^l - c\right) G\left(u - p_d^l\right) + (u - c - f)\left(1 - G\left(u - p_d^l\right)\right) \geq 0.$$

In this case, the low-quality seller maximizes its chances of being shown by setting $p_m^l = u$ and setting p_d^l as high as possible subject to the non-negative profit constraint above. The derivation of the equilibrium will have to account for this subsidization strategy. Of course, this strategy no longer works if f is sufficiently large. Then the low-quality seller cannot make non-negative total profits, in which case it sets $p_m^l > u$ and is effectively irrelevant, i.e. the high-quality seller acts as a monopolist since it knows that it will get recommended by M no matter what (given our assumptions above).

We can fully characterize the equilibrium once we make our standard distributional assumption on switching costs. Relegating the details of the equilibrium characterization to the appendix, we obtain the following proposition.

Proposition 7. *Suppose there are two competing sellers, a high-quality seller with value v and a low-quality seller with value $u \leq v$, switching costs are distributed according to $G(s) = \frac{s}{\mu}$ with $\overline{s} = \mu$, and the marketplace can steer buyers to either seller. In equilibrium, the high-quality seller always makes all sales on both channels. The marketplace's profit is always weakly higher than both the profit with two competing sellers and no steering, and the profit with the high-quality seller only (i.e. the baseline). Moreover, the marketplace's profit*

[17] Alternatively, we could assume that when neither seller is offering non-negative surplus to buyers that purchase via the marketplace, M does not show either seller. In Online Appendix D, we redo the analysis in this section with this alternative assumption and show Proposition 7 continues to hold, even though the precise expressions for the optimal fee and M's profit change somewhat.

is weakly increasing in u.

Proposition 7 shows that M's profit is always weakly higher with steering. Note, however, that this is not entirely obvious a priori, since M cannot commit to steer when setting f.[18] The reason for this result is that sellers compete to get favored by M, which means they have an incentive to reduce leakage, which always benefits M. We show in the proof of Proposition 7 that, fixing u, if the switching cost μ is high enough, there is always positive leakage in equilibrium. However, when μ is low enough, there is no leakage. This is in contrast to the case without steering, where there is always positive leakage in equilibrium.

Taken together with Proposition 6, Proposition 7 implies steering is crucial for the introduction of a competing seller to benefit M.[19] Indeed, recall that without steering, M's profit was weakly decreasing in the competitiveness of the low-quality seller (measured by u), and therefore it was lower with competing sellers than with a monopoly seller. By contrast, with steering, M's profit is now weakly increasing in the competitiveness of the low-quality seller, so M always benefits from its presence (relative to the baseline monopoly seller case).

To better understand why the effect of seller competition on M's profit changes with steering, it is useful to consider two limit cases. When $u \to c$, the low-quality seller becomes irrelevant, so the optimal fee f^* and M's profit Π^* are the same as without competition, and this would be true with or without steering. On the other hand, when $u \to v$, i.e. the two sellers' products become perfect substitutes, both sellers end up setting the same price on and off M in an effort to get recommended, so there is no leakage, and M optimally sets $f^* = v - c$ to extract the entire surplus. This last result contrasts to the case without steering, in which the equilibrium prices are $p_m = c + f$ and $p_d = c$ when $u \to v$, so there is positive leakage still.

6 Managerial implications

While our analysis has focused on the solutions a platform can employ to deal with leakage, it is useful to start by asking what conditions make leakage more likely to be a

[18]Hagiu et al. (2022) show how a platform can benefit from the ability to steer consumers towards its own product in a setting where consumers can switch and buy directly if the price on the platform is too high. The threat of steering relaxes the leakage constraint on the platform's commission, thus increasing its profit. Our setting here is different since steering is among different (vertically differentiated) third-party sellers rather than between a seller and the platform's own product, and steering arises in equilibrium in our setting.

[19]In Online Appendix E, we confirm this also holds in a (simpler) variant of our setup with competing sellers, in which the low-quality seller is only active on M, i.e. does not have a direct channel.

concern. These can be thought of as factors that determine how high are switching costs (e.g. the parameter μ that we focused on):

a) Propensity of repeat transactions with the same counterparty

Repeated transactions between the same counterparties not only increase the amount of transaction fees the parties can save by completing transactions off the marketplace, but they also make it possible to get to know one another, build trust, and so be more willing to transact directly. Marketplaces for babysitters (e.g. Care.com), dog walkers (e.g. Wag!), coaches (e.g. CoachUp), tutors (e.g. Preply), home cleaners (e.g. Handy), and gardeners (e.g. Lawn Love) all grapple with this problem.

b) Ability to clearly specify transaction scope on the marketplace

For some services, such as fixing a car or resolving electrical or plumbing issues, the parties have to meet and communicate before they settle on what the transaction will involve (e.g. inspect a problem and give a quote for the work required). In such cases, leakage becomes a lot easier as the parties share information, gain trust and can make payments directly before the marketplace is ever in a position to try to charge a fee for the transaction.

c) In-person vs. remote/online transactions

Other things equal, leakage is harder to avoid when transactions are conducted in-person (TaskRabit) than online (Fiverr). The reasons are obvious: in-person meetings make it easier for the parties to share information, gain trust, and coordinate payment in cash, or via one of the many available peer-to-peer payment apps (e.g. PayPal, Venmo or WeChat).

After determining the severity of the leakage problem, companies and managers can prioritize the various solutions to it based on our analysis.

In general, providing transaction benefits beyond discovery should be the first and main priority. As we have seen, such benefits are particularly effective when switching costs to transacting directly are either very low or very high, i.e. when leakage is either very easy or very difficult (based on the criteria laid out above). When switching costs are very low, transaction benefits are crucial for the marketplace to be able to extract any transaction fees. When switching costs are very high, the marketplace can extract the sellers' full margin and so has a strong incentive to invest in such benefits. In addition to increasing transaction benefits, marketplaces can and should also add features that increase buyer switching costs to the relevant direct channels (e.g. loyalty programs, group discounts for multiple buyers as pioneered by Pinduoduo).

Second, marketplaces should encourage seller competition, provided their steering of buyers is sufficiently effective to make competition reduce leakage. Specifically, marketplaces should steer buyers towards sellers with direct prices that do not induce too much, if any, leakage. In settings where direct prices are not observed but consumers make repeat transactions, this means steering towards sellers with a high number of repeat transactions conducted through the marketplace. In our model, we have assumed steering can perfectly conceal sellers which are not shown from buyers. In reality, however, steering is not perfect. So marketplaces need to determine how effective their steering efforts are in influencing buyers' awareness and shopping behavior (which can be done via A/B testing). The more effective steering is, the closer we are to the situation in Section 5.5, where competition leads to less leakage and higher marketplace profits. In these situations, marketplaces should make design decisions that increase the competitive intensity among sellers. If on the other hand steering is ineffective, then more intense competition leads to more leakage and lower marketplace profits, as in Section 5.5.

That being said, in practice there are other reasons why a marketplace may still want to allow some seller competition even if steering is not possible. For instance, the rival seller may produce a different or better variety of the same product; the lower on-marketplace prices resulting from seller competition induce more buyers to come to the marketplace in the first place. In these situations, our finding implies marketplaces should take into account that increased head-to-head competition among sellers will exacerbate leakage when weighing up how much seller competition they wish to allow.

Third, limiting communication between participants can sometimes be effective up to a point, especially when it doesn't result in too much information loss for buyers, or when the underlying buyer uncertainty about the product's desirability is small. Furthermore, marketplaces should only consider limiting communication between buyers and sellers if the scope for leakage (based on the factors discussed above) is high.

Fourth, while imposing price parity would in principle seem like the cleanest way to eliminate leakage, in reality it is oftentimes problematic. Indeed, as we have discussed, price parity can be deemed anti-competitive for large marketplaces and is hard to implement for smaller marketplaces.

Fifth, where the scope for leakage is high, and the above strategies are not very effective in dealing with it, marketplaces should consider changing their business model to one based on referral fees instead of transaction fees. As we have discussed, this is a more drastic measure, which has the advantage of sidestepping leakage altogether, but at the cost of giving up the ability to extract value via price discrimination among sellers with very different buyer demands.

Finally, we should note an important pitfall that fast growing marketplaces (and their investors) need to be aware of when evaluating the risk of leakage. Initially, when a marketplace is growing fast and users on either side are mainly discovering new transaction partners on the other side, rather than doing repeat transactions with existing partners, leakage does not appear to be a big issue and may lead to complacency, e.g. under-investment in the features designed to minimize leakage incentives. Once users have settled down on a few counterparts they transact with often, then the leakage problem can become a lot more serious. Consequently, marketplaces should try to think about getting the right strategies in place for combatting leakage from the very beginning.

7 Future directions

Some of our results suggest natural avenues for empirical researchers to pursue. We have provided predictions regarding a marketplace's choice of strategy to combat leakage (investment in transaction benefits, the type of fee used, whether communication is limited, whether price parity clauses are adopted, whether competition between sellers is encouraged), depending on various underlying factors (the value of transactions, the level of switching costs, the importance of uncertainty in buyer demand, the ability to steer buyers among competing sellers and the degree of substitutability between these sellers' products)—some of these predictions can and should be tested.

Another interesting avenue to explore (theoretically) is the idea that leakage could also create an adverse selection problem on marketplaces. If consumers prefer to buy repeatedly from the same seller once their product or service has been proven to be good, and if sellers have a capacity constraint, then high-quality sellers will be more likely to leave the marketplace after they have built a large-enough base of buyers. Consequently, the marketplace will be left with a disproportionate fraction of low-quality sellers, at which point buyers will stop using the marketplace. This was exactly the problem that caused the demise of Homejoy, a marketplace for home cleaners: the most popular cleaners left the marketplace once they had amassed a sufficient number of customers that they could serve regularly. It would be interesting to explore whether adverse selection created by leakage can make it optimal for marketplaces to adopt a more radical change in business model, namely to stop being marketplaces altogether, and provide the product or service themselves instead.

One can also extend our analysis to competing marketplaces. To the extent that fees are pinned down by competition between marketplaces and sellers do not multihome across marketplaces (or if they do, they can set separate direct prices to target buyers from each marketplace), one could apply our equilibrium analysis of seller pricing in the subgame for a given fee to obtain the effects of leakage. However, the way in which competition determines marketplace fees, and the resulting effects on the choice of different strategies to address leakage will be more complicated than in our monopoly marketplace setting. This would be an interesting direction that future research should explore.

Finally, ours is a static framework, which means we have not explicitly modeled the possibility of return customers, which give a marketplace the option to adjust transaction fees based on the number of times buyers return to purchase from the same seller on the marketplace. For instance, CoachUp charges coaches transaction fees based on a sliding scale depending on the number of sessions completed with the same athlete, as does Upwork with respect to the value of the work, and Preply with respect to the value of tutoring done within a month. It would be interesting to explore the effectiveness of such pricing strategies in dealing with leakage.

8 Appendix

We include proofs of propositions not established in the text.

8.1 Proof of Lemma 1

If M were to set $f > v - c$, S would make a loss selling through M, and as a result would simply set some price $p_m > v$, so that it makes no sales on M.[20] This in turn implies M would make zero profits in this case. So we must have $f \leq v - c$. And it must be that $f > 0$, otherwise M makes no profits, which cannot be optimal.

Second, given $f \leq v - c$, we must have $p_m = v$. If $p_m > v$, then S benefits (weakly) from switching to $p_m = v$, so that buying from S through M now gives buyers the same net payoff as the outside option (zero). Indeed, buyers who were buying from S's direct channel will keep doing so. Meanwhile, if the buyers with the highest switching cost $s = \overline{s}$ were not buying in the direct channel, then there is now a positive measure of buyers who were not buying anything from S at $p_m > v$ and who will now buy at $p_m = v$. If instead $p_m < v$, then S's profits are

$$(p_d - c)\, G\left(p_m - p_d\right) + \left(p_m - c - f\right)\left(1 - G\left(p_m - p_d\right)\right),$$

where $G\left(p_m - p_d\right) = 0$ whenever $p_m \leq p_d$. In this case S can increase profits by slightly increasing p_m and p_d by the same amount.

And third, S sets $p_d < v$. Indeed, given $p_m = v$ and $f > 0$, S does better by shifting from $p_d \geq v$ to $p_d = v - \varepsilon$, where $\varepsilon < f$: this shifts buyers with low enough switching cost from M to the direct channel, where S's margin is higher by $f - \varepsilon$ (it no longer pays the fee on those buyers).

[20] Note that if we take the interpretation of our model in which S needs to be present on M in order to be discovered by buyers, setting $p_m > v$ ensures no buyer purchases from S through M, yet S is discovered so buyers can purchase from it in its direct channel.

8.2 Proof of Proposition 1

Given (5) and noting second-order conditions hold for any $f > 0$ and $\mu > 0$, we have that M solves

$$\arg\max_f \left\{ f\left(1 - \frac{f}{2\mu}\right)\right\} = \mu < 2\mu.$$

Taking into account that $f \leq v - c$, this implies the level of f^* and Π^* given in Proposition 1.

First, note that Π^* is always increasing in μ. To determine the direct price, note that if $\mu \leq v - c$, then $f^* = \mu \leq 2\mu$, so in this case using (4) we get $p_d^* = v - \frac{\mu}{2}$. If $\mu \geq v - c$, then $f^* = v - c \leq \mu \leq 2\mu$, so in this case using (4) we get $p_d^* = v - \frac{v-c}{2}$. Combining these two results implies p_d^* in Proposition 1. Clearly, p_d^* is strictly decreasing in μ when $\mu < v - c$, but for $\mu \geq v - c$, p_d^* is constant in μ.

The equilibrium extent of leakage is given by $\frac{f^*}{2\mu}$. Given $f^* = \min\{\mu, v - c\}$, the extent of leakage is $\min\left\{\frac{1}{2}, \frac{v-c}{2\mu}\right\}$. This is initially constant, and then decreasing in μ.

8.3 Proof of Proposition 2

With $G(s) = \frac{s}{\mu}$ and $K(b) = \frac{k}{2}b^2$, we have

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad f \geq b + 2\mu \\ v - \frac{1}{2}(f - b) & \text{if} \quad b \leq f \leq b + 2\mu \end{cases}.$$

This implies that M's profit is

$$\Pi^* = \max_{\substack{b \geq 0 \\ b \leq f \leq v - c + b}} \Pi(b, f),$$

where

$$\Pi(b, f) = \begin{cases} -\frac{kb^2}{2} & \text{if} \quad f \geq b + 2\mu \\ f\left(1 - \frac{1}{2\mu}(f - b)\right) - \frac{kb^2}{2} & \text{if} \quad b \leq f \leq b + 2\mu \\ b - \frac{kb^2}{2} & \text{if} \quad f = b \end{cases}.$$

Recall that we also have the constraint $f \leq v + b - c$, since at any higher fee S will not want to list on M. The FOC for the middle expression implies

$$f^* = \mu + \frac{b}{2} < 2\mu + b.$$

Thus, we have

$$\Pi(b) = \begin{cases} b - \frac{kb^2}{2} & \text{if} \quad b \geq 2\mu \\ \frac{(2\mu+b)^2}{8\mu} - \frac{kb^2}{2} & \text{if} \quad \max\{2\mu - 2(v-c), 0\} \leq b \leq 2\mu \\ (v-c+b)\left(1 - \frac{v-c}{2\mu}\right) - \frac{kb^2}{2} & \text{if} \quad 0 \leq b \leq \max\{2\mu - 2(v-c), 0\} \end{cases},$$

which we can optimize over b.

Suppose first $\mu \leq v - c$. Then

$$\Pi(b) = \begin{cases} b - \frac{kb^2}{2} & \text{if} \quad b \geq 2\mu \\ \frac{(2\mu+b)^2}{8\mu} - \frac{kb^2}{2} & \text{if} \quad 0 \leq b \leq 2\mu \end{cases},$$

so

$$\Pi'(b) = \begin{cases} 1 - kb & \text{if} \quad b \geq 2\mu \\ \frac{1}{2} - \left(k - \frac{1}{4\mu}\right)b & \text{if} \quad 0 \leq b \leq 2\mu \end{cases}.$$

If $k > \frac{1}{4\mu}$, then $\Pi''(b) < 0$ everywhere, so there is a unique maximizer given by

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \frac{1}{4\mu} \leq k \leq \frac{1}{2\mu} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad k \geq \frac{1}{2\mu} \end{cases},$$

which leads to profits

$$\Pi^* = \begin{cases} \frac{1}{2k} & \text{if} \quad \frac{1}{4\mu} \leq k \leq \frac{1}{2\mu} \\ \frac{\mu k}{2k - \frac{1}{2\mu}} & \text{if} \quad k \geq \frac{1}{2\mu} \end{cases}.$$

If $k < \frac{1}{4\mu}$, then $\Pi'(b)$ is increasing in b over $b \in [0, 2\mu]$, and decreasing in b for $b \geq 2\mu$. Since $\Pi'(0) = \frac{1}{2} > 0$, the profit-maximizing choice of b is $b^* = \frac{1}{k}$. The bottomline for the case $\mu \leq v - c$:

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad k \leq \frac{1}{2\mu} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad k \geq \frac{1}{2\mu} \end{cases},$$

Now suppose $\mu > v - c$. We have

$$\Pi'(b) = \begin{cases} 1 - kb & \text{if} \quad b \geq 2\mu \\ \frac{1}{2} - \left(k - \frac{1}{4\mu}\right)b & \text{if} \quad 2\mu - 2(v-c) \leq b \leq 2\mu \\ 1 - \frac{v-c}{2\mu} - kb & \text{if} \quad 0 \leq b \leq 2\mu - 2(v-c) \end{cases}$$

Note that $\Pi'(b)$ is continuous in b.

Here too, there are two cases. If $k \geq \frac{1}{4\mu}$, then $\Pi'(b)$ is decreasing for all b, so $\Pi'(b) = 0$

has a unique solution b^*, which is the maximizer of $\Pi(b)$. The profit-maximizing b is

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \frac{1}{4\mu} \leq k \leq \frac{1}{2\mu} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2\mu} \leq k \leq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \\ \frac{1}{k} - \frac{v-c}{2k\mu} & \text{if} \quad k \geq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \end{cases}$$

and the maximum profit is

$$\Pi^* = \begin{cases} \frac{1}{2k} & \text{if} \quad \frac{1}{4\mu} \leq k \leq \frac{1}{2\mu} \\ \frac{\mu k}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2\mu} \leq k \leq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \\ (v-c)\left(1 - \frac{v-c}{2\mu}\right) + \frac{1}{2k}\left(1 - \frac{v-c}{2\mu}\right)^2 & \text{if} \quad k \geq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \end{cases}$$

If $k < \frac{1}{4\mu}$, then $\Pi'(b)$ is decreasing for $b \in [0, 2\mu - 2(v-c)]$, increasing for $b \in [2\mu - 2(v-c), 2\mu]$, and decreasing for $b \geq 2\mu$. And we have

$$\begin{aligned} \Pi'(2\mu - 2(v-c)) &= 1 - \frac{1}{2\mu}(v-c) - k(2\mu - 2(v-c)) \\ &> 1 - \frac{1}{2\mu}(v-c) - \frac{1}{4\mu}(2\mu - 2(v-c)) \\ &= \frac{1}{2} > 0 \end{aligned}$$

So in this case the profit maximizing solution is $b^* = \frac{1}{k}$.

The bottomline for the case $\mu > v - c$:

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad k \leq \frac{1}{2\mu} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2\mu} \leq k \leq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \\ \frac{1}{k} - \frac{v-c}{2k\mu} & \text{if} \quad k \geq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))} \end{cases}$$

If $\mu > (v-c)$, then it is straightforward to verify that $\frac{1}{2\mu} \leq k \leq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))}$ is equivalent to $\frac{1}{2k} \leq \mu \leq \overline{\mu}$, where

$$\overline{\mu} = \frac{v-c}{2} + \frac{1}{4k} + \sqrt{\left(\frac{v-c}{2}\right)^2 + \frac{1}{16k^2}}.$$

And $k \geq \frac{2\mu-(v-c)}{4\mu(\mu-(v-c))}$ is equivalent to $\mu \geq \overline{\mu}$. Note that

$$\overline{\mu} > \max\left\{v - c, \frac{1}{2k}\right\}.$$

Thus, we can write the optimal solution as a function of μ as follows:

- if $\mu \leq v - c$, then

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \mu \leq \frac{1}{2k} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \mu \geq \frac{1}{2k} \end{cases},$$

- if $\mu > v - c$, then

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \mu \leq \frac{1}{2k} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ \frac{1}{k} - \frac{v-c}{2k\mu} & \text{if} \quad \mu \geq \overline{\mu} \end{cases}$$

Distinguishing the cases $v - c \leq \frac{1}{2k}$ and $v - c \geq \frac{1}{2k}$, it is easily seen that all these cases collapse to

$$b^* = \begin{cases} \frac{1}{k} & \text{if} \quad \mu \leq \frac{1}{2k} \\ \frac{1}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ \frac{1}{k} - \frac{v-c}{2k\mu} & \text{if} \quad \mu \geq \overline{\mu} \end{cases}.$$

This implies

$$f^* = \begin{cases} b^* & \text{if} \quad \mu \leq \frac{1}{2k} \\ \mu + \frac{b^*}{2} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ v + b^* - c & \text{if} \quad \mu \geq \overline{\mu} \end{cases}$$

$$p_d^* = \begin{cases} v & \text{if} \quad \mu \leq \frac{1}{2k} \\ v - \frac{\mu(2k\mu - 1)}{4k\mu - 1} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ \frac{v+c}{2} & \text{if} \quad \mu \geq \overline{\mu} \end{cases}$$

$$\Pi^* = \begin{cases} \frac{1}{2k} & \text{if} \quad \mu \leq \frac{1}{2k} \\ \frac{\mu k}{2k - \frac{1}{2\mu}} & \text{if} \quad \frac{1}{2k} \leq \mu \leq \overline{\mu} \\ (v - c)\left(1 - \frac{v-c}{2\mu}\right) + \frac{1}{2k}\left(1 - \frac{v-c}{2\mu}\right)^2 & \text{if} \quad \mu \geq \overline{\mu} \end{cases}.$$

8.4 Proof of Proposition 3

A standard result for the generalized Pareto distribution is that the expected value is

$$\int_{\underline{q}}^{\overline{q}} q dH(q) = \underline{q} + \frac{\sigma}{1 - \varepsilon}. \tag{12}$$

Taking the first-order condition of $\max_y \{y(1 - H(y))\}$, we have

$$y^* = \max\left\{\frac{\sigma - \varepsilon \underline{q}}{1 - \varepsilon}, \underline{q}\right\},$$

which implies[21]

$$\max_y \{y(1-H(y))\} = \begin{cases} \sigma^{\frac{1}{\varepsilon}} \left(\frac{\sigma-\varepsilon \underline{q}}{1-\varepsilon}\right)^{1-\frac{1}{\varepsilon}} & \text{if} \quad \sigma \geq \underline{q} \\ \underline{q} & \text{if} \quad \sigma \leq \underline{q} \end{cases}. \tag{13}$$

Substituting (12) and (13) into the expression for the tradeoff in (7), we obtain (8)-(9).

From the expression for Π^* in Proposition 8, we know as $\mu \to \infty$, $\Pi^* \to v-c$. Given $\int_{\underline{q}}^{\overline{q}} q dH(q) > \max_y \{y(1-H(y))\}$, M will do strictly better with a transaction fee. Similarly, as $\mu \to 0$, $\Pi^* \to 0$, so M will do strictly better with a referral fee.

Taking $\sigma \to 0$ when $\underline{q} > 0$, (9) becomes

$$\frac{\Pi^*}{v-c} > 1,$$

which can never hold, so in this case M must do better with a referral fee.

Next consider the comparative static results on (8) and (9). Note the case with $\varepsilon = 0$ simply corresponds to the limit as $\varepsilon \to 0$.

The result on μ in Proposition 2 follows directly from Proposition 1. If $\underline{q} \geq \sigma$, then the right-hand side of (9) is clearly increasing in $\underline{q}$, and decreasing in σ and ε. If $\underline{q} \leq \sigma$, then the derivative of the right-hand side of (8) with respect to $\underline{q}$ is equal to

$$\frac{\sigma^{\frac{1}{\varepsilon}}(1-\varepsilon)\underline{q}}{\left(\frac{\sigma-\underline{q}\varepsilon}{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} \left(\underline{q}(1-\varepsilon)+\sigma\right)^2} > 0,$$

and with respect to σ is equal

$$-\frac{(1-\varepsilon)\underline{q}^2}{\sigma^{1-\frac{1}{\varepsilon}} \left(\frac{\sigma-\underline{q}\varepsilon}{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} \left((1-\varepsilon)\underline{q}+\sigma\right)^2} < 0.$$

Finally, consider ε when $0 \leq \underline{q} < \sigma$. We let $\underline{q} = \lambda\sigma$ where $0 \leq \lambda < 1$, and rewrite the right-hand side of (8) as

$$\frac{(1-\lambda\varepsilon)^{1-\frac{1}{\varepsilon}}(1-\varepsilon)^{\frac{1}{\varepsilon}}}{1+\lambda(1-\varepsilon)} = \left(\frac{1-\lambda\varepsilon}{1+\lambda(1-\varepsilon)}\right)\left(\frac{1-\varepsilon}{1-\lambda\varepsilon}\right)^{\frac{1}{\varepsilon}}. \tag{14}$$

[21] In case $\varepsilon = 0$, the limit of this expression as $\varepsilon \to 0$ corresponds to the case with the exponential distribution; i.e., the right-hand side equals $\max\left\{\sigma e^{-\frac{\sigma-\underline{q}}{\sigma}}, \underline{q}\right\}$.

Note the derivative of the term $\frac{1-\lambda\varepsilon}{1+\lambda(1-\varepsilon)}$ is

$$-\frac{\lambda^2}{\left(1+\lambda\left(1-\varepsilon\right)\right)^2} < 0.$$

Taking the log of the second term in ε, we get

$$\frac{1}{\varepsilon}\ln\left(1-\varepsilon\right) - \frac{1}{\varepsilon}\ln\left(1-\lambda\varepsilon\right).$$

The derivative of this with respect to ε is

$$-\frac{1}{\varepsilon^2}\ln\left(1-\varepsilon\right) - \frac{1}{\varepsilon\left(1-\varepsilon\right)} + \frac{1}{\varepsilon^2}\ln\left(1-\lambda\varepsilon\right) + \frac{\lambda}{\varepsilon\left(1-\lambda\varepsilon\right)},$$

which is clearly increasing in λ and equals zero when $\lambda = 1$, meaning it is negative for all $0 \leq \lambda < 1$. Thus, since both terms in (14) are positive but decreasing in ε, the right-hand side of (8) is decreasing in ε for $\varepsilon < 1$ and $\sigma > \underline{q}$.

8.5 Proof of Proposition 4

If $qv < c$, then $\Pi_L(\eta) < 0$ for all $\eta \in [0,1]$, so M chooses option 1 for all $\eta \in [0,1]$.

Suppose now $qv > c$. We have

$$\Pi_H(0) = 0 < \Pi_L(0) = qv - c.$$

and

$$\lim_{\eta\to1}\Pi_H(\eta) > 0 > \lim_{\eta\to1}\Pi_L(\eta) = -\infty.$$

Furthermore, Π_H is obviously increasing in η and Π_L is decreasing in η:

$$\frac{d\Pi_L}{d\eta} = -\frac{\left(1-q\right)qv}{\left(1-\eta\right)^2}\left(\left(2-\eta\right)q\eta + 2\left(1-\eta\right)^2\right) + c\left(1-q\right),$$

which is negative under $qv > c$ since $\left(2-\eta\right)q\eta + 2\left(1-\eta\right)^2 > \left(1-\eta\right)^2$.

Thus, when $qv > c$, there exists a unique $\eta^* \in (0,1)$ such that M prefers option 1 ($\Pi_H(\eta) \geq \Pi_L(\eta)$) when $\eta \geq \eta^*$ and option 2 ($\Pi_H(\eta) \leq \Pi_L(\eta)$) when $\eta \leq \eta^*$, where

$$\eta^* = \frac{qv\left(2-q\right) - c - q\sqrt{\left(1-q\right)v\left(v-c\right)}}{qv\left(3\left(1-q\right)+q^2\right) - c}.$$

This means M's profits with communication banned are decreasing and then increasing in

η, with the minimum occurring at $\eta = \eta^*$. When $\eta = 0$, profits are $qv - c$, and when $\eta = 1$, profits are $q(v-c)$.

We now need to compare these profits to M's profit when it allows communication, which is $q\Pi^*$. First note that M prefers to ban communications when $\eta > \frac{\Pi^*}{v-c}$, regardless of q. Indeed, $\eta > \frac{\Pi^*}{v-c}$ implies $\Pi_H(\eta) > q\Pi^*$ for all $q > 0$. Second, if $v - \frac{c}{q} \leq \Pi^*$, then $\Pi_L(\eta) \leq q\Pi^*$ for all η, so M prefers to ban communications if and only if $\eta > \frac{\Pi^*}{v-c}$. (Note this includes the case in which $qv - c < 0$ noted at the start.) And third, if $v - \frac{c}{q} > \Pi^*$, then M also prefers to ban communications when $\eta < \eta_L$, where $\eta_L > 0$ is uniquely defined by $\Pi_L(\eta_L) = q\Pi^*$. In this case, if $\eta^* > \frac{\Pi^*}{v-c}$, then M prefers to ban communications for all η and q. Since $\lim_{q\to 1}\eta^* = 1$ and $qv > c$ holds if q is close enough to one, this implies M always prefers to ban communications if q is close enough to one.

8.6 Proof of Proposition 5

Denote M's optimal fee without price parity by f^*. We have

$$\begin{aligned}\max_{c\leq p_d\leq v}\{(p_d-c)\,G(v-p_d)\} &\leq \max_{p_d\leq v}\{(p_d-c)\,G(v-p_d)+(v-c-f^*)(1-G(v-p_d))\}\\ &= (p_d^*(f^*)-c)\,G(v-p_d^*(f^*))+(v-c-f^*)(1-G(v-p_d^*(f^*)))\end{aligned}$$

and

$$v-c \geq (p_d^*(f^*)-c)\,G(v-p_d^*(f^*))+(v-c)(1-G(v-p_d^*(f^*)))\,.$$

Taking the difference between these last two inequalities, we obtain (11).

8.7 Proof of Proposition 6

We begin by proving the following lemma.

Lemma 2. *If $u - c \leq \frac{v-c}{2}$, then M's optimal fee and resulting profits are:*

$$f^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq \left(4-2\sqrt{2}\right)(u-c) \\ 2(u-c) & \text{if} \quad \left(4-2\sqrt{2}\right)(u-c) < \mu \leq 2(u-c) \\ \mu & \text{if} \quad 2(u-c) < \mu \leq v-c \\ v-c & \text{if} \quad \mu > v-c \end{cases}$$

$$\Pi^* = \begin{cases} \frac{\mu}{4} & \text{if} \quad \mu \leq \left(4 - 2\sqrt{2}\right)(u-c) \\ 2(u-c)\left(1 - \frac{u-c}{\mu}\right) & \text{if} \quad \left(4 - 2\sqrt{2}\right)(u-c) \leq \mu \leq 2(u-c) \\ \frac{\mu}{2} & \text{if} \quad 2(u-c) \leq \mu \leq v-c \\ (v-c)\left(1 - \frac{v-c}{2\mu}\right) & \text{if} \quad \mu \geq v-c \end{cases}.$$

If $\frac{v-c}{2} \leq u-c \leq v-c$, *then M's optimal fee and resulting profits are:*

$$f^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right) \\ v-c & \text{if} \quad \mu > 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right) \end{cases}$$

$$\Pi^* = \begin{cases} \frac{\mu}{4} & \text{if} \quad \mu \leq 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right) \\ (v-c)\left(1 - \frac{u-c}{\mu}\right) & \text{if} \quad \mu \geq 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right) \end{cases}$$

Proof of Lemma 2. In equilibrium, it must be that the high-quality seller (S_h) makes all sales on both channels. Indeed, suppose the low-quality seller (S_l) makes sales at a price $p_d^l \in [c, u]$ in the direct channel. Then S_h can profitably deviate by setting $p_d^h = p_d^l + v - u \in [c + v - u, v]$ and keeping p_m^h unchanged. This will not affect its profit on M since it doesn't induce any more buyers to switch to buy directly, and allows it to make additional profit from the direct channel. And similarly if S_l makes sales on M at a price $p_m^l \in [c + f, u]$. This means there is no way for S_l to make profitable sales anywhere in equilibrium. Specifically, it can't be that it makes a loss in one channel since it would need to make profits in the other channel, which is not possible. Thus, S_h makes all sales.

Given f, S_l's prices are $p_d^l = c$ and $p_m^l = c + f$. Note that when $f > u - c$, we have $p_m^l > u$, so S_l becomes irrelevant on M in that case.

In equilibrium, we must have $f \leq v - c$. Otherwise, if $f > v - c$, then S_h would set $p_m^h > v$ to avoid selling at a loss on M (and S_l would set $p_m^l > u$), resulting in zero profits for M.

Next, for S_h, we can restrict attention to $p_d^h \leq p_m^h$ without loss of generality. Indeed, if $p_d^h > p_m^h$, then no buyer would buy from S_h in the direct channel, so S_h can do at least as well with $p_d^h \leq p_m^h$. And we must have $p_m^h \leq \min\{c + f + v - u, v\}$ and $p_d^h \leq \min\{c + v - u, v\} = c + v - u$, otherwise S_l could deviate and make some positive profits. S_h's profits are

$$\left(p_d^h - c\right) G\left(p_m^h - p_d^h\right) + \left(p_m^h - c - f\right)\left(1 - G\left(p_m^h - p_d^h\right)\right).$$

For this to be an equilibrium, we must have $p_m^h = \min\{c + f + v - u, v\}$ or $p_d^h = c + v - u$, otherwise S_h could increase profits by raising both prices by the same very small amount.

Suppose the binding constraint is $p_d^h = c + v - u$ and we have $p_m^h < \min\{c + f + v - u, v\}$.

Then the margin that S_h makes on direct channel sales is $v-u$, while the margin on sales through M is $p_m^h - c - f$, which is strictly less than $v-u$, because $p_m^h - c - f < \min\{v-u, v-c-f\}$. This means that S_h can strictly increase profits by slightly increasing p_m^h, which shifts sales to the higher margin channel. Thus, in equilibrium, we must have $p_m^h = \min\{c+f+v-u, v\}$.

There are then two main cases.

1. If $f \leq u-c$, then $p_m^h = c+f+v-u$ and S_h solves

$$\max_{p_d^h \leq c+v-u} \left\{ \left(p_d^h - c\right) \frac{\min\left\{c+f+v-u-p_d^h, \mu\right\}}{\mu} + (v-u)\left(\frac{\mu - \min\left\{c+f+v-u-p_d^h, \mu\right\}}{\mu}\right)\right\}.$$

   It is easily verified that the solution is always $p_d^h = c+v-u$, which implies that in this case M's profits are

$$\Pi(f) = f\left(1 - G\left(p_m^h - p_d^h\right)\right) = f\left(1 - \frac{\min\{f, \mu\}}{\mu}\right).$$

2. If $f > u-c$, then $p_m^h = v$ and S_h solves

$$\max_{p_d^h \leq c+v-u} \left\{ \left(p_d^h - c\right) \frac{\min\left\{v-p_d^h, \mu\right\}}{\mu} + (v-c-f)\left(\frac{\mu - \min\left\{v-p_d^h, \mu\right\}}{\mu}\right)\right\}.$$

   There are two subcases:

   - If $\mu \leq u-c$, then S_h optimally sets $p_d^h = c+v-u$, resulting in profits $v-u$ for S_h and zero for M.
   - If $\mu \geq u-c$, then S_h optimally sets

$$p_d^h(f) = \begin{cases} c+v-u & \text{if} \quad u-c < f \leq 2(u-c) \\ v - \frac{f}{2} & \text{if} \quad 2(u-c) \leq f \leq 2\mu \\ v-\mu & \text{if} \quad f \geq 2\mu \end{cases},$$

     resulting in profits for M of

$$\Pi(f) = \begin{cases} f\left(1 - \frac{u-c}{\mu}\right) & \text{if} \quad u-c < f \leq 2(u-c) \\ f\left(1 - \frac{f}{2\mu}\right) & \text{if} \quad 2(u-c) \leq f \leq 2\mu \\ 0 & \text{if} \quad f \geq 2\mu \end{cases}.$$

We now proceed to solve for M's optimal choice of f.

Suppose $\mu \leq u - c$. Then M must set $f \leq u - c$ to obtain positive profits, so it solves $\max_{f \leq u-c} \left\{ f \left(1 - \frac{f}{\mu}\right) \right\}$. This implies $f^* = \frac{\mu}{2} < u - c$ and $\Pi^* = \frac{\mu}{4}$.

Now consider $\mu > u - c$. There are three relevant choices for M to obtain positive profits when $\mu > u - c$: (i) M can set $f \leq u - c$ and obtain $f\left(1 - \frac{f}{\mu}\right)$, so the optimal fee on this interval is $f = \min\left\{\frac{\mu}{2}, u - c\right\}$; (ii) M can set $u - c \leq f \leq \min\{2(u-c), v - c\}$ and obtain $f\left(1 - \frac{u-c}{\mu}\right)$, so the optimal fee on this interval is $f = \min\{2(u-c), v-c\}$; (iii) M can set $2(u-c) \leq f \leq \min\{2\mu, v-c\}$ and obtain $f\left(1 - \frac{f}{2\mu}\right)$, which is only possible when $2(u-c) \leq \min\{v - c, 2\mu\}$.

We need to distinguish two cases for $\mu > u - c$.

1. First suppose $v - c \leq 2(u-c)$. And suppose $\mu \leq 2(u-c)$. Then M chooses between (i) $\frac{\mu}{4}$ (obtained by setting $f = \frac{\mu}{2}$) and (ii) $(v-c)\left(1 - \frac{u-c}{\mu}\right)$ (obtained by setting $f = v - c$). Comparing (i) and (ii), M will set $f^* = \frac{\mu}{2}$ if $\mu < 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right)$ and otherwise will set $f^* = v - c$. Note given u lies between c and v, it can be verified that
$$u - c \leq 2(v-c)\left(1 - \sqrt{\frac{v-u}{v-c}}\right) \leq 2(u-c).$$
Next suppose $\mu > 2(u-c)$. Then M chooses between (i) $(u-c)\left(1 - \frac{u-c}{\mu}\right)$ and (ii) $(v-c)\left(1 - \frac{u-c}{\mu}\right)$, and clearly will choose the latter so $f^* = v - c$ in this case too. Thus, we have confirmed the results in the Proposition when $\mu > u - c$ and $v - c \leq 2(u-c)$.

2. Second, suppose $v - c > 2(u-c)$. We can first consider the case $\mu \leq 2(u-c)$. So M can choose between (i) $\frac{\mu}{4}$ and (ii) $2(u-c)\left(1 - \frac{u-c}{\mu}\right)$. (Note M's profit $f\left(1 - \frac{f}{2\mu}\right)$ under (iii) is decreasing in f for $f \geq 2(u-c)$ under these assumptions, which is why M can ignore option (iii)). Comparing (i) and (ii), M will set $f^* = \frac{\mu}{2}$ if $\mu < 2\left(2 - \sqrt{2}\right)(u-c)$ and otherwise will set $f^* = 2(u-c)$. Next suppose $\mu > 2(u-c)$. Then M chooses between (i) $(u-c)\left(1 - \frac{u-c}{\mu}\right)$, (ii) $2(u-c)\left(1 - \frac{u-c}{\mu}\right)$ and (iii) $f^* = \min\{\mu, v-c\}$ with M obtaining either $\frac{\mu}{2}$ and $(v-c)\left(1 - \frac{v-c}{2\mu}\right)$ respectively. Clearly (ii) dominates (i). If $2(u-c) < \mu < v - c$, then $f^* = \mu$ under option (iii) so M obtains $\frac{\mu}{2}$ which is better than option (ii) given $\mu > 2(u-c)$. Finally, if $\mu > v - c$, then $f^* = v - c$ under option (iii) so M obtains $(v-c)\left(1 - \frac{v-c}{2\mu}\right)$ which is better than option (ii) given that $f\left(1 - \frac{f}{2\mu}\right)$ is increasing in f at $f = v - c$ when $\mu > v - c$.

Thus, we have proven each of the different cases for f^* and Π^* given in the text of the lemma.

■

We can now use the expressions of f^* and Π^* provided in the Lemma 2 to prove Proposition 6. To determine the effect of u on M's profits, suppose first $u-c \geq \frac{v-c}{2}$. Note that $2(v-c)\left(1-\sqrt{\frac{v-u}{v-c}}\right)$ is decreasing in u. If $\mu < 2(v-c)\left(1-\sqrt{\frac{v-u}{v-c}}\right)$, then a slight increase in u leaves Π^* unchanged. If $\mu \geq 2(v-c)\left(1-\sqrt{\frac{v-u}{v-c}}\right)$, then a slight increase in u decreases Π^*. Now suppose $u-c \leq \frac{v-c}{2}$. If $\mu \leq \left(4-2\sqrt{2}\right)(u-c)$ or $\mu > 2(u-c)$, then a slight increase in u leaves Π^* unchanged. If $\left(4-2\sqrt{2}\right)(u-c) < \mu \leq 2(u-c)$, then a slight increase in u decreases Π^* because $2(u-c)\left(1-\frac{u-c}{\mu}\right)$ is decreasing in $(u-c)$ when $\mu \leq 2(u-c)$. Thus, in all cases Π^* is weakly decreasing in u. And it is easily seen that Π^* converges to the expression of M's profit with a monopoly high-quality seller given by (5) when $u \to c$.

8.8 Proof of Proposition 7

We start by proving the following lemma.

Lemma 3. *If $u-c \leq \frac{v-c}{2}$, then M's optimal fee and resulting profits are*

$$f^* = \begin{cases} u-c & \text{if} \quad \mu \leq 2(u-c) \\ \mu & \text{if} \quad 2(u-c) < \mu \leq v-c \\ v-c & \text{if} \quad \mu > v-c \end{cases}$$

$$\Pi^* = \begin{cases} u-c & \text{if} \quad \mu \leq 2(u-c) \\ \frac{\mu}{2} & \text{if} \quad 2(u-c) \leq \mu \leq v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu \geq v-c \end{cases} . \tag{15}$$

If $\frac{v-c}{2} \leq u-c \leq v-c$, then M's optimal fee and resulting profits are

$$f^* = \begin{cases} u-c & \text{if} \quad \mu \leq \frac{(v-c)^2}{2(v-u)} \\ v-c & \text{if} \quad \mu > \frac{(v-c)^2}{2(v-u)} \end{cases}$$

$$\Pi^* = \begin{cases} u-c & \text{if} \quad \mu \leq \frac{(v-c)^2}{2(v-u)} \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu \geq \frac{(v-c)^2}{2(v-u)} \end{cases} \tag{16}$$

Proof of Lemma 3. As explained in the main text, given f and seller prices, M recommends the seller that induces the least amount of leakage (i.e. with the lowest non-negative difference between price on M and direct price), subject to offering non-negative utility to buyers that buy via M. We define $\overline{p}_d^l(f)$ as the direct price that maximizes the low-quality seller's chance to get recommended while allowing it to make non-negative total profits:

$$\overline{p}_d^l(f) = \max_{\substack{p_d \le u \\ (p_d - c)\frac{\min\{u-p_d,\mu\}}{\mu} + (u-c-f)\left(1-\frac{\min\{u-p_d,\mu\}}{\mu}\right)\ge 0}} \{p_d\}$$

Indeed, the low-quality seller's (i.e. S_l's) best chance to get recommended is when it sets $p_m^l = u$ and p_d^l as close as possible to u, while still making non-negative profits. We define $\overline{p}_d^h(f)$ in the same way for the high-quality seller (S_h):

$$\overline{p}_d^h(f) = \max_{\substack{p_d \le v \\ (p_d - c)\frac{\min\{v-p_d,\mu\}}{\mu} + (v-c-f)\left(1-\frac{\min\{v-p_d,\mu\}}{\mu}\right)\ge 0}} \{p_d\}$$

First, we prove that[22]

$$\overline{p}_d^l(f) = \begin{cases} u & \text{if} \quad f \le u-c \\ u - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2} & \text{if} \quad \begin{array}{c} \mu \le u-c \le f \text{ or} \\ \mu > u-c \text{ and } u-c \le f \le 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \end{array} \\ -\infty & \text{if} \quad \mu > u-c \text{ and } f > 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \end{cases}$$

and

$$\overline{p}_d^h(f) = \begin{cases} v & \text{if} \quad f \le v-c \\ v - \frac{f-\sqrt{f^2-4\mu(f-(v-c))}}{2} & \text{if} \quad \begin{array}{c} \mu \le v-c \le f \text{ or} \\ \mu > v-c \text{ and } v-c \le f \le 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) \end{array} \\ -\infty & \text{if} \quad \mu > v-c \text{ and } f > 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right). \end{cases}$$

Focus on S_l first. If $f \le u-c$, then it is easily seen that $\overline{p}_d^l(f) = u$. If $\mu \le u-c$, then $\overline{p}_d^l(f)$ is well-defined for all $f \ge u-c$, because for $p_d = u-\mu$ we have

$$(p_d - c)\frac{\min\{u-p_d,\mu\}}{\mu} + (u-c-f)\left(1-\frac{\min\{u-p_d,\mu\}}{\mu}\right) = u-c-\mu \ge 0.$$

In this case

$$\overline{p}_d^l(f) = u - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2}. \tag{17}$$

Indeed, $\mu \le u-c$ implies $f^2 \ge 4\mu(f-(u-c))$ for all f and $\overline{p}_d^l(f) \ge u-\mu$.

If $\mu > u-c$, then $\overline{p}_d^l(f)$ is well-defined (i.e. has a finite value) iff $f^2 \ge 4\mu(f-(u-c))$

[22] It is easily verified that $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) > u-c$ whenever $\mu > u-c$ and $2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) > v-c$ whenever $\mu > v-c$.

and $f \leq 2\mu$, which is equivalent to

$$0 \leq f \leq 2\mu \left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right).$$

Thus, $\overline{p}_d^l(f)$ is well-defined iff $f \leq u - c$ or $\mu \leq u - c \leq f$ or $\mu > u - c$ and $u - c \leq f \leq 2\mu \left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$. Indeed, $u - c < 2\mu \left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$ when $\mu > u - c$. Otherwise, S_l cannot make non-negative profits, so $\overline{p}_d^l(f) = -\infty$. When it is well-defined, $\overline{p}_d^l(f)$ is given by (17) above.

The proof for the expression of $\overline{p}_d^h(f)$ is the same, replacing u by v.

Next, we show that in equilibrium, S_h makes all sales. Indeed, from the expressions of $\overline{p}_d^l(f)$ and $\overline{p}_d^h(f)$ above, if $f \leq u-c$, then both are willing to make all sales via M, so each of them sets the same price on and off M, meaning S_h makes all sales (M breaks ties in favor of S_h).

Suppose $\mu \leq u - c \leq f$ or $\mu > u - c$ and $u - c \leq f \leq 2\mu \left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$. Then

$$u - \overline{p}_d^l(f) = \frac{f - \sqrt{f^2 - 4\mu(f - (u - c))}}{2}.$$

Note that in this case we cannot have $\mu > v - c$ and $f > 2\mu \left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right)$, because $v - c \geq u - c$. Thus $v - \overline{p}_d^h(f)$ is either equal to zero or to $\frac{f - \sqrt{f^2 - 4\mu(f-(v-c))}}{2}$. And in both cases we have $v - \overline{p}_d^h(f) \leq u - \overline{p}_d^l(f)$, so S_h can offer a lower difference in prices and will make all sales.

Finally, if $\mu > u - c$ and $f > 2\mu \left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then $u - \overline{p}_d^l(f) = +\infty$, so S_l cannot make non-negative profits by selling via M, and will set $p_m^l > u$. If $v - \overline{p}_d^h(f)$ is finite, then S_h will choose p_d^h to maximize profits ignoring the presence of S_l, which is irrelevant. If in addition $f > v - c$, then S_h will set $p_m^h > v$ and

$$p_d^h = \arg\max_{p_d \leq v} \left\{ (p_d - c) \min \left\{ \frac{v - p_d}{\mu}, 1 \right\} \right\}.$$

Indeed, this is profit-maximizing for S_h, given that we have assumed M recommends S_h when $p_m^l > u$ and $p_m^h > v$. So M makes zero profits in this case. Finally, if $v - \overline{p}_d^h(f) = u - \overline{p}_d^l(f) = +\infty$, then neither seller can make non-negative profits with positive sales via M, so once again $p_m^l > u$ and $p_m^h > v$, so M makes zero profits. Thus, if $u - \overline{p}_d^l(f) = +\infty$, then we must have $f \leq v - c$ for M to make positive profits. And in this case S_h will make all sales once again.

Given f, S_l's prices are

$$\left(p_m^l, p_d^l\right) = \begin{cases} \left(u, \overline{p}_d^l(f)\right) & \text{if } \overline{p}_d^l(f) \text{ is well-defined} \\ (u+\varepsilon, -\infty) & \text{if } \overline{p}_d^l(f) = -\infty \end{cases},$$

where ε is an arbitrary positive number. In other words, S_l becomes irrelevant when $\overline{p}_d^l(f) = -\infty$.

There are therefore two cases:

1. If $\mu > u - c$ and $f > 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f) = -\infty$, which means S_l has no chance of making non-negative total profits if it sells anything via M. This implies it might as well price at $p_d^l = p_m^l > u$, which makes it irrelevant.[23] In this case, if $f \leq v - c$, then S_h does best by setting $p_m^h = v$ and $p_d^h = p_d^*(f) = v - \min\left\{\frac{f}{2}, \mu\right\}$, so it gets recommended by M, makes positive profits, and M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$ as in the baseline model. If $f > v - c$, then S_h sets $p_m^h > v$ to avoid making sales at a loss on M (as in the baseline model), so M makes zero profits.

2. If $\mu \leq u - c$ or $\mu > u - c$ and $0 \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f)$ exists. In this case, S_l sets $p_m^l = u$ and $p_d^l = \overline{p}_d^l(f)$, while S_h sets $p_m^h = v$ and p_d^h to maximize profits subject to $v - p_d^h \leq u - \overline{p}_d^l(f)$ (so that it is recommended by M), i.e.

$$\begin{aligned} p_d^h &= \arg \max_{p_d \geq v - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2}} \left\{ (p_d - c)\frac{\min\{v - p_d, \mu\}}{\mu} + (v - c - f)\left(1 - \frac{\min\{v - p_d, \mu\}}{\mu}\right)\right\} \\ &= v - \frac{f - \sqrt{f^2 - 4\mu(f - (u-c))}}{2}, \end{aligned}$$

where the last equality follows because $v - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2} \geq \max\left\{v - \frac{f}{2}, v - \mu\right\}$ under the conditions that define this case. Also, we know that at these prices, S_h must make non-negative profits because if $\overline{p}_d^l(f)$ is well-defined, then so is $\overline{p}_d^h(f)$. This implies M's profit in this case is

$$f\left(1 - \frac{f - \sqrt{f^2 - 4\mu(f - (u-c))}}{2\mu}\right).$$

If M sets $f \leq u - c$, then $\overline{p}_d^l(f) = u$ and $\overline{p}_d^h(f) = v$. In this case, S_l's best chance to be recommended and make non-negative profits is to set $p_d^l = p_m^l = u$. The best response of S_h

[23] Thus, we are implicitly making the assumption (standard in Bertrand competition games) that S_l does not play a dominated strategy off the equilibrium path (e.g. $p_d^l = p_m^l = u$, which would yield negative profits if M were to recommend it).

is then to set $p_d^h = p_m^h = v$, which ensures that it is recommended by M (given M breaks ties in favor of S_h). This leads all buyers to purchase from S_h on M, so M's profits are equal to f. As a result, M does best in this range to set $f = u - c$, yielding a profit equal to $u - c$.

We can therefore restrict attention to $f \geq u - c$.

Suppose $\mu \leq u - c$, so we are in the second case above. The derivative of M's profit with respect to f is

$$\frac{-\left(2\mu - f + \sqrt{f^2 - 4\mu\left(f - (u-c)\right)}\right)\left(f - \sqrt{f^2 - 4\mu\left(f - (u-c)\right)}\right)}{2\mu\sqrt{f^2 - 4\mu\left(f - (u-c)\right)}},$$

which is non-positive because $f > \sqrt{f^2 - 4\mu\left(f - (u-c)\right)}$ and $u - c \geq \mu$ imply $2\mu - f + \sqrt{f^2 - 4\mu\left(f - (u-c)\right)} \geq 0$. This means M wants to set f as low as possible subject to $f \geq u - c$. Thus, we have proven that when $\mu \leq u - c$, the optimal solution for M is to set $f^* = u - c$, resulting in $p_d^h = p_m^h = v$, no leakage and $\Pi^* = u - c$.

Now suppose $\mu > u - c$.

- If $u - c \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then we are once again in the second case above. And once again $2\mu - f + \sqrt{f^2 - 4\mu\left(f - (u-c)\right)} \geq 0$ because $2\mu - f \geq 0$, so M's best option on this range is to set $f = u - c$, resulting in profit $u - c$.

- If $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \geq v - c$, then M makes zero profits for all $f > 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$ (see the first case above).

- If $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \leq v - c$, then M can set f such that $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) < f \leq v - c$, so that we are in the first case above. S_l is irrelevant, and M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$. Setting $f > v - c$ in this case results in zero profits for M.

Thus, if $\mu > u - c$ and $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \geq v - c$, then M's optimal solution is $f^* = u - c$, resulting in profit $u - c$.

Note that $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \geq v - c$ is equivalent to $\mu \geq \frac{v-c}{2}$ and $\mu \leq \frac{(v-c)^2}{4(v-u)}$. This is possible iff $\frac{v-c}{2} \leq \frac{(v-c)^2}{4(v-u)}$, i.e. iff $u - c \geq \frac{v-c}{2}$. So, assuming $u - c \geq \frac{v-c}{2}$, we have that $\mu > u - c$ and $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \geq v - c$ is equivalent to $u - c < \mu \leq \frac{(v-c)^2}{4(v-u)}$. Indeed, $u - c < \frac{(v-c)^2}{4(v-u)}$ whenever $u < v$.

Finally, if $\mu > u - c$ and $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \leq v - c$, then M chooses between $u - c$ and

$$\max_{2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) < f \leq v - c} \left\{f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)\right\} = \max_{2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) < f \leq v - c} \left\{f\left(1 - \frac{f}{2\mu}\right)\right\}.$$

Note that $\mu > u-c$ and $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \leq v-c$ is equivalent to $\mu > u-c$ and ($\mu \leq \frac{v-c}{2}$ or $\mu \geq \frac{(v-c)^2}{4(v-u)}$). Furthermore,

$$2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)\sqrt{1-\frac{u-c}{\mu}} < u-c$$

for all $\mu > u-c$ and

$$(v-c)\left(1-\frac{v-c}{2\mu}\right) \leq u-c \Longleftrightarrow \mu \leq \frac{(v-c)^2}{2(v-u)}.$$

We can therefore distinguish the following possibilities when $\mu > u-c$ and $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \leq v-c$:

- If $u-c < \frac{v-c}{2}$, then $\frac{(v-c)^2}{4(v-u)} < \frac{v-c}{2}$ so $\mu > u-c$ and $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \leq v-c$ is equivalent to $\mu > u-c$. In this case, it is easily verified that

$$\max_{\substack{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f \\ f\leq v-c}} \left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)\sqrt{1-\frac{u-c}{\mu}} & \text{if} \quad u-c<\mu\leq\frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if} \quad \frac{4(u-c)}{3}\leq\mu\leq v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu\geq v-c \end{cases},$$

  so:

  - if $u-c < \mu \leq 2(u-c)$, then M's optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$
  - if $2(u-c) \leq \mu \leq v-c$, then M's optimal solution is $f^* = \mu$, yielding $\Pi^* = \frac{\mu}{2}$
  - if $\mu \geq v-c$, then M's optimal solution is $f^* = v-c$, yielding $\Pi^* = (v-c)\left(1-\frac{v-c}{2\mu}\right)$

- If $u-c \geq \frac{v-c}{2}$, then $\mu > u-c$ and $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) \leq v-c$ is equivalent to $\mu \geq \frac{(v-c)^2}{4(v-u)}$. And $u-c \geq \frac{v-c}{2}$ implies

$$\frac{(v-c)^2}{4(v-u)} > \frac{4(u-c)}{3} \Leftrightarrow \frac{4(u-c)}{3} > v-c.$$

  In this case:

– if $\frac{v-c}{2} \leq u-c \leq \frac{3(v-c)}{4}$, then

$$\max_{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f\leq v-c}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)\sqrt{1-\frac{u-c}{\mu}} & \text{if} \quad u-c<\mu\leq\frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if} \quad \frac{4(u-c)}{3}\leq\mu\leq v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu\geq v-c \end{cases},$$

so:

* if $u-c \leq \mu \leq \frac{(v-c)^2}{2(v-u)}$, then M's optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$
* if $\mu \geq \frac{(v-c)^2}{2(v-u)}$, then M's optimal solution is $f^* = v-c$, yielding $\Pi^* = (v-c)\left(1-\frac{v-c}{2\mu}\right)$

– if $\frac{3(v-c)}{4} < u-c \leq v-c$, then $\frac{(v-c)^2}{4(v-u)} > v-c$ and therefore

$$\max_{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f\leq v-c}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = (v-c)\left(1-\frac{v-c}{2\mu}\right),$$

so once again:

* if $u-c \leq \mu \leq \frac{(v-c)^2}{2(v-u)}$, then M's optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$
* if $\mu \geq \frac{(v-c)^2}{2(v-u)}$, then M's optimal solution is $f^* = v-c$, yielding $\Pi^* = (v-c)\left(1-\frac{v-c}{2\mu}\right)$

We have thus proven the expressions of f^* and Π^* given in the text of lemma 3. ■

We can now use these expressions to prove Proposition 7.

To determine the effect of u on M's profits, suppose first $u-c > \frac{v-c}{2}$. Note that $\frac{(v-c)^2}{2(v-u)}$ is increasing in u. If $\mu \leq \frac{(v-c)^2}{2(v-u)}$, then a slight increase in u increases Π^*. If $\mu > \frac{(v-c)^2}{2(v-u)}$, then a slight increase in u leaves Π^* unchanged. Now suppose $u-c \leq \frac{v-c}{2}$. If $\mu \leq 2(u-c)$, then a slight increase in u increases Π^*. If $\mu > 2(u-c)$, then a slight increase in u leaves Π^* unchanged. Thus, in all cases Π^* is weakly increasing in u. And it is easily seen that Π^* converges to the expression of M's profit with a monopoly high-quality seller given by (5) when $u \to c$. Since M's profit with steering is weakly increasing in u, while M's profit without steering is weakly decreasing in u, and both profits converge to M's profit with a monopoly high-quality seller when $u \to c$, we can conclude that M's profit with steering is everywhere weakly higher than the profit without steering.

Finally, it is easily seen from the expressions of M's optimal fee and profits (15) and (16) that in both cases there is no leakage when μ is small and positive leakage when μ is large.

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Online Appendix: Marketplace leakage

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A Two-part tariff

Suppose M can also extract a share $\beta \in [0,1]$ of S's net profits through a fixed (upfront) fee (β can be thought of as M's bargaining power) that is set at the same time as f. We find that provided $\beta < 1$, our insights go through: there is equilibrium leakage and M's profits are increasing in μ.

Since the fixed fee is paid upfront, it doesn't change S's pricing, which is given by

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad f \geq 2\mu \\ v - \frac{f}{2} & \text{if} \quad f \leq 2\mu \end{cases}$$

as before. S's corresponding profit from participating (before fixed fee) is

$$\pi(f) = (p_d(f) - c)\left(\frac{v - p_d(f)}{\mu}\right) + (v - c - f)\left(1 - \frac{v - p_d(f)}{\mu}\right)$$

The corresponding profit for M is

$$\Pi(f) = \begin{cases} \beta\pi(f) & \text{if} \quad f \geq 2\mu \\ f\left(1 - \frac{f}{2\mu}\right) + \beta\pi(f) & \text{if} \quad f \leq 2\mu \end{cases}.$$

Since $f \leq v - c$, we have $\pi(f) > 0$, so S always participates, since its net profit from an ex-ante perspective is $(1-\beta)\pi(f)$.

Thus,

$$\Pi(f) = \begin{cases} \beta(v - \mu - c) & \text{if} \quad f \geq 2\mu \\ f\left(1 - \frac{f}{2\mu}\right) + \beta\left(\frac{f^2}{4\mu} + (v - c - f)\right) & \text{if} \quad f \leq 2\mu \end{cases}.$$

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And we therefore have

$$\begin{aligned} f^* &= \min\left\{\frac{2\mu(1-\beta)}{2-\beta}, v-c\right\} \\ \Pi^* &= f^*\left(1-\frac{f^*}{2\mu}\right)+\beta\left(\frac{f^{*2}}{4\mu}+(v-c-f^*)\right) \\ &= \begin{cases} \frac{\mu(1-\beta)^2}{(2-\beta)}+\beta(v-c) & \text{if } \frac{2\mu(1-\beta)}{2-\beta}\leq v-c \\ (v-c)-\frac{(v-c)^2}{2\mu}\left(1-\frac{\beta}{2}\right) & \text{if } \frac{2\mu(1-\beta)}{2-\beta}\geq v-c \end{cases}. \end{aligned}$$

Of course, when $\beta = 0$, we obtain the f^* and Π^* from our baseline case in the main text. Meanwhile, when $\beta \to 1$, we obtain

$$\begin{aligned} f^* &= 0 \\ \Pi^* &= v-c. \end{aligned}$$

This means that if M can extract S's entire profit via a fixed fee, then there is no reason to charge a transaction fee (since it induces leakage), and M obtains the maximum profit $v-c$ (all transactions are conducted on M). However, provided $\beta < 1$, there will be positive leakage in equilibrium. Furthermore, it is easily seen that Π^* is increasing in μ for all $\beta < 1$.

B Ad-valorem fee

Let $0 < \rho < 1$ be the ad-valorem (proportional) fee charged by M, so that S retains $(1-\rho)p_m$ and pays ρp_m to M. Given prices, the buyers' choices are the same. First, note that M will never set ρ so that $(1-\rho)v < c$, i.e. $\rho > 1-\frac{c}{v}$. If it did, S would make a loss when selling through M at the highest possible price of v, and as a result would simply set some price $p_m > v$, so that it makes no sales on M. This in turn implies M would make zero profits in this case. Second, given that $\rho \leq 1-\frac{c}{v}$, S will set $p_m = v$. The logic is the same as before. And third, S sets $p_d \leq v$, again for the same reason as before.

Given these observations, S's pricing problem reduces to setting $p_d \leq v$ to maximize its profit

$$\pi = (p_d - c)\,G(v-p_d) + ((1-\rho)v-c)(1-G(v-p_d)).$$

S's optimal p_d is such that $v - \overline{s} \leq p_d \leq v$.

Denote by $p_d(f)$ the unique solution in p_d to the first-order condition (FOC)

$$G(v-p_d) - g(v-p_d)(p_d-(1-\rho)v) = 0,$$

so that

$$p_d(\rho) = (1-\rho)v + \frac{G(v - p_d(\rho))}{g(v - p_d(\rho))}.$$

S's profit maximizing price $p_d^*(\rho)$ is then given by

$$p_d^*(\rho) = \begin{cases} v - \overline{s} & \text{if} \quad p_d(\rho) \le v - \overline{s} \\ p_d(\rho) & \text{if} \quad v - \overline{s} \le p_d(\rho) \le v \\ v & \text{if} \quad p_d(\rho) \ge v \end{cases}.$$

The corresponding profit for M is

$$\Pi^* = \max_{\rho \le 1 - \frac{c}{v}} \left\{ \rho v \left(1 - G\left(v - p_d^*(\rho)\right)\right) \right\}.$$

Assuming $G(s) = \frac{s}{\mu}$ on $s \in [0, \mu]$, and assuming $\rho \le 1 - \frac{c}{v}$, we have

$$p_d(\rho) = v\left(1 - \frac{\rho}{2}\right)$$

and therefore S's profit maximizing price $p_d^*(f)$ is given by

$$v\rho = 2\mu$$

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad \rho \ge \frac{2\mu}{v} \\ v\left(1 - \frac{\rho}{2}\right) & \text{if} \quad \rho \le \frac{2\mu}{v} \end{cases}.$$

The corresponding profit for M is

$$\Pi(\rho) = \begin{cases} 0 & \text{if} \quad f \ge \frac{2\mu}{v} \\ \rho v \left(1 - \frac{v\rho}{2\mu}\right) & \text{if} \quad f \le \frac{2\mu}{v} \end{cases}.$$

Recalling that M will always set $\rho \le 1 - \frac{c}{v}$ and following the same steps as the proof of Proposition 1, the optimal ad-valorem fee is $\rho^*(\mu) = \min\left\{\frac{\mu}{v}, 1 - \frac{c}{v}\right\}$. Substituting this back into the above pricing and profit formula, we obtain the identical equilibrium pricing $p_d^*(\mu)$ and profit functions $\Pi^*(\mu)$ as in Proposition 1, proving the results are identical.

C Power function distribution

We repeat our baseline analysis when $G(s) = \frac{1}{\mu}s^\alpha$ for $s \in \left[0, \mu^{\frac{1}{\alpha}}\right]$ to allow for any $\alpha > 0$. Note an increase in the parameter μ still corresponds to an increase in switching costs. By increasing μ, we will increase the expected switching cost which is $\frac{\alpha}{1+\alpha}\mu^{\frac{1}{\alpha}}$, with

$G_2(s)$ stochastically dominating $G_1(s)$ if $\mu_2 > \mu_1 > 0$. We obtain the following proposition which generalizes Proposition 1 in the main text to allow for $\alpha \neq 1$.

Proposition 8. *The optimal transaction fee, direct price and marketplace profits are as follows:*

$$\begin{aligned} f^*(\mu) &= \min\left\{\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}}, v-c\right\} \\ p_d^*(\mu) &= v - \min\left\{\left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}, \frac{\alpha}{1+\alpha}(v-c)\right\} \\ \Pi^*(\mu) &= \begin{cases} \left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}} & \text{if } \mu \leq \frac{\alpha^\alpha (v-c)^\alpha}{(1+\alpha)^{\alpha-1}} \\ (v-c)\left(1-\frac{\alpha^\alpha (v-c)^\alpha}{\mu(1+\alpha)^\alpha}\right) & \text{if } \mu > \frac{\alpha^\alpha (v-c)^\alpha}{(1+\alpha)^{\alpha-1}} \end{cases} \end{aligned}$$

The extent of leakage is $\frac{1}{\mu}(v - p_d^*(\mu))^\alpha$. In response to an increase in switching costs (an increase in μ), the marketplace's fee weakly increases, S's direct price weakly decreases, the extent of leakage weakly decreases, and the marketplace's profit increases. There is always positive but partial leakage.

Proof. Assuming $f \leq v - c$, we have

$$p_d(f) = v - \frac{\alpha}{1+\alpha} f$$

and therefore S's profit maximizing price $p_d^*(f)$ is given by

$$p_d^*(f) = \begin{cases} v - \mu^{\frac{1}{\alpha}} & \text{if } f \geq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \\ v - \frac{\alpha}{1+\alpha} f & \text{if } f \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \end{cases}.$$

The corresponding profit for M is

$$\Pi(f) = \begin{cases} 0 & \text{if } f \geq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \\ f\left(1 - \frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha} f^\alpha\right) & \text{if } f \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \end{cases}.$$

Noting second-order conditions hold for any $f > 0$, $\alpha > 0$ and $\mu > 0$, we have

$$\arg\max_f \left\{ f\left(1 - \frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha} f^\alpha\right)\right\} = \frac{1+\alpha}{\alpha}\frac{1}{(1+\alpha)^{\frac{1}{\alpha}}}\mu^{\frac{1}{\alpha}} < \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}}.$$

Taking into account that $f \leq v - c$, this implies the level of f^* and Π^* given in Proposition 8.

First, note that Π^* is always increasing in μ. To determine the direct price, note that if $\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq v - c$, then

$$f^* = \frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}},$$

so in this case

$$p_d^* = v - \left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}.$$

If $\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \geq v - c$, then

$$f^* = v - c \leq \frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}},$$

so in this case

$$p_d^* = v - \frac{\alpha}{1+\alpha}(v-c).$$

Combining these two results implies p_d^* in Proposition 8. Since $\left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}$ is everywhere increasing in μ, p_d^* is strictly decreasing in μ below a threshold level of μ, but above that threshold, p_d^* is constant in μ.

The equilibrium extent of leakage is given by

$$\frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha}(f^*)^\alpha.$$

Given $f^* = \min\left\{\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}}, v-c\right\}$, the extent of leakage is

$$\min\left\{\frac{1}{1+\alpha}, \frac{\alpha^\alpha(v-c)^\alpha}{\mu(1+\alpha)^\alpha}\right\}.$$

This is weakly decreasing in μ; initially constant, and then decreasing in μ. □

D Alternative tie-breaking assumption

Here we redo the analysis of steering under the assumption that if neither seller is offering non-negative surplus to buyers that purchase via M (i.e. $p_m^l > u$ and $p_m^h > v$), then M does not show either seller.

We first prove the following lemma.

Lemma 4 *If* $u - c \leq \frac{2(v-c)}{3}$*, then*

$$f^* = \begin{cases} u - c & \text{if} \quad \mu \leq 2(u-c) \\ \mu & \text{if} \quad 2(u-c) \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}$$

$$\Pi^* = \begin{cases} u - c & \text{if} \quad \mu \leq 2(u-c) \\ \frac{\mu}{2} & \text{if} \quad 2(u-c) \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}$$

If $\frac{2(v-c)}{3} \leq u - c \leq v - c$*, then*

$$f^* = \begin{cases} u - c & \text{if} \quad \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \\ 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \end{cases}$$

$$\Pi^* = \begin{cases} u - c & \text{if} \quad \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \end{cases}$$

Proof of Lemma 4

Here too, given f and seller prices, M recommends the seller that induces the least amount of leakage (i.e. with the lowest non-negative difference between price on M and direct price), subject to offering non-negative utility to buyers that buy via M. We define $\overline{p}_d^l(f)$ and $\overline{p}_d^h(f)$ as in the proof of Lemma 3:

$$\overline{p}_d^l(f) = \max_{\substack{p_d \leq u \\ (p_d - c)\frac{\min\{u-p_d,\mu\}}{\mu} + (u-c-f)\left(1 - \frac{\min\{u-p_d,\mu\}}{\mu}\right) \geq 0}} \{p_d\}$$

$$\overline{p}_d^h(f) = \max_{\substack{p_d \leq v \\ (p_d - c)\frac{\min\{v-p_d,\mu\}}{\mu} + (v-c-f)\left(1 - \frac{\min\{v-p_d,\mu\}}{\mu}\right) \geq 0}} \{p_d\}.$$

First, because $\overline{p}_d^l(f)$ and $\overline{p}_d^h(f)$ are defined in the same way, we still have

$$\overline{p}_d^l(f) = \begin{cases} u & \text{if} \quad f \leq u - c \\ u - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2} & \text{if} \quad \begin{array}{c} \mu \leq u - c \leq f \text{ or} \\ \mu > u - c \text{ and } u - c \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \end{array} \\ -\infty & \text{if} \quad \mu > u - c \text{ and } f > 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \end{cases}$$

$$\overline{p}_d^h(f) = \begin{cases} v & \text{if} & f \leq v-c \\ v - \frac{f-\sqrt{f^2-4\mu(f-(v-c))}}{2} & \text{if} & \begin{array}{c} \mu \leq v-c \leq f \text{ or} \\ \mu > v-c \text{ and } v-c \leq f \leq 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) \end{array} \\ -\infty & \text{if} & \mu > v-c \text{ and } f > 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) \end{cases}$$

Second, S_h still makes all sales in equilibrium by the same reasoning as in the proof of Lemma 3. The only slight difference is when $v - \overline{p}_d^h(f) = u - \overline{p}_d^l(f) = +\infty$, i.e. neither seller can make non-negative profits with positive sales via M. In this case the two sellers set $p_m^l > u$ and $p_m^h > v$, which means M doesn't show either of them and makes zero profits. This means M would never set such an f in equilibrium in the first place.

There are therefore two cases:

Case 1) If $\mu > u-c$ and $f > 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f) = -\infty$, which means S_l has no chance of making non-negative profits. This implies it might as well price at $p_d^l = p_m^l > u$, which makes it irrelevant. In this case, if $\overline{p}_d^h(f)$ is well-defined (i.e. not equal to $-\infty$), then S_h does best by setting $p_m^h = v$ and $p_d^h = p_d^*(f) = v - \min\left\{\frac{f}{2}, \mu\right\}$, so it gets recommended by M, makes positive profits, and M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$. If $\overline{p}_d^h(f) = -\infty$, then S_h cannot make non-negative profits selling through M, so it sets $p_m^h > v$ and M makes zero profits.

Case 2) If $\mu \leq u-c$ or $\mu > u-c$ and $0 \leq f \leq 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f)$ exists. In this case, S_l sets $p_m^l = u$ and $p_d^l = \overline{p}_d^l(f)$, while S_h sets $p_m^h = v$ and p_d^h to maximize profits subject to $v - p_d^h \leq u - \overline{p}_d^l(f)$ (so that it is recommended by M), i.e.

$$\begin{aligned} p_d^h &= \arg \max_{p_d \geq v - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2}} \left\{(p_d - c)\frac{\min\{v-p_d,\mu\}}{\mu} + (v-c-f)\left(1 - \frac{\min\{v-p_d,\mu\}}{\mu}\right)\right\} \\ &= v - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2}, \end{aligned}$$

where the last equality follows because $v - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2} \geq \max\left\{v-\frac{f}{2}, v-\mu\right\}$ under the conditions that define case 2). Also, we know that at these prices, S_h must make non-negative profits because if $\overline{p}_d^l(f)$ is well-defined, then so is $\overline{p}_d^h(f)$.

This implies M's profit in this case is

$$f\left(1 - \frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2\mu}\right).$$

If M sets $f \leq u-c$, then $\overline{p}_d^l(f) = u$ and $\overline{p}_d^h(f) = v$. In this case, S_l's best chance to be recommended and make non-negative profits is to set $p_d^l = p_m^l = u$. The best response of

S_h is then to set $p_d^h = p_m^h = v$, which ensures that it is recommended by M (we assume M breaks ties in favor of S_h). This leads all buyers to purchase from S_h on M, so M's profits are equal to f. As a result, M does best in this range to set $f = u - c$, yielding a profit equal to $u - c$.

We can therefore restrict attention to $f \geq u - c$.

Suppose $\mu \leq u - c$, so we are in case 2) above. The derivative of M's profit with respect to f is

$$\begin{aligned} \frac{d\left(f\left(1-\frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2\mu}\right)\right)}{df} &= \frac{-\left(2\mu - f + \sqrt{f^2 - 4\mu\left(f-(u-c)\right)}\right)\left(f - \sqrt{f^2-4\mu\left(f-(u-c)\right)}\right)}{2\mu\sqrt{f^2-4\mu\left(f-(u-c)\right)}} \\ &\leq 0, \end{aligned}$$

where the last inequality follows because $f > \sqrt{f^2 - 4\mu\left(f-(u-c)\right)}$ and $u - c \geq \mu$ imply $2\mu - f + \sqrt{f^2 - 4\mu\left(f-(u-c)\right)} \geq 0$. This means M wants to set f as low as possible subject to $f \geq u - c$. Thus, we have proven that when $\mu \leq u - c$, the optimal solution for M is to set $f^* = u - c$, resulting in $p_d^h = p_m^h = v$, no leakage and $\Pi^* = u - c$.

Now suppose $\mu > u - c$.

- If $u - c \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then we are once again in case 2) above. And once again $2\mu - f + \sqrt{f^2 - 4\mu\left(f-(u-c)\right)} \geq 0$ because $2\mu - f \geq 0$, so M's best option on this range is to set $f = u - c$, resulting in profit $u - c$.
- If $\mu \leq v - c$, then M can set f such that $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) < f \leq 2\mu$ to obtain profits $f\left(1 - \frac{f}{2\mu}\right)$ (case 1) above)
- If $\mu > v - c$, then M can set f such that $2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) < f \leq 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right)$ to obtain profits $f\left(1 - \frac{f}{2\mu}\right)$ (case 1) above)

Thus, if $u - c < \mu \leq v - c$, then M chooses between $u - c$ and

$$\max_{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f\leq 2\mu}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\sqrt{1-\frac{u-c}{\mu}}\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) & \text{if } \mu \leq \frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if } \mu \geq \frac{4(u-c)}{3} \end{cases}$$

And if $\mu > v - c$, then M chooses between $u - c$ and

$$\max_{\substack{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f \\ f\leq 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)}}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\sqrt{1-\frac{u-c}{\mu}}\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) & \text{if } \mu \leq \frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if } \frac{4(u-c)}{3} \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) & \text{if } \mu \geq \frac{4(v-c)}{3} \end{cases}$$

Suppose $u-c \leq \frac{v-c}{2}$. Then:

- if $u-c < \mu \leq 2(u-c)$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $2(u-c) \leq \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = \mu$, yielding $\Pi^* = \frac{\mu}{2}$.
- if $\mu \geq \frac{4(v-c)}{3}$, the optimal solution is $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$, yielding

$$\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right).$$

Suppose $\frac{v-c}{2} \leq u-c \leq \frac{2(v-c)}{3} < \frac{3(v-c)}{4}$. Then:

- if $u-c < \mu \leq v-c$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $v-c < \mu \leq 2(u-c)$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $2(u-c) < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = \mu$, yielding $\Pi^* = \frac{\mu}{2}$.
- if $\mu \geq \frac{4(v-c)}{3}$, the optimal solution is $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$, yielding

$$\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right).$$

Suppose $\frac{2(v-c)}{3} < u-c \leq \frac{3(v-c)}{4}$. Then $2(u-c) > \frac{4(v-c)}{3}$ and:

- if $u-c < \mu \leq v-c$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $v-c < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $\mu \geq \frac{4(v-c)}{3}$ then M chooses between $u-c$ and $2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$. We have

$$\begin{aligned} 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) &\geq u-c \\ \sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) &\geq \frac{u-c}{2\mu} \end{aligned}$$

  Clearly, if $\mu \leq 2(u-c)$, then last inequality above cannot hold because the LHS is less than or equal to $\frac{1}{4}$. So if $\frac{4(v-c)}{3} \leq \mu \leq 2(u-c)$, the optimal solution continues to be $f^* = u - c$, yielding $\Pi^* = u - c$. So suppose $\mu > 2(u-c)$. Let $x = \sqrt{1 - \frac{v-c}{\mu}}$ and $y = \frac{u-c}{2\mu} < \frac{1}{4}$. Then the last inequality above is equivalent to

$$\begin{aligned} x^2 - x + y &\leq 0 \\ \frac{1-\sqrt{1-4y}}{2} &\leq x \leq \frac{1+\sqrt{1-4y}}{2} \\ \frac{1-\sqrt{1-\frac{2(u-c)}{\mu}}}{2} &\leq \sqrt{1-\frac{v-c}{\mu}} \leq \frac{1+\sqrt{1-\frac{2(u-c)}{\mu}}}{2} \end{aligned}$$

The LHS inequality always holds when $\mu > 2(u-c) > \frac{4(v-c)}{3}$. Indeed,

$$\frac{1-\sqrt{1-\frac{2(u-c)}{\mu}}}{2} \leq \sqrt{1-\frac{v-c}{\mu}}$$

is equivalent to

$$1 - 2\frac{v-c}{\mu} + \frac{(u-c)}{\mu} + \sqrt{1-\frac{2(u-c)}{\mu}} \geq 0,$$

which holds because

$$1 - 2\frac{v-c}{\mu} + \frac{(u-c)}{\mu} \geq 1 - 2\frac{v-c}{\mu} + \frac{2(v-c)}{3\mu} = 1 - \frac{4(v-c)}{3\mu} > 0.$$

Thus, when $\mu > 2(u-c)$ the inequality

$$\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) \geq \frac{u-c}{2\mu}$$

is equivalent to

$$\sqrt{1-\frac{v-c}{\mu}} \leq \frac{1+\sqrt{1-\frac{2(u-c)}{\mu}}}{2},$$

i.e.

$$\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$$

And it can be verified that

$$\frac{(2(v-c)-(u-c))^2}{4(v-u)} > 2(u-c).$$

Bottomline for this case is that solution is $f^* = u - c$ and $\Pi^* = u - c$ for $\mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$, and $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$ and $\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$

for $\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$.

Suppose $\frac{3(v-c)}{4} < u - c \leq v - c$. Then:

- if $u - c < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = u - c$, yielding $\Pi^* = u - c$
- if $\frac{4(v-c)}{3} \leq \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$, the optimal solution is $f^* = u - c$, yielding $\Pi^* = u - c$
- if $\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$ then M chooses $f^* = 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right)$ and

$$\Pi^* = 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right).$$

We have thus proven the expressions of f^* and Π^* given above.

■

Using the expressions from the text of Lemma 4, we now verify that the same results stated in Proposition 7 from the main text continue to hold here.

First, the proof of Lemma 4 has already shown that S_h makes all sales.

Second, it is easily seen that Π^* is weakly increasing in u.

Third, it is easily seen that when $u \to c$, we have

$$\Pi_s^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}.$$

This is the profit with a single seller of value v (the low-quality seller with value u is irrelevant when $u \to c$), assuming M can steer and hides the seller when it sets $p_m > v$. Note the difference with the baseline model in the paper, where we have assumed no steering or, if steering is possible, that M shows the seller when $p_m > v$ (in that case, M is indifferent between showing and not showing the seller). The monopoly profit in the baseline was

$$\Pi_{ns}^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq v - c \\ (v-c)\left(1 - \frac{v-c}{2\mu}\right) & \text{if} \quad \mu \geq v - c \end{cases}.$$

Comparing the two profit expressions, we have $\Pi_s^* \geq \Pi_{ns}^*$ for all μ. To see this, note that $(v-c)\left(1 - \frac{v-c}{2\mu}\right) \leq \frac{\mu}{2}$ for all μ and

$$2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) > (v-c)\left(1 - \frac{v-c}{2\mu}\right)$$

is equivalent to

$$\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)>\left(1-\frac{v-c}{2\mu}\right)\frac{v-c}{2\mu}.$$

The last inequality is true when $\mu \geq \frac{4(v-c)}{3}$ because in that case

$$1-\frac{v-c}{2\mu}>\sqrt{1-\frac{v-c}{\mu}}\geq\frac{1}{2}.$$

Thus, since the profit expression in Lemma 4 is increasing in u and equal to Π_s^* when $u \to c$, while the profit with two sellers and no steering from Lemma 2 is decreasing in u and equal to Π_{ns}^* when $u \to c$, we can conclude that here too, M's profit with steering is always higher than M's profit without steering.

E Competing sellers with low-quality seller only active on the marketplace

Here we assume the low-quality seller (whose product offers utility u) does not have a direct channel so is only active on M. The high-quality seller is still active in both channels.

For the case without steering, we prove the following result.

Lemma 5 If the low-quality seller is only active on M and M does not (or cannot) steer, then M obtains the exact same profits as in the baseline, i.e. in the absence of the low-quality seller.

Proof of Lemma 5

Using the same reasoning as in the proof of Proposition 6 in the main text, the high-quality seller (S_h) must make all the sales on and off M in equilibrium. Given this, the low-quality seller (S_l)'s price on M is $p_m^l = c+f$ (this is the only price it sets). Again, by similar arguments as in the proof of Proposition 6, $f \leq v-c$ and $p_d^h \leq p_m^h$. And we must also have $p_m^h \leq \min\{c+f+v-u, v\}$ or $p_d^h \leq v$, which implies either $p_m^h = \min\{c+f+v-u, v\}$ or $p_d^h = v$. It is then easily verified that here too we must always have $p_m^h = \min\{c+f+v-u, v\}$.

There are then two cases:

a) If $f \leq u-c$, then $p_m^h = c+f+v-u$ and S_h solves

$$\max_{p_d^h \leq c+f+v-u}\left\{\left(p_d^h-c\right)\frac{\min\left\{c+f+v-u-p_d^h,\mu\right\}}{\mu}+(v-u)\left(\frac{\mu-\min\left\{c+f+v-u-p_d^h,\mu\right\}}{\mu}\right)\right\},$$

where the $p_d^h \leq c+f+v-u$ constraint comes from $p_d^h \leq p_m^h$. It is easily verified that the solution is $p_d^{h*} = c+\frac{f}{2}+v-u+\max\left\{0, \frac{f}{2}-\mu\right\}$, so M's profits in this case are $f\left(1-\min\left\{\frac{f}{2\mu},1\right\}\right)$.

b) If $f > u-c$, then $p_m^h = v$ and S_h solves

$$\max_{p_d^h \leq v}\left\{\left(p_d^h - c\right)\frac{\min\left\{v-p_d^h,\mu\right\}}{\mu} + (v-c-f)\left(\frac{\mu-\min\left\{v-p_d^h,\mu\right\}}{\mu}\right)\right\}$$

It is easily verified the solution is $p_d^{h*} = v - \min\left\{\frac{f}{2},\mu\right\}$, so M's profits in this case are once again $f\left(1-\min\left\{\frac{f}{2\mu},1\right\}\right)$.

Thus, in all cases M makes $f\left(1-\min\left\{\frac{f}{2\mu},1\right\}\right)$, which is the exact same profit (as a function of f) as in the baseline, i.e. when the low-quality seller was absent. Optimizing over f leads to the same solution for M.

■

Thus, the presence of a low-quality seller without a direct channel has no impact on leakage and marketplace profits when steering is not possible. To understand this, note that in case b) above, $f > u-c$ renders the low-quality seller irrelevant. Meanwhile, in case a), the low-quality seller does constrain the high-quality seller's pricing on M, but because the high-quality seller is a monopolist in the direct channel, it can adjust its direct price downwards, so the amount of leakage is independent of u. Indeed, note that $p_m^h - p_d^h = \min\left\{\frac{f}{2},\mu\right\}$, which is exactly the same as in the baseline. This means that the high-quality seller makes lower profits than in the baseline model, but induces the same amount of leakage.

Consider now the case when M can steer. We make the same assumptions about M's steering decision as in the main text: given a set of prices chosen by the two sellers, M shows the seller that induces the least amount of leakage subject to offering non-negative utility to buyers via M, and when indifferent, it shows the high-quality seller.

We first prove the following result.

Lemma 6 If the low-quality seller is only active on M and M does not (or cannot) steer, then M's profits are

$$\Pi^* = \begin{cases} u-c & \text{if} \quad \mu \leq u-c \\ \frac{\mu}{2} & \text{if} \quad u-c < \mu \leq v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu > v-c \end{cases}. \tag{18}$$

Proof of Lemma 6

Again, the high-quality seller makes all sales on and off M (similar argument to that in the proof of Proposition 7 in the main text, but simpler). Given that S_l does not have a direct channel, everything is as if it had one but chose to set $p_m^l = p_d^l$. Furthermore, there is no reason for S_l to set $p_m^l < u$, so S_l sets $p_m^l = \max\{u, c+f\}$.

If $f > u - c$, then S_l is irrelevant, so everything is as in the baseline, meaning M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$ provided $f \leq v - c$.

If $f \leq u-c$, then S_l is relevant and induces no leakage. Thus, in order to be recommended, S_h must induce no leakage either, meaning we must have $p_d^h = p_m^h = v$. This means M will recommend S_h and make profits equal to f.

So M's profits as a function of f are

$$\mathbf{\Pi}(f) = \begin{cases} f & \text{if} \quad f \leq u - c \\ f\left(1 - \frac{f}{2\mu}\right) & \text{if} \quad u - c < f \leq \min\{2\mu, v - c\} \\ 0 & \text{if} \quad f > 2\mu \end{cases}$$

Optimizing over f, it is easily seen that we obtain the profit expression Π^* given in the text of the Lemma.

■

First, note that the profit expression Π^* given in (18) is increasing in u, which confirms that even without a direct channel, a more competitive low-quality seller is better for M when it can steer.

Second, comparing with M's profit in the baseline given by (6), it is apparent that the two profits are equal except in the range $\mu \leq u - c$, where the profit with a competing low-quality seller without a direct channel is strictly higher. So adding the low-quality seller is weakly better for M.

Finally, it can be easily verified that M's profit with steering when the low-quality seller does not have a direct channel (18) is weakly lower than M's profit with steering when the low-quality seller has a direct channel (16) and (15).