Pricing distortions in multi-sided platforms*

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Abstract

We show that in the context of pricing by a multi-sided platform, in addition to the classical market-power and Spence distortions described by Weyl (2010), other distortions exist. With pure interaction heterogeneity, we show conditions under which these distortions can completely offset the Spence distortions so that prices are only higher than socially efficient levels due to classical market-power distortions. With pure membership heterogeneity, we show that even though the Spence distortions are zero, membership fees can be distorted well above or below the levels implied by classical market-power distortions. Implications for public policy are discussed.

1 Introduction

One of the most important contributions to the theory of pricing in multi-sided platforms is Weyl (2010), who integrates and extends the earlier work of Rochet and Tirole (2003), Armstrong (2006) and Rochet and Tirole (2006).[1] By focusing on a monopoly platform that uses insulating tariffs to avoid a coordination failure, Weyl is able to obtain a number of new results on how platforms set prices.

In standard (one-sided) settings, profit maximization by a monopoly firm leads to a classical market-power distortion—restricting output to increase price. One of the key insights of Weyl's article is that in multi-sided platform settings, "Profit maximization leads to classical and Spence

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[1] See also the seminal contribution of Caillaud and Julien (2003).

distortions" (p. 1657), with the Spence distortion providing a new explanation for why prices may sometimes be too high (or too low) on platforms.

Profit maximization distorts in the spirit of Spence (1975) by internalizing only network externalities to marginal users rather than all participating users.[2] In particular, Weyl argues that after adjusting for the classical market-power distortion, a monopoly platform will set its price too high on side $\mathcal{I}$ compared to the socially optimal price on side $\mathcal{I}$ when the interaction values of average users is greater than the interaction value of marginal users on side $\mathcal{J}$ of the platform (see equation (7) in his paper). Thus, he argues, AmEx sets its price too high to merchants in part because loyal cardholders value the participation of merchants more than those indifferent between AmEx and another payment form do. He writes (p.1642) "Given its limited ability to price discriminate, AmEx fails to fully internalize the preferences of loyal users, putting too little effort into attracting merchants and charging them a higher price than would be socially optimal."

In Tan and Wright (2018) we show that in directly comparing the first-order conditions for profit maximizing and socially efficient allocations, Weyl misses that these first-order conditions are evaluated at different allocations. In this paper we do a comparison of pricing outcomes. This reveals that additional distortions also play a role in explaining the difference between privately and socially optimal prices (or tariffs). A fundamental additional distortion that arises when there is heterogeneity in interaction values is what we call the "displacement distortion". It reflects that the profit maximizing price on side $\mathcal{I}$ is based on the interaction benefits enjoyed by marginal users on side $\mathcal{J}$ (and so the price on side $\mathcal{J}$) at the monopoly outcome, which are generally different from the interaction benefits enjoyed by marginal users on side $\mathcal{J}$ at the socially optimal outcome. If the socially optimal outcome entails higher participation than in the monopoly outcome, the marginal users in the socially optimal outcome will have lower interaction benefits compared to in the monopoly outcome, so the displacement distortion will be negative. When the interaction values of average users is higher than marginal users, so the Spence distortion is positive, this displacement distortion will tend to offset the Spence distortion. The net effect of the Spence and displacement distortions can therefore go in either direction.

In the simplest case with heterogeneity only in interaction benefits, which is when the Spence distortion is at its largest, we show the displacement distortion is always in the opposite direction to the Spence distortion on at least one side of the market. Moreover, in the case interaction benefits on each side are distributed according to the generalized Pareto distribution with log-concave demand (which includes the case demands are linear), we show that the displacement distortion exactly offsets the Spence distortion. In such a case, monopoly pricing in multi-sided platforms only involves the classical market-power distortion and nothing else. This suggests the high merchant fees in the payment industry are not well explained by a Spence-type distortion.[3]

[2] Spence (1975) finds that for a given quantity, a monopolist's choice of quality of a product reflects its impact on the value of marginal consumers rather than on the average valuation of consumers, whereas its the impact on the average valuation that matters for the socially optimal choice of quality. In a two-sided platform setting, for each side, the participation rate on that side is analogous to the quantity and the participation rate on the other side is analogous to the quality.

[3] High merchant fees for card payments have been explained in Rochet and Tirole (2002) and Wright (2012) by merchant internalization due to price coherence, by Wright (2004) due to asymmetries in pass-through rates, and in

When heterogeneity is purely in membership benefits, Weyl notes there is no Spence distortion. From his general decomposition of possible market failures into the classical distortion and Spence distortion, this suggests only a classical distortion remains. However, even though interaction benefits are constant (and therefore the same for both privately and socially optimal outcomes, as well as for marginal and average users), an additional distortion can still arise in the tariff charged per participant, which we call the scale distortion. It reflects that the profit a monopolist can extract from side $\mathcal{J}$ from interactions with a participant on side $\mathcal{I}$ depends on the number of participants on side $\mathcal{J}$, which can be different in the privately optimal outcome compared to the socially optimal outcome. This means the incentive a monopolist platform has to increase the tariff on side $\mathcal{I}$ can be excessive or insufficient (depending upon the relative magnitude of the interaction cost and the constant interaction benefit on side $\mathcal{J}$). This distortion in tariffs can be as important as (or more important than) the classical market-power distortion.

We also consider the case with both types of heterogeneity. Based on our decomposition of the distortions between privately and socially optimal tariffs, all four possible distortions can arise—classical, Spence, displacement and scale. We provide a numerical example to show using Weyl's preferred scale-income model of heterogeneity, that it may well be the case that the regulator does better reducing prices opposite the side with the smaller Spence distortion and allowing them to increase on the side opposite that with the larger Spence distortion. Thus, we question Weyl's advice that "... the novel element in two-sided markets is that regulators should focus most on reducing price opposite a side with a large Spence distortion" (p. 1666).

2 Model

For expositional purposes, Weyl develops much of his analysis in the context of Rochet and Tirole (2006), and we do likewise. For brevity, we skip technical details and the necessary second-order conditions for first-order conditions to characterize the solutions, and refer the reader to Weyl's article for these, as well as a comprehensive discussion of the modelling assumptions.

In Weyl's benchmark model, there is a continuum of potential users on each side $\mathcal{I} = \mathcal{A}, \mathcal{B}$ of the market, with mass normalized to 1. A typical user i on side $\mathcal{I}$ obtains a membership benefit of $B_i^{\mathcal{I}}$, which is independent of the number of users participating on the other side. User i also obtains an interaction (usage) benefit $b_i^{\mathcal{I}}$ for every user that participates on the other side. Thus, user i on side $\mathcal{I}$ joins the platform if

$$B_i^{\mathcal{I}} + b_i^{\mathcal{I}} N^{\mathcal{J}} \geq P^{\mathcal{I}}, \tag{1}$$

where $N^{\mathcal{J}}$ is the number of users participating on side $\mathcal{J}$ (the other side than $\mathcal{I}$) and $P^{\mathcal{I}}$ is the tariff set by the platform prescribing how much users on side $\mathcal{I}$ must pay (or will be paid) to participate on side $\mathcal{I}$, which can be conditional on the number of participants on side $\mathcal{J}$. The platform (or planner) chooses an allocation $(N^{\mathcal{A}}, N^{\mathcal{B}})$, or equivalently insulating tariffs $P^{\mathcal{A}}\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right)$ and $P^{\mathcal{B}}\left(N^{\mathcal{B}}, N^{\mathcal{A}}\right)$

Bedre-Defolie and Calvano (2013) by an asymmetry in price discrimination possibilities due to the fact consumers decide which payment method to use.

to achieve this allocation, as defined by Weyl (p.1648). Assume $\left(B^{\mathcal{I}}, b^{\mathcal{I}}\right)$ is distributed according to the well behaved probability density function $f^{\mathcal{I}}\left(B^{\mathcal{I}}, b^{\mathcal{I}}\right)$. Finally, the platform faces two types of costs: a per-member cost $C^{\mathcal{I}}$ and a per-interaction cost c.

This model incorporates two special cases: (i) pure interaction heterogeneity in which $B_i^{\mathcal{I}}$ is the same for all users, which corresponds to the situation in Rochet and Tirole (2003) (with the normalization $B_i^{\mathcal{I}} = 0$ and $C^{\mathcal{I}} = 0$), and (ii) pure membership heterogeneity in which $b_i^{\mathcal{I}} \equiv b^{\mathcal{I}}$, which corresponds to the situation in Armstrong (2006).

3 Weyl's result

Consider first the welfare created by the platform on both sides. This is

$$V\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right) = V^{\mathcal{A}}\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right) + V^{\mathcal{B}}\left(N^{\mathcal{B}}, N^{\mathcal{A}}\right) - C^{\mathcal{A}}N^{\mathcal{A}} - C^{\mathcal{B}}N^{\mathcal{B}} - cN^{\mathcal{A}}N^{\mathcal{B}},$$

where

$$V^{\mathcal{I}}\left(N^{\mathcal{I}}, N^{\mathcal{J}}\right) = \int_{-\infty}^{\infty}\int_{P^{\mathcal{I}}(N^{\mathcal{I}},N^{\mathcal{J}})-b^{\mathcal{I}}N^{\mathcal{J}}}^{\infty} \left[B^{\mathcal{I}} + b^{\mathcal{I}}N^{\mathcal{J}}\right] f^{\mathcal{I}}\left(B^{\mathcal{I}}, b^{\mathcal{I}}\right) dB^{\mathcal{I}}db^{\mathcal{I}}.$$

Differentiating V with respect to $N^{\mathcal{I}}$, and using that the derivative of $V^{\mathcal{I}}$ with respect to $N^{\mathcal{I}}$ equals the price $P^{\mathcal{I}}$, Weyl shows that the socially optimal price is given by

$$P_S^{\mathcal{I}} = \underbrace{C^{\mathcal{I}} + cN_S^{\mathcal{J}}}_{cost} \quad - \quad \underbrace{\overline{b_S^{\mathcal{J}}}N_S^{\mathcal{J}}}_{external\ benefit\ on\ side\ \mathcal{J}}, \tag{2}$$

where

$$\overline{b_S^{\mathcal{J}}} = \frac{\int_{-\infty}^{\infty}\int_{P^{\mathcal{J}}\left(N_S^{\mathcal{J}},N_S^{\mathcal{I}}\right)-b^{\mathcal{J}}N_S^{\mathcal{I}}}^{\infty} b^{\mathcal{J}} f^{\mathcal{J}}\left(B^{\mathcal{J}}, b^{\mathcal{J}}\right) dB^{\mathcal{J}}db^{\mathcal{J}}}{\int_{-\infty}^{\infty}\int_{P^{\mathcal{J}}\left(N_S^{\mathcal{J}},N_S^{\mathcal{I}}\right)-b^{\mathcal{J}}N_S^{\mathcal{I}}}^{\infty} f^{\mathcal{J}}\left(B^{\mathcal{J}}, b^{\mathcal{J}}\right) dB^{\mathcal{J}}db^{\mathcal{J}}} \tag{3}$$

is the average interaction benefit across all participating users on side $\mathcal{J}$, evaluated at the socially optimal allocation.[4] Unlike Weyl, we have added subscripts S on each term to emphasize that each term is evaluated at the socially optimal allocation, which will be in contrast to the monopoly allocation below.

Weyl contrasts this with the monopoly solution. The monopoly platform obtains profit of

$$\pi\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right) = \left(P^{\mathcal{A}}\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right) - C^{\mathcal{A}}\right)N^{\mathcal{A}} + \left(P^{\mathcal{B}}\left(N^{\mathcal{A}}, N^{\mathcal{B}}\right) - C^{\mathcal{B}}\right)N^{\mathcal{B}} - cN^{\mathcal{A}}N^{\mathcal{B}}.$$

Differentiating π with respect to $N^{\mathcal{I}}$, Weyl finds that the profit maximizing tariff is given by

$$P^{\mathcal{I}} = \underbrace{C^{\mathcal{I}} + cN^{\mathcal{J}}}_{cost} \; + \; \underbrace{\frac{P^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{markup} \; - \; \underbrace{\widetilde{b^{\mathcal{J}}}N^{\mathcal{J}}}_{external\ benefit\ on\ side\ \mathcal{J}}, \tag{4}$$

[4] Note Weyl's corresponding expression has an obvious typo in that it uses $f^{\mathcal{I}}\left(B^{\mathcal{J}}, b^{\mathcal{J}}\right)$ instead of $f^{\mathcal{J}}\left(B^{\mathcal{J}}, b^{\mathcal{J}}\right)$.

where

$$\widetilde{b^{\mathcal{J}}} = \frac{\int_{-\infty}^{\infty} b^{\mathcal{J}} f^{\mathcal{J}} \left(P^{\mathcal{J}}\left(N^{\mathcal{J}}, N^{\mathcal{I}}\right) - b^{\mathcal{J}} N^{\mathcal{I}}, b^{\mathcal{J}}\right) db^{\mathcal{J}}}{\int_{-\infty}^{\infty} f^{\mathcal{J}} \left(P^{\mathcal{J}}\left(N^{\mathcal{J}}, N^{\mathcal{I}}\right) - b^{\mathcal{J}} N^{\mathcal{I}}, b^{\mathcal{J}}\right) db^{\mathcal{J}}} \tag{5}$$

is the average interaction benefit across marginal users on side J evaluated at the profit maximizing allocation, and

$$\epsilon^{\mathcal{I}} = -\frac{\partial N^{\mathcal{I}}}{\partial P^{\mathcal{I}}} \frac{P^{\mathcal{I}}}{N^{\mathcal{I}}}$$

is the price elasticity of demand evaluated at the profit maximizing allocation. Note we denote the classical market-power term as "markup" for short.

As noted in Tan and Wright (2018), Weyl then directly compares the first-order conditions (2) and (4), overlooking the fact that they are evaluated at different allocations, which we have recognized by subscripting the solution to (2) but not the solution to (4). Thus, ignoring the subscript S on the solution to (2), a direct comparison suggests we can write the profit maximizing tariff as

$$P^{\mathcal{I}} = \underbrace{C^{\mathcal{I}} + cN^{\mathcal{J}} - \overline{b^{\mathcal{J}}} N^{\mathcal{J}}}_{\text{socially optimal price}} + \underbrace{\frac{P^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{\text{markup}} + \underbrace{\left(\overline{b^{\mathcal{J}}} - \widetilde{b^{\mathcal{J}}}\right) N^{\mathcal{J}}}_{\text{Spence distortion}}. \tag{6}$$

This corresponds to equation (7) in Weyl (2010). As a result, Weyl concludes that there are only two distortions in two-sided markets, a classical market-power distortion (the markup term) and a Spence distortion.

The mistake Weyl makes in writing (6) is to ignore that the socially optimal tariff is obtained by evaluating interaction benefits and the level of participation at the socially optimal allocation and not at the privately optimal allocation. This means (2) and (4) cannot be directly compared. Taking into account this difference by using the subscript S to denote socially optimal allocations, we can write the profit maximizing tariff as

$$P^{\mathcal{I}} = \underbrace{C^{\mathcal{I}} + cN_S^{\mathcal{J}} - \overline{b_S^{\mathcal{J}}} N_S^{\mathcal{J}}}_{\text{socially optimal price}} + \underbrace{\frac{P^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{\text{markup}} + \underbrace{\left(\overline{b_S^{\mathcal{J}}} - \widetilde{b_S^{\mathcal{J}}}\right) N_S^{\mathcal{J}}}_{\text{Spence distortion}} + \underbrace{\left(\widetilde{b_S^{\mathcal{J}}} - \widetilde{b^{\mathcal{J}}}\right) N_S^{\mathcal{J}}}_{\text{displacement distortion}} + \underbrace{\left(\widetilde{b^{\mathcal{J}}} - c\right)\left(N_S^{\mathcal{J}} - N^{\mathcal{J}}\right)}_{\text{scale distortion}}. \tag{7}$$

This result says that there are four ways in which the privately optimal tariff differs from the socially optimal tariff. Apart from the classical markup and Spence distortions emphasized by Weyl, two additional distortions arise—a displacement distortion and a scale distortion.

The displacement distortion reflects that the interaction benefits of marginal users on the opposite side can differ when comparing the privately optimal outcome with the socially optimal outcome. This, in turn, reflects that monopoly prices will generally differ from efficient prices. To the extent that the monopoly price exceeds the efficient price on side $\mathcal{J}$, the interaction benefit of the marginal user on side $\mathcal{J}$ will be higher for the monopoly outcome, and so the displacement distortion defined in (7) will be negative. This makes sense. The monopoly two-sided platform considers the effect of its pricing on side $\mathcal{I}$ on the marginal user's interaction benefit on side $\mathcal{J}$ since this determines how much the monopolist can extract on side $\mathcal{J}$. If the marginal user's interaction benefit on side $\mathcal{J}$ is distorted upwards due to monopoly pricing on side $\mathcal{J}$, then the platform has a reason to lower its price on side $\mathcal{I}$, thereby reducing its incentive to set higher prices on side $\mathcal{I}$.

The last distortion noted in (7) is the scale distortion. It reflects that even after controlling for the Spence and displacement distortions, so that a monopolist platform focusing on the marginal user's interaction benefit (or price) on the other side is appropriate, there is still a distortion since the number of participants on the other side is different in the monopoly outcome compared to the socially optimal outcome. This reflects that the profit (or surplus) a monopolist (or planner) can extract from interactions with a side $\mathcal{I}$ participant on side $\mathcal{J}$ depends on the number of participants on side $\mathcal{J}$, which is generally different in the privately optimal outcome compared to the socially optimal outcome. In particular, the monopolist considers surplus (or profit) $\widetilde{b^{\mathcal{J}}} - c$ over $N^{\mathcal{J}}$ participants on side $\mathcal{J}$ whereas after controlling for the Spence and displacement distortions, the planner measures the surplus $\widetilde{b^{\mathcal{J}}} - c$ over $N_S^{\mathcal{J}}$ participants. Thus, the scale distortion arises because there may be a different number of users in the privately optimal outcome compared to the socially optimal outcome. It will be positive on side $\mathcal{I}$ if the surplus created by interaction benefits for the marginal users on side $\mathcal{J}$ is positive at the monopoly prices, and if there is more participation in the socially optimal outcome on side $\mathcal{J}$.

In some cases, the researcher may be interested instead in the distortion of the per-interaction price. Indeed, this is the conventional per-transaction price that have been the focus of attention in the two-sided market settings in which there is pure interaction heterogeneity (e.g. Rochet and Tirole (2003)), reflecting that the insulating tariff then is simply a constant per-transaction price paid on each interaction. With this in mind, let $p^{\mathcal{I}} \equiv P^{\mathcal{I}}/N^{\mathcal{J}}$ be the price charged to users on side $\mathcal{I}$ for each interaction with the $N^{\mathcal{J}}$ participants on side J.[5] Then dividing through by $N_S^{\mathcal{J}}$, (2) becomes

$$p_S^{\mathcal{I}} = \underbrace{\frac{C^{\mathcal{I}}}{N_S^{\mathcal{J}}} + c}_{cost} - \underbrace{\overline{b_S^{\mathcal{J}}}}_{external\ benefit\ on\ side\ \mathcal{J}} \tag{8}$$

while dividing through by $N^{\mathcal{J}}$, (4) becomes

$$p^{\mathcal{I}} = \underbrace{\frac{C^{\mathcal{I}}}{N^{\mathcal{J}}} + c}_{cost} + \underbrace{\frac{p^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{markup} - \underbrace{\widetilde{b^{\mathcal{J}}}}_{external\ benefit\ on\ side\ \mathcal{J}}. \tag{9}$$

Note we can still define $\epsilon^{\mathcal{I}} = -\frac{\partial N^{\mathcal{I}}}{\partial P^{\mathcal{I}}}\frac{P^{\mathcal{I}}}{N^{\mathcal{I}}}$, as before, since $\frac{\partial N^{\mathcal{I}}}{\partial P^{\mathcal{I}}} = \frac{\partial N^{\mathcal{I}}}{\partial p^{\mathcal{I}}}\frac{\partial p^{\mathcal{I}}}{\partial P^{\mathcal{I}}} = \frac{\partial N^{\mathcal{I}}}{\partial p^{\mathcal{I}}}\frac{1}{N^{\mathcal{J}}}$, so that $-\frac{\partial N^{\mathcal{I}}}{\partial P^{\mathcal{I}}}\frac{P^{\mathcal{I}}}{N^{\mathcal{I}}} = -\frac{\partial N^{\mathcal{I}}}{\partial p^{\mathcal{I}}}\frac{1}{N^{\mathcal{J}}}\frac{p^{\mathcal{I}}N^{\mathcal{J}}}{N^{\mathcal{I}}} = -\frac{\partial N^{\mathcal{I}}}{\partial p^{\mathcal{I}}}\frac{p^{\mathcal{I}}}{N^{\mathcal{I}}}$. Comparing (8) with (9), the profit maximizing price can be written as

$$p^{\mathcal{I}} = \underbrace{\frac{C^{\mathcal{I}}}{N_S^{\mathcal{J}}} + c - \overline{b_S^{\mathcal{J}}}}_{socially\ optimal\ price} + \underbrace{\frac{p^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{markup} + \underbrace{\overline{b_S^{\mathcal{J}}} - \widetilde{b_S^{\mathcal{J}}}}_{Spence\ distortion} + \underbrace{\widetilde{b_S^{\mathcal{J}}} - \widetilde{b^{\mathcal{J}}}}_{displacement\ distortion} + \underbrace{\frac{C^{\mathcal{I}}}{N^{\mathcal{J}}} - \frac{C^{\mathcal{I}}}{N_S^{\mathcal{J}}}}_{cost\ distortion}. \tag{10}$$

By writing tariffs on a per-interaction basis, we can ignore the scale distortion that arises when comparing the total tariff paid by each user across the privately and socially optimal outcomes.

[5] Note Weyl defines $p^{\mathcal{I}} \equiv P^{\mathcal{I}}/N^{\mathcal{I}}$, but this is a typo.

However, doing so introduces a cost distortion to the extent that there are membership costs which will be allocated over a different number of interactions when $N^{\mathcal{J}} \neq N_S^{\mathcal{J}}$.

To further interpret these different distortions it helps to focus on some special cases. These allow us to obtain a sharp contrast between our findings and those of Weyl, and to show that allowing for these additional distortions can lead to dramatically different results. We first consider the case with pure interaction heterogeneity.

4 Pure interaction heterogeneity

Rochet and Tirole (2003) consider the case with pure interaction heterogeneity. Weyl notes (p.1652) that in this case, the Spence distortion is exactly equal to the per-interaction surplus, which is necessarily positive. We confirm the Spence distortion, properly measured, is indeed the per-interaction surplus, but show that in this setting, it can be completely offset (or in some cases more than offset) by other distortions.

Without membership benefits or costs to consider, users on side $\mathcal{I}$ make use of the platform if $b_i^{\mathcal{I}} \geq p^{\mathcal{I}}$, which follows from (1). Let $f^{\mathcal{I}}\left(b^{\mathcal{I}}\right)$ be the density function for $b^{\mathcal{I}}$. Then the number of participating users on side $\mathcal{I}$ is given by $N^{\mathcal{I}} = \int_{p^{\mathcal{I}}}^{\infty} f^{\mathcal{I}}\left(b^{\mathcal{I}}\right) db^{\mathcal{I}}$. Then (3) simplifies to

$$\overline{b_S^{\mathcal{J}}} = \frac{\int_{p_S^{\mathcal{J}}}^{\infty} b^{\mathcal{J}} f^{\mathcal{J}}\left(b^{\mathcal{J}}\right) db^{\mathcal{J}}}{\int_{p_S^{\mathcal{J}}}^{\infty} f^{\mathcal{J}}\left(b^{\mathcal{J}}\right) db^{\mathcal{J}}},$$

while (5) can simply be written as $\widetilde{b^{\mathcal{J}}} = p^{\mathcal{J}}$, since by construction, the interaction benefit of the marginal user on side $\mathcal{J}$ just equals the per-interaction price on side $\mathcal{J}$. Apart from these simplifications, the only other change in (10) is that the cost distortion disappears given $C^{\mathcal{I}} = 0$.

With no membership benefits or costs, it is natural to focus on distortions in the per-interaction price. Thus, we get from (10) that

$$p^{\mathcal{I}} = \underbrace{c - \overline{b_S^{\mathcal{J}}}}_{\text{socially optimal price}} + \underbrace{\frac{p^{\mathcal{I}}}{\epsilon^{\mathcal{I}}}}_{\text{markup}} + \underbrace{\overline{b_S^{\mathcal{J}}} - \widetilde{b_S^{\mathcal{J}}}}_{\text{Spence distortion}} + \underbrace{\widetilde{b_S^{\mathcal{J}}} - \widetilde{b^{\mathcal{J}}}}_{\text{displacement distortion}}. \tag{11}$$

Given at least one of the monopoly prices must be higher than the efficient price, we are able to obtain the following result.

Proposition 1. With only interaction benefits and focusing on prices, the Spence distortion is always positive on both sides, while the displacement distortion is negative on at least one side.

Proof. First note that $\overline{b_S^{\mathcal{J}}} - \widetilde{b_S^{\mathcal{J}}} > 0$ follows from the definition of $\overline{b_S^{\mathcal{J}}}$ and that $\widetilde{b_S^{\mathcal{J}}} = p_S^{\mathcal{J}}$. Thus, the Spence distortion is always positive. Given $\widetilde{b_S^{\mathcal{J}}} = p_S^{\mathcal{J}}$ and $\widetilde{b^{\mathcal{J}}} = p^{\mathcal{J}}$, (11) with $\mathcal{I} = \mathcal{A}$ implies

$$p^{\mathcal{A}} + p^{\mathcal{B}} = p_S^{\mathcal{A}} + p_S^{\mathcal{B}} + \frac{p^{\mathcal{A}}}{\epsilon^{\mathcal{A}}} + \overline{b_S^{\mathcal{B}}} - \widetilde{b_S^{\mathcal{B}}}.$$

Since the classical market-power and Spence distortions are positive, $p^{\mathcal{A}}+p^{\mathcal{B}} > p_S^{\mathcal{A}}+p_S^{\mathcal{B}}$. Therefore, it must be that $\widetilde{b_S^{\mathcal{A}}} - \widetilde{b^{\mathcal{A}}} < 0$ and/or $\widetilde{b_S^{\mathcal{B}}} - \widetilde{b^{\mathcal{B}}} < 0$. □

To obtain more specific results, we consider the case in which interaction benefits are distributed according to the generalized Pareto distribution. Specifically, assume $f^{\mathcal{I}}(b) = \lambda^{\mathcal{I}}\left(1-\lambda^{\mathcal{I}}\sigma^{\mathcal{I}}\left(b-\mu^{\mathcal{I}}\right)\right)^{\frac{1-\sigma^{\mathcal{I}}}{\sigma^{\mathcal{I}}}}$ for $\mathcal{I}=\mathcal{A},\mathcal{B}$ with $b \in \left[\mu^{\mathcal{I}}, \mu^{\mathcal{I}}+\frac{1}{\lambda^{\mathcal{I}}\sigma^{\mathcal{I}}}\right]$, where $\mu^{\mathcal{I}}$ is a shift parameter, $\lambda_{\mathcal{I}}$ is a scale parameter and $\sigma_{\mathcal{I}}$ is a shape parameter. The demand on side I is then $N^{\mathcal{I}}(p) = \left(1-\lambda^{\mathcal{I}}\sigma^{\mathcal{I}}\left(p-\mu^{\mathcal{I}}\right)\right)^{\frac{1}{\sigma^{\mathcal{I}}}}$ provided $\mu^{\mathcal{I}} \leq p^{\mathcal{I}} \leq \mu^{\mathcal{I}}+\frac{1}{\lambda^{\mathcal{I}}\sigma^{\mathcal{I}}}$. We assume $\sigma^{\mathcal{I}}>0$ so that demand is log concave on side $\mathcal{I}$. Note that $\sigma^{\mathcal{I}}=1$ captures the case with linear demand on side $\mathcal{I}$.

Noting that $\overline{b_S^{\mathcal{J}}} = \frac{1+\lambda^{\mathcal{J}}\left(p_S^{\mathcal{J}}+\mu^{\mathcal{J}}\sigma^{\mathcal{J}}\right)}{\lambda^{\mathcal{J}}\left(1+\sigma^{\mathcal{J}}\right)}$ and solving (8) for the socially optimal prices, we find

$$p_S^{\mathcal{I}} = \frac{\lambda^{\mathcal{J}} - \lambda^{\mathcal{I}}\left(1+\sigma^{\mathcal{I}}\right)\left(1-c\lambda^{\mathcal{J}}\sigma^{\mathcal{J}}\right) - \lambda^{\mathcal{I}}\lambda^{\mathcal{J}}\left(\mu^{\mathcal{J}}\left(1+\sigma^{\mathcal{I}}\right)\sigma^{\mathcal{J}} - \mu^{\mathcal{I}}\sigma^{\mathcal{I}}\right)}{\lambda^{\mathcal{I}}\lambda^{\mathcal{J}}\left(\sigma^{\mathcal{I}}+\sigma^{\mathcal{J}}+\sigma^{\mathcal{I}}\sigma^{\mathcal{J}}\right)}$$

for $\mathcal{I}=\mathcal{A},\mathcal{B}$. We can compare this to the price implied by profit maximization, which is determined by solving (9) for $\mathcal{I}=\mathcal{A},\mathcal{B}$. The solution can be written as

$$p^{\mathcal{I}} = p_S^{\mathcal{I}} + m,$$

where

$$m = \frac{p^{\mathcal{I}}}{\epsilon^{\mathcal{I}}} = \frac{p^{\mathcal{J}}}{\epsilon^{\mathcal{J}}} = \frac{\lambda^{\mathcal{I}}\sigma^{\mathcal{I}} + \lambda^{\mathcal{J}}\sigma^{\mathcal{J}} + \lambda^{\mathcal{I}}\lambda^{\mathcal{J}}\sigma^{\mathcal{I}}\sigma^{\mathcal{J}}\left(\mu^{\mathcal{I}}+\mu^{\mathcal{J}}-c\right)}{\lambda^{\mathcal{I}}\lambda^{\mathcal{J}}\left(\sigma^{\mathcal{I}}+\sigma^{\mathcal{J}}+\sigma^{\mathcal{I}}\sigma^{\mathcal{J}}\right)}$$

is the symmetric markup that applies to both sides in the profit maximizing solution. Then based on the decomposition in (10), we find the Spence distortion is also equal m, and the displacement distortion equals $-m$, so there is perfect offset. The only difference between the privately optimal and socially optimal prices is the classical markup term m. We state this result as a proposition.

Proposition 2. With only interaction benefits, the Spence distortion is always exactly offset by the displacement distortion when the usage benefits on each side are distributed according to the generalized Pareto distribution with log-concave demand.

Proposition 2 is interesting since the case in which there is only heterogeneity in interaction benefits represents the case in which the Spence distortion is at its maximum. Indeed, under the generalized Pareto distribution, the Spence distortion is the same magnitude as the classical market-power distortion. Based on Weyl's argument that these are the only two distortions in monopoly pricing, one might conclude that for this class of demands, monopoly prices involve double the normal mark-up due to market-power. Instead, as shown in Proposition 2, taking into account the displacement distortion, there is in fact no additional distortion in the prices on each side beyond the usual markup due to market power.

One may wonder if the result in Proposition 2 in which the Spence distortion and displacement distortion exactly offset holds generally. The following example based on the power distribution shows that it does not. In the example, $f^{\mathcal{I}}(b) = \lambda^{\mathcal{I}} b^{\lambda^{\mathcal{I}}-1}$ with $b \in [0,1]$, so the demand function on side $\mathcal{I}$ is given by $N^{\mathcal{I}}(p) = 1 - p^{\lambda^{\mathcal{I}}}$. Let $\lambda^{\mathcal{A}} = 1/3$, $\lambda^{\mathcal{B}} = 2/3$, and $c = 1$. Using straightforward calculations, the decomposition in (11) is different on each side, and becomes

$$\underbrace{0.639}_{\textit{privately optimal } p^{\mathcal{A}}} = \underbrace{0.328}_{\textit{socially optimal } p^{\mathcal{A}}} + \underbrace{0.308}_{\textit{markup}} + \underbrace{0.295}_{\textit{Spence distortion}} - \underbrace{0.294}_{\textit{displacement distortion}}$$

for side $\mathcal{A}$ and

$$\underbrace{0.669}_{\textit{privately optimal } p^{\mathcal{B}}} = \underbrace{0.377}_{\textit{socially optimal } p^{\mathcal{B}}} + \underbrace{0.308}_{\textit{markup}} + \underbrace{0.295}_{\textit{Spence distortion}} - \underbrace{0.311}_{\textit{displacement distortion}}$$

for side $\mathcal{B}$. For this example, the displacement distortion does not quite fully offset the Spence distortion on side $\mathcal{A}$, but more than offsets the Spence distortion on side $\mathcal{B}$.

If we instead consider the distortions arising in tariffs (i.e. the total amount a user pays for all her interactions), from the comparison in (7) we can see that the additional scale distortion remains. This reflects that while monopoly pricing can be explained purely by the classical markup on top of the socially optimal prices, this can lead to an additional positive or negative distortion for users' total tariff depending on whether the monopoly price on the opposite side exceeds marginal cost and whether there is more or less participation on the other side at the socially optimal outcome than the monopoly outcome. Since the marginal cost is incurred per interaction, but the platform sets a price on each side of the interaction, it is quite possible that $\widetilde{b^{\mathcal{J}}} - c$ or equivalently $p^{\mathcal{J}} - c$, is negative on one or both sides. As a result, the scale distortion can be positive, negative, or zero, even when assuming symmetric demand.

For the total tariff, the comparisons based on the power distribution with $\lambda^{\mathcal{A}} = 1/3$, $\lambda^{\mathcal{B}} = 2/3$, and $c = 1$ implies

$$\underbrace{0.150}_{\textit{privately optimal } P^{\mathcal{A}}} = \underbrace{0.157}_{\textit{socially optimal } P^{\mathcal{A}}} + \underbrace{0.073}_{\textit{markup}} + \underbrace{0.141}_{\textit{Spence distortion}} - \underbrace{0.140}_{\textit{displacement distortion}} - \underbrace{0.081}_{\textit{scale distortion}}$$

for side $\mathcal{A}$ and

$$\underbrace{0.093}_{\textit{privately optimal } P^{\mathcal{B}}} = \underbrace{0.117}_{\textit{socially optimal } P^{\mathcal{B}}} + \underbrace{0.043}_{\textit{markup}} + \underbrace{0.092}_{\textit{Spence distortion}} - \underbrace{0.097}_{\textit{displacement distortion}} - \underbrace{0.062}_{\textit{scale distortion}}$$

for side $\mathcal{B}$. In this example, the displacement and scale distortions together more than offset the positive classical and Spence distortion on each of the sides, so that the monopoly tariffs are actually lower than the socially optimal tariffs on each of the sides.[6] If instead, $\lambda^{\mathcal{A}} = 1/2$, $\lambda^{\mathcal{B}} = 1/2$, and $c = 4/9$, then the scale distortion is exactly zero, although in this case the displacement distortion more than offsets the Spence distortion. Finally, when $\lambda^{\mathcal{A}} = 1/2$, $\lambda^{\mathcal{B}} = 1/2$, and $c = 4/10$, the scale distortion is positive, but the sum of the Spence, displacement and scale distortions still remains negative.

[6] It is still the case that the (per-unit) price is higher in the monopoly solution. The (total) tariffs are the product of the (per-unit) prices and the participation rates on the opposite side (which are lower under monopoly in this example).

5 Pure membership heterogeneity

Armstrong (2006) considers the case with pure membership heterogeneity. In this case, $b_i^{\mathcal{I}}$ is constant across all users on side $\mathcal{I} = \mathcal{A}, \mathcal{B}$, so the Spence distortion and displacement distortion disappear. The scale distortion remains, however, which as noted previously, can be positive (reinforcing the usual classical market-power distortion) or negative (reducing the classical distortion). This means Weyl is incorrect to conclude (p. 1658), that in this case "the only distortions caused by market power are the classic, familiar ones of any multiproduct monopolist."

With $b_i^{\mathcal{I}} = b^{\mathcal{I}}$, users on side $\mathcal{I}$ make use of the platform if $B_i^{\mathcal{I}} + b^{\mathcal{I}} N^{\mathcal{J}} \geq P^{\mathcal{I}}$, which follows from (1). Let $f^{\mathcal{I}}\left(B^{\mathcal{I}}\right)$ be the density function for $B^{\mathcal{I}}$. Then the number of participating users on side $\mathcal{I}$ is given by $N^{\mathcal{I}} = \int_{P^{\mathcal{I}} - b^{\mathcal{I}} N^{\mathcal{J}}}^{\infty} f^{\mathcal{I}}\left(B^{\mathcal{I}}\right) dB^{\mathcal{I}}$. To illustrate, consider a completely symmetric setting in which $b_i^{\mathcal{A}} = b_i^{\mathcal{B}} = b$, $f^{\mathcal{I}}(B) = 1$ for $\mathcal{I} = \mathcal{A}, \mathcal{B}$ over $[0, 1]$, and $C^{\mathcal{A}} = C^{\mathcal{B}} = C < 1$. Assume $\frac{c}{2} < b < \frac{c+C}{2}$, which ensures that the total interaction benefit to both sides exceeds the cost of an interaction (i.e. $2b > c$), second-order conditions hold for the planner's problem and the platform's problem, and not all consumers participate at the socially optimal solution and the platform's solution. Then the scale distortion becomes

$$\left(\frac{b - c}{1 + c - 2b} \right) m, \tag{12}$$

where $m = \frac{1-C}{2+c-2b} > 0$ is the classical market-power distortion in (4). Clearly, (12) can be positive, zero, or negative, depending on whether b is greater than, equal to, or less than c. Note in particular, if $b = 1$, then the scale distortion equals $-m$, so the monopoly platform sets its membership tariffs at the socially efficient level since the scale distortion completely offsets the classical market-power distortion. On the other hand, if $b = \frac{1}{3} + \frac{2c}{3}$, then the scale distortion equals m, so the overall markup is double the usual classical markup.

6 Scale-income model

In general, users on each side $\mathcal{I}$ differ in both their membership benefit $B^{\mathcal{I}}$ and their usage benefit $b^{\mathcal{I}}$. The general analysis of Section 3 then applies. To say something more concrete, we adopt Weyl's preferred Scale-Income (SI) model. Specifically, in the SI model, $b^{\mathcal{I}} = \beta^{\mathcal{I}} B^{\mathcal{I}}$, where $\beta^{\mathcal{I}}$ is a constant on side $\mathcal{I}$. Thus, a (high income) user that obtains a high value from belonging to the platform (membership) also puts a high (low) value on interacting on the platform if $\beta > 0$ (< 0). In this way, users still vary in how much they value both membership and usage, but with just one dimension of heterogeneity.

We use the SI model to consider the claim of Weyl, that (p.1666) "the novel element in two-sided markets is that regulators should focus most on reducing price opposite a side with a large Spence distortion. Thus regulators of ISPs should focus on limiting prices to Web sites (net neutrality) if there is more (interaction) surplus among loyal users than among highly profitable Web sites. But if the situation is reversed, forcing ISPs to reduce prices and build more line to consumer homes may be a higher priority."

As we have seen above, a large Spence distortion on one side may be offset by a negative displacement and/or scale distortion on that side. In this case, there is no reason in general why a regulator will want to lower prices on the side opposite that which generates the largest Spence distortion. We illustrate with a numerical example motivated by the regulation of ISPs, in which $\mathcal{A}$ represents users and $\mathcal{B}$ represents websites. Assume $\beta^{\mathcal{A}} > 0$ (high-income consumers value participation and interactions more) and $\beta^{\mathcal{B}} < 0$ (high-value websites value interactions more but face higher fixed costs of participating). We consider parameter values such that the Spence distortion produced by websites is larger than that produced by users which according to Weyl's claim, implies regulators of ISPs should focus on limiting prices to users rather than Web sites.

In particular, if $f^{\mathcal{A}}(b^{\mathcal{A}}) = \frac{1}{7.5}\left(1 - \frac{b^{\mathcal{A}}}{5}\right)^{-\frac{1}{3}}$ with $b^{\mathcal{A}} \in [0,5]$, $f^{\mathcal{B}}(b^{\mathcal{B}}) = \frac{1}{3}\left(1 - \frac{b^{\mathcal{B}}}{6}\right)$ with $b^{\mathcal{B}} \in [0,6]$, $\beta^{\mathcal{A}} = 1$, $\beta^{\mathcal{B}} = -6$, $C^{\mathcal{A}} = 3$, $C^{\mathcal{B}} = 0.5$ and $c = 3.5$, then

$$\underbrace{4.847}_{\textit{privately optimal } P^{\mathcal{A}}} = \underbrace{3.998}_{\textit{socially optimal } P^{\mathcal{A}}} + \underbrace{1.796}_{\textit{markup}} + \underbrace{1.557}_{\textit{Spence distortion}} - \underbrace{2.350}_{\textit{displacement distortion}} - \underbrace{0.154}_{\textit{scale distortion}}$$

for side $\mathcal{A}$ and

$$\underbrace{0.564}_{\textit{privately optimal } P^{\mathcal{B}}} = \underbrace{0.249}_{\textit{socially optimal } P^{\mathcal{B}}} + \underbrace{0.237}_{\textit{markup}} + \underbrace{1.165}_{\textit{Spence distortion}} - \underbrace{1.263}_{\textit{displacement distortion}} + \underbrace{0.176}_{\textit{scale distortion}}$$

for side $\mathcal{B}$. Note the Spence distortion $\left(\overline{b_S^{\mathcal{B}}} - \widetilde{b_S^{\mathcal{B}}}\right) N_S^{\mathcal{B}}$ generated from side $\mathcal{B}$ (1.557) is larger than that generated from side $\mathcal{A}$ (1.165), but the total distortion generated from side $\mathcal{A}$ (0.078) is larger than that generated from side $\mathcal{B}$ (-0.948). Consistent with the logic based on the total of the distortions generated from the other side rather than just the Spence distortion, we find that the welfare maximizing regulated tariffs subject to the platform breaking even involve regulating down the tariff on side $\mathcal{B}$ (from 0.564 to 0.537) and allowing the tariff on side $\mathcal{A}$ to increase (from 4.847 to 4.892). Thus, while focusing on the Spence distortions alone may suggest regulating the tariffs charged to users, in this example the Ramsey optimal tariffs actually involve regulating the tariff charged to websites and allowing the tariff charged to users to increase. This result holds despite the fact that the example was constructed so that the classical markup is much larger on the user side, which might suggest regulation on the user side should dominate. This illustrates the potential importance for regulatory policy of accounting for more than just the classical markup and Spence distortion in multi-sided platforms.

7 Conclusion

Weyl (2010) has provided a general framework to study pricing by a monopoly platform which integrates earlier contributions. By formulating the platform's problem in terms of its choice of allocation, rather than prices, his work has greatly simplified and generalized the analysis of platform pricing so as to better explain how the normative properties and comparative statics of two-sided platforms depend on the underlying sources of user heterogeneity.

One of Weyl's key normative findings is that the tendency of a monopoly platform to focus on the surplus of marginal rather than average users (a so-called Spence distortion) provides a fundamental reason why multi-sided platforms with market power distort prices away from efficient levels, with the other reason being the classical market-power distortion. Indeed, Weyl notes (p.1652) "This Spence distortion is likely more important in two-sided markets than the contexts for which it was originally conceived." We show that in comparing first-order conditions between the profit maximizing and socially efficient allocations, Weyl missed additional distortions that arise when the interaction benefits of marginal users and/or the number of transactions differs at the monopoly outcome from that of the planner's. We find for reasonable cases, these additional distortions can completely offset the Spence distortion so that only the classical market-power distortion remains, create positive or negative distortions aside from the classical ones in settings where the Spence distortion does not exist, or reverse the pattern of distortions implied by focusing only on the Spence distortion, thereby potentially reversing policy implications. Ultimately, however, the direction and size of the different distortions we have documented is an empirical matter.

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