Online Appendix

Bo Shen* and Julian Wright†

In this online appendix, we provide some additional results referred to in the paper "Why (don't) firms free ride on an intermediary's advice?".

A Assumption (1) is relaxed

In this section, we analyze how our main results extend when assumption (1) is relaxed. For brevity, we focus on the case with observable commissions. The analysis for the case in which commissions are unobservable and consumers' beliefs are naive follows in a similar way.

When firms set commissions, consider the proposed equilibrium characterized in Proposition 1 where both firms offer no discounts and set zero commissions.[1] Then the equilibrium cutoff is $q^D = 1/2$, and the equilibrium profits of firms are $\pi_i^D = \frac{1}{2}(P(1/2) - c_i)$ for $i = A, B$. Consider the possibility that firm A deviates by setting its price sufficiently low so that consumers would always want to buy from firm A irrespective of M's recommendation. For any deviating commission and discount with $(f_A, \Delta p_A)$, the optimal intermediated price to induce all consumers to buy from firm A is $p_A^m = \int_0^{\bar{q}_A} v_A(q)\frac{g(q)}{G(\bar{q}_A)}dq$, where $\bar{q}_A = \frac{1}{2} - \frac{(1-H(\Delta p_A))f_A}{2w}$. Then firm A's deviation profit is

$$\pi_A = \int_0^{\bar{q}_A} v_A(q)\frac{g(q)}{G(\bar{q}_A)}dq - (1 - H(\Delta p_A))f_A - \Delta p_A H(\Delta p_A) - c_A.$$

Following our argument of effective commissions, the optimal deviation also involves no discount, i.e., $\Delta p_A = 0$. Then the optimal deviating profit becomes

$$\pi'_A = \max_{f_A}\left\{\int_0^{\bar{q}_A} v_A(q)\frac{g(q)}{G(\bar{q}_A)}dq - f_A - c_A\right\},$$

where $\bar{q}_A = \frac{1}{2} - \frac{f_A}{2w}$. Note that we have $\pi'_A < \int_0^1 v_A(q)dG(q) - c_A$. Therefore, firm A does not want to deviate when $\pi_A^D \geq \pi'_A$, i.e.,

$$\frac{1}{2}(P(1/2) - c_A) \geq \max_{f_A}\left\{\int_0^{\bar{q}_A} v_A(q)\frac{g(q)}{G(\bar{q}_A)}dq - f_A - c_A\right\},$$

*Economics and Management School, Wuhan University

†Department of Economics, National University of Singapore

[1]The result that firms set zero commissions is formally derived in Section D below.

which is weaker than condition (1). Similarly, firm B does not want to deviate when

$$\frac{1}{2}(P(1/2) - c_B) \geq \max_{f_B} \left\{ \int_{\bar{q}_B}^{1} v_B(q) \frac{g(q)}{1 - G(\bar{q}_B)} dq - f_B - c_B \right\},$$

which is again weaker than condition (1).

Next consider the case M sets commissions. Consider the proposed equilibrium in Proposition 4 where M sets positive commissions f_i^* and both firms offer positive discounts Δp_i^*. Denote the equilibrium cutoff by q^*, and firm A's equilibrium profit by π_A^*, which should be non-negative. Suppose firm A deviates by setting its intermediated prices sufficiently low so that consumers would always want to buy from it. Then firm A's optimal deviating profit becomes

$$\pi_A' = \max_{\Delta p_A} \left\{ \int_0^{\bar{q}_A} v_A(q) \frac{g(q)}{G(\bar{q}_A)} dq - (1 - H(\Delta p_A)) f_A^* - \Delta p_A H(\Delta p_A) - c_A \right\},$$

where $\bar{q}_A = \frac{1}{2} + \frac{(1-H(\Delta p_B^*))f_B^* - (1-H(\Delta p_A))f_A^*}{2w}$. Then firm A does not want to deviate when $\pi_A^* \geq \pi_A'$. Provided that $\pi_A' < \int_0^1 v_A(q) dG(q) - c_A$, the condition which ensures firm A has no incentive to deviate in this way is weaker than (1).

B M does not care about suitability

In the main paper, we assume that M gets additional utility when a consumer purchases the more suitable product. In this section, we show our main results still hold even if M does not care about suitability, but rather only recommends the firm which is a better match when it is indifferent, provided commissions are observable.

When M does not care about suitability, its expected payoff from recommending firm A is $[1 - H(\Delta p_A)]f_A$ and that from recommending firm B is $[1 - H(\Delta p_B)]f_B$. Therefore, M recommends whichever firm gives it a higher effective commission $F_i = [1 - H(\Delta p_i)]f_i$. In case M is indifferent (i.e., $F_A = F_B$), we assume it recommends A if and only if $q \geq \frac{1}{2}$ and recommends B otherwise.

First consider the case M sets commissions. We aim to show that there is an equilibrium in which firms want to set positive discounts. Note that the following is an equilibrium: M sets commissions $f_A = f_B = f^* \equiv P\left(\frac{1}{2}\right) - c_B$, and firms set the prices $p_i^m = P\left(\frac{1}{2}\right)$ and Δp_i, where Δp_i solves $H(\Delta p_i) = \max\left\{1 - (1 - H(\Delta))\left(\frac{f^*}{f_i}\right), 0\right\}$ for some constant Δ which is no more than $P\left(\frac{1}{2}\right) - c_B$. Thus, each firm sets its discount to keep the effective commission at the constant level $(1 - H(\Delta)) f^*$, or if that is not possible because f_i is too low, it sets no discount. In equilibrium, the profits of the two firms are

$$\begin{aligned}
\pi_A &= \frac{1}{2}\left(P\left(\frac{1}{2}\right) - c_A - (1 - H(\Delta))\left(P\left(\frac{1}{2}\right) - c_B\right) - \Delta H(\Delta)\right) \\
\pi_B &= \frac{1}{2}\left(P\left(\frac{1}{2}\right) - c_B - (1 - H(\Delta))\left(P\left(\frac{1}{2}\right) - c_B\right) - \Delta H(\Delta)\right) \\
&= \frac{1}{2}H(\Delta)\left(P\left(\frac{1}{2}\right) - c_B - \Delta\right).
\end{aligned}$$

Note $\pi_A \geq \pi_B \geq 0$. If either firm tries to discount more, it will not be recommended at all, and so will not obtain any positive profit. If either firm tries to discount less, it will always be recommended, but then consumers will not be able to infer anything from M's recommendation, which follows from $p_A^m \leq P(0) < c_A$ and $p_B^m \leq P(1) < c_A \leq c_B$. This implies the deviating firm cannot obtain any positive profit. Moreover, M cannot gain by offering a higher commission for one firm relative to the other, since then it will always want to recommend that firm, and consumers will not be able to infer anything from M's recommendation. Finally, M cannot gain by increasing both commissions since according to the proposed equilibrium strategies, each firm will adjust its discounting to keep its effective commission unchanged.

Consider next the case the firms set commissions and consider whether the above equilibrium can still be sustained. Firm A can then do better by lowering its discount to zero and lowering f_A so that $f_A = (1 - H(\Delta))\left(P\left(\frac{1}{2}\right) - c_B\right)$. Since its effective commission remains the same as firm B, M is still willing to recommend it whenever $q \geq \frac{1}{2}$. Firm A's deviation profit is then

$$\begin{aligned}
& \frac{1}{2}\left(P\left(\frac{1}{2}\right) - c_A - (1 - H(\Delta))\left(P\left(\frac{1}{2}\right) - c_B\right)\right) \\
> \; & \frac{1}{2}\left(P\left(\frac{1}{2}\right) - c_A - (1 - H(\Delta))\left(P\left(\frac{1}{2}\right) - c_B\right) - \Delta H(\Delta)\right),
\end{aligned}$$

since it saves on the discount given to inframarginal direct consumers. Based on this argument, the only possible equilibria will have zero discount. Thus, for example, without discounting, there is an equilibrium in which $f_A = f_B = P\left(\frac{1}{2}\right) - c_B$, and $\pi_A = \frac{1}{2}(c_B - c_A) \geq 0$ and $\pi_B = 0$.

C M's additional utility

In the baseline model, we assume that the additional utility M gets is the same regardless of whether consumers complete purchases through M or directly. In this section, we show robustness of our main results to some natural variations from this assumption. For brevity, we focus on the case in which commissions are unobservable and consumers' beliefs are naive. For brevity, we also just focus on showing the less obvious result, that firms do not offer discounts in equilibrium when firms set commissions. The other result, in which firms offer discounts when M sets commissions, continues to apply in each of the following cases.

C.1 Less concern for suitability for a direct transaction

Suppose M's concern for suitability is less for a direct transaction than an intermediated transaction. Our specification allows for the possibility that M has no concern for direct sales.

To explore what happens in this case, we assume $w_h - w_l = w$, $w_h - w_h^d = \delta$, $w_l^d = w_l \geq 0$, and $w \geq \delta > 0$. Thus, we have $w_h - w_l > w_h^d - w_l^d \geq 0$, so M gets less additional utility from recommending the good match when the sale is direct. In particular, when $w = \delta$, M only cares about suitability for intermediated purchase but not for direct purchase since this implies $w_h^d = w_l^d = w_l$, which note can equal zero.

M's expected payoff from recommending firm A is $[1 - H(\Delta p_A)]f_A + qw_h + (1-q)w_l - H(\Delta p_A)q\delta$, and from recommending firm B is $[1-H(\Delta p_B)]f_B + (1-q)w_h + qw_l - H(\Delta p_B)(1-q)\delta$. When both products are recommended with positive probability, M recommends firm A rather than firm B if $q \geq \bar{q}$, where the cutoff is

$$\bar{q} = \frac{1}{2} + \frac{(1 - H(\Delta p_B))\, f_B - \frac{1}{2}H(\Delta p_B)\delta - (1 - H(\Delta p_A))\, f_A + \frac{1}{2}H(\Delta p_A)\delta}{2w - (H(\Delta p_A) + H(\Delta p_B))\,\delta}. \tag{C.1}$$

If $w = \delta$ and $H(\Delta p_A) = H(\Delta p_B) = 1$ so that the denominator of (C.1) is zero, the cutoff can be defined as $\bar{q} = 1/2$.

Given $p_i^m = P_i(q^*)$, firms' profits are

$$\pi_A = [P_A(q^*) - F_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q})), \tag{C.2}$$

$$\pi_B = [P_B(q^*) - F_B - c_B - \Delta p_B H(\Delta p_B)]G(\bar{q}), \tag{C.3}$$

where the cutoff $\bar{q}$ is determined by (C.1). We argue that price-parity must hold. We show it by contradiction. Suppose there exists an equilibrium in which $\Delta p_A^* > 0$, $\Delta p_B^* \geq 0$, $f_A^* \geq 0$, and $f_B^* \geq 0$. Denote the proposed equilibrium cutoff by q^*, where

$$q^* = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))\, f_B^* - \frac{1}{2} H(\Delta p_B^*)\delta - (1 - H(\Delta p_A^*))\, f_A^* + \frac{1}{2} H(\Delta p_A^*)\delta}{2w - (H(\Delta p_A^*) + H(\Delta p_B^*))\, \delta},$$

and $0 < q^* < 1$ since otherwise there will be no sales given condition (1) from the main paper.

We want to argue that firm A can always deviate by lowering Δp_A and at the same time lowering f_A while keeping the effective commission F_A constant. Suppose firm A deviates by setting $0 \leq \Delta p_A' < \Delta p_A^*$ and $f_A' = \frac{(1-H(\Delta p_A^*))f_A^*}{(1-H(\Delta p_A'))} \geq 0$,

$$q' = \frac{1}{2} + \frac{(1 - H(\Delta p_B^*))\, f_B^* - \frac{1}{2} H(\Delta p_B^*)\delta - (1 - H(\Delta p_A'))\, f_A' + \frac{1}{2} H(\Delta p_A')\delta}{2w - (H(\Delta p_A') + H(\Delta p_B^*))\, \delta},$$

Such a deviation is always profitable. On one hand, it increases firm A's profit margin by $\Delta p_A^* H(\Delta p_A^*) - \Delta p_A' H(\Delta p_A') > 0$. Note this makes use of the fact that p_A^m optimally remains equal to $P_A(q^*)$ given commissions are unobservable and consumers hold naive beliefs (i.e. consumers' expectation of $\bar{q}$ is unaffected by the lower Δp_A). On the other hand, firm A's deviation also leads to an increase in its market share; i.e. $\bar{q}$ is lower. To show this, by substituting the expressions for q' and q^*, and rearranging, we get

$$q' - q^* = \frac{\delta(H(\Delta p_A') - H(\Delta p_A^*))q^*}{2w - (H(\Delta p_A') + H(\Delta p_B^*))\, \delta} < 0,$$

where the inequality uses that $\Delta p_A' < \Delta p_A^*$, $q^* > 0$ and $2w - (H(\Delta p_A') + H(\Delta p_B^*))\, \delta > 0$. Thus, with naive beliefs, there exists no equilibrium in which firm A offers a positive discount. A similar argument applies to firm B. As a result, the only possible equilibrium is the one in which price-parity holds. That is, we can still show that when firms set commissions, firm i does best by setting $\Delta p_i = 0$.

C.2 Less utility from a direct transaction

Suppose the additional utility that M enjoys from a completed transaction is less for a direct transaction than an intermediated transaction. Formally, assume M's additional utility for a direct transaction is given by $w_k^d = w_k - \eta$, where η is a positive constant and $k \in \{h, l\}$. Elsewhere in the paper we have assumed $\eta = 0$. When the additional utility reflects a combination of M's private benefits (e.g. cross-selling and repeat purchase opportunities from making a recommendation) and the penalty arising from recommending a bad match, then a positive η could reflect that M's private benefits are lower for direct transactions.

M's expected payoff from recommending firm A is $[1-H(\Delta p_A)]f_A - H(\Delta p_A)\eta + qw_h + (1-q)w_l$ and from recommending firm B is $[1-H(\Delta p_B)]f_B - H(\Delta p_B)\eta + (1-q)w_h + qw_l$. When both products are recommended with positive probability, M recommends firm A rather than firm B if $q \geq \bar{q}$, where the cutoff can also be written as

$$\bar{q} = \frac{1}{2} + \frac{\bar{F}_B - \bar{F}_A}{2w}, \tag{C.4}$$

and $\bar{F}_i = (1 - H(\Delta p_i)f_i - H(\Delta p_i)\eta$ is the "adjusted effective commission". The adjusted effective commission takes into account the probability that the commission is collected by M (i.e. consumers do not switch to buy directly) and the loss of additional utility η in case of direct purchase.

Given $p_i^m = P_i(q^*)$, firms' profits are given by (C.2) and (C.3), where the cutoff $\bar{q}$ is determined by (C.4). Following a similar logic as in Section (5.2.1) of the main paper, firm i would never set $\Delta p_i > 0$. A firm i with $\Delta p_i > 0$ can always do better by lowering the price discount Δp_i and at the same time lowering the commission such that the effective commission $F_i = (1 - H(\Delta p_i))f_i$ remains the same. This implies that the adjusted effective commission $\bar{F}_i$ actually increases when $\eta > 0$, so that firm i is recommended more often by M and its market share is higher (i.e. $\bar{q}$ is lower for firm A and higher for firm B). Moreover, firm i's profit margin is also higher, since only the term $\Delta p_i H(\Delta p_i)$ is affected, which is lower. As a result, we can still show that when firms set commissions, firm i does best by setting $\Delta p_i = 0$ and the price-parity result holds.

C.3 M's additional utility is from recommendations

Suppose by recommending a product, M gets additional utility w only when this product turns out to be a good match for a consumer, regardless of whether the consumer actually purchases the good (either directly or through M). This is motivated by M incurring a psychic benefit of recommending a product in proportion to the probability of that product being a good match (or equivalently, a psychic cost of recommending a product in proportion to the probability of that product being a bad match). Therefore, if M recommends product A, with a probability q, A is more suitable for consumers and M gets w; otherwise, M receives nothing. Then M's expected payoff from recommending firm A is $[1 - H(\Delta p_A)]f_A + qw$. A similar argument applies for firm B. Then for all of the cases considered in Sections 5-6 of the main paper, M's recommendations are always followed. Since M's additional utility in the benchmark model of the main paper was assumed to be the same regardless of whether consumers buy directly or through M, by setting $w_h = w$ and $w_l = 0$, the same equilibrium analysis applies in this case too.

D Different equilibrium selection rule

In the stage-2 subgame, Inderst and Ottaviani (2012a) select the equilibrium in which advice is informative (meaning consumers follow M's recommendations). We have done the same provided both firms' intermediated prices were not so high that consumers would never be willing to purchase from them through M (i.e. even for a consumer with an infinitely high shopping cost s of buying directly). If either firm sets their price this high, we selected the trivial babbling equilibrium in the stage-2 subgame (i.e. consumers treat M's recommendation as noise and M always recommends the same firm). This avoided having to make additional assumptions which are sufficient but not necessary to rule out firms wanting to deviate in this way (i.e. to a very high intermediated price).

Suppose instead we always select an informative equilibrium in the stage-2 subgame. If firm A's intermediated price is set so high that consumers never buy from firm A through M when it is recommended, this implies (i) M would never obtain any commission from firm A and (ii) M would only get the additional utility $qw_h + (1-q)w_l$ from recommending firm A on the fraction $H(P_A(q_A^e) - p_A^d)$ of consumers who are willing to buy directly. While this makes M less likely to recommend firm A, it may also raise consumers' willingness to pay for firm A's product when they buy directly, and greatly complicates the analysis given the cutoff rule requires solving a fixed-point problem (i.e. the cutoff $\bar{q}_A$ now depends on consumers' expectation of the cutoff rule through the term $H(P_A(q_A^e) - p_A^d))$. We can no longer rule out in general that firm A wants to deviate in this way even though we view such extreme pricing as unrealistic in practice. However, even if we always select the informative equilibrium in the stage-2 subgame (i.e. even in case firms set $p_i^m > P_i(q^e)$), we can still rule out such a deviation under reasonable restrictions on the underlying parameters of the model.[2]

Note that if M sets commissions, the possibility of firms setting $p_i^m > P_i(q_i^e)$ and relying on discounts to sell directly would not change our main result in Section 6 that any equilibrium involves firms setting discounts. Therefore, the rest of this section focuses on the case firms set commissions.

Consider any equilibrium that is characterized in Section 5 where the commissions are f_A^*, f_B^* and prices are $p_A^m = P_A(q^*)$, $p_B^m = P_B(q^*)$, $\Delta p_A^* = \Delta p_B^* = 0$. We provide some

[2] If instead M gets the additional utility w_h or w_l when it makes a recommendation regardless of whether consumers actually purchase through M or not, then the only case we would need these additional restrictions is the case with sophisticated beliefs.

analysis that applies generally if firm i deviates from the above proposed equilibrium by setting $p_i^m > P_i(q_i^e)$. Suppose firm A sets $p_A^m > P_A(q_A^e)$. Then no consumers will purchase from firm A through M, and only a fraction of $H(P_A(q_A^e) - p_A^d)$ consumers will purchase from firm A directly. Given that we have selected an informative equilibrium in stage 2 subgame (i.e. consumers will follow M's recommendation of which firm to buy from in the equilibrium in the stage-3 subgame), M's expected payoff from recommending firm A is $H(P_A(q_A^e) - p_A^d)(qw_h + (1-q)w_l)$, while the expected payoff from recommending firm B remains equal to $f_B^* + qw_l + (1-q)w_h$. Thus, the cutoff used by M solves

$$\bar{q}_A = \frac{f_B^* + w + (1 - H(P_A(q_A^e) - p_A^d)w_l}{(1 + H(P_A(q_A^e) - p_A^d))w}, \tag{D.1}$$

when $\bar{q}_A < 1$ (otherwise $\bar{q}_A = 1$). Note that

$$\frac{1}{2} + \frac{f_B^*}{2w} \le \bar{q}_A \le \frac{f_B^* + w + w_l}{w},$$

which follows from

$$\bar{q}_A - \left(\frac{1}{2} + \frac{f_B^*}{2w}\right) = \frac{(1 - H(P_A(q_A^e) - p_A^d))(f_B^* + w_h + w_l)}{2(1 + H(P_A(q_A^e) - p_A^d))w} \ge 0$$

and

$$\bar{q}_A - \frac{f_B^* + w + w_l}{w} = \frac{H(P_A(q_A^e) - p_A^d)(f_B^* + w_h)}{(1 + H(P_A(q_A^e) - p_A^d))w} \ge 0,$$

which in turn follow from the fact $0 \le H(P_A(q_A^e) - p_A^d) \le 1$, $f_B^* \ge 0$, $w_h + w_l \ge 0$, and $w_h \ge 0$. When $f_B^* \ge w$, we always have $\bar{q}_A = 1$, for any p_A^d. Otherwise, we have $\frac{1}{2} + \frac{f_B^*}{2w} \le \bar{q}_A \le \min\left\{1, \frac{f_B^*+w+w_l}{w}\right\}$, where we could have $\frac{f_B^*+w+w_l}{w} < 1$ since we can allow $w_l < 0$.

D.1 Firms set observable commissions

Given the proposed equilibrium in Section 5.1, we provide conditions under which firm i does not want to set $p_i^m > P_i(q_i^e)$ so as to steer consumers to buy directly even if the informative equilibrium is selected in the stage-2 subgame.

We first prove the following lemma.

Lemma 1. *In the unique pure strategy informative equilibrium characterized in Section 5.1, both firms set zero commissions.*

Proof. In Section IV of Inderst and Ottaviani (2012a), under observable commissions, the firms' expected profits are

$$\begin{aligned}\pi_A &= \left[P_A(\bar{q}) - f_A - c_A\right]\left[1 - G(\bar{q})\right], \\ \pi_B &= \left[P_B(\bar{q}) - f_B - c_B\right] G(\bar{q}).\end{aligned}$$

First-order derivatives are

$$\begin{aligned}\frac{d\pi_A}{df_A} &= \frac{g(\bar{q})}{2w}\left(v_A(\bar{q}) - f_A - c_A - \frac{2w(1 - G(\bar{q}))}{g(\bar{q})}\right), \\ \frac{d\pi_B}{df_B} &= \frac{g(\bar{q})}{2w}\left(v_B(\bar{q}) - f_B - c_B - \frac{2wG(\bar{q})}{g(\bar{q})}\right).\end{aligned}$$

First, we claim that $\frac{d\pi_A}{df_A} < 0$ for $\bar{q} \leq \frac{1}{2}$ and $f_A \geq 0$, which follows from

$$\frac{d\pi_A}{df_A} \leq \frac{g(\bar{q})}{2w}\left(v_A(1/2) - f_A - c_A - \frac{2w(1 - G(1/2))}{g(1/2)}\right) < 0,$$

where the first inequality follows from the monotonicity of $v_A(q)$ and the increasing hazard rate of $G(q)$, and the second inequality follows from (1). A similar logic implies that $\frac{d\pi_B}{df_B} < 0$ for $\bar{q} \geq \frac{1}{2}$ and $f_B \geq 0$. This implies there is an equilibrium with $f_A^* = f_B^* = 0$ and $q^* = \frac{1}{2}$. Firm A cannot profitably deviate given that $\frac{d\pi_A}{df_A} < 0$ for any $f_A > 0$, since $\bar{q} = \frac{1}{2} + \frac{f_B^* - f_A}{2w} < \frac{1}{2}$. A similar argument applies to firm B. Moreover, any proposed equilibrium with $f_A^* > f_B^* \geq 0$ must have $q^* < \frac{1}{2}$ which from above implies $\frac{d\pi_A}{df_A} < 0$ and so can be ruled out. Similarly, any proposed equilibrium with $f_B^* > f_A^* \geq 0$ or $f_A^* = f_B^* > 0$ can be ruled out. Thus, the only pure strategy informative equilibrium has $f_A^* = f_B^* = 0$ and $q^* = \frac{1}{2}$. □

From Lemma 1, in equilibrium, the prices are $f_i^* = 0$, $\Delta p_i = 0$, $p_i^m = P_i(q^*)$, the cutoff is $q^* = 1/2$, and firms' profits are $\pi_i^* = (P_i(1/2) - c_i)(1/2)$ for $i = A, B$. Suppose firm A deviates from the equilibrium by setting $p_A^m > P_A(q_A^e)$. Then the cutoff $\bar{q}_A$ used by M is determined by (D.1), where q_A^e is replaced with $\bar{q}_A$ given that commissions are observable. Given $f_B^* = 0$, the cutoff is

$$\bar{q}_A = \frac{w + (1 - H(P_A(\bar{q}_A) - p_A^d)w_l}{(1 + H(P_A(\bar{q}_A) - p_A^d))w}, \tag{D.2}$$

Then if firm A deviates in this way, the optimal deviation profit is

$$\pi_A' = \max_{p_A^d}[p_A^d - c_A]H(P_A(\bar{q}_A) - p_A^d))(1 - G(\bar{q}_A)), \tag{D.3}$$

where $\bar{q}_A$ is given by (D.2). To ensure firm A cannot profitably deviate in this way, i.e. to show $\pi'_A \leq \pi^*_A$, a sufficient condition is that

$$w_l > 0 \text{ and } H(v_h - P_A(1/2)) \leq \frac{w_l}{w_h}.$$

To show this, first note that the optimal deviating direct price that solves (D.3) must satisfy $p^d_A \leq P_A(1/2)$. This is because any $p^d_A > P_A(1/2)$ would always result in $\pi'_A = 0$ given that

$$\begin{aligned}
\bar{q}_A &= \frac{w + (1 - H(P_A(\bar{q}_A) - p^d_A)w_l}{(1 + H(P_A(\bar{q}_A) - p^d_A))w} \\
&> \frac{w + (1 - H(v_h - P_A(1/2)))w_l}{(1 + H(v_h - P_A(1/2)))w} \\
&\geq 1,
\end{aligned}$$

where the first inequality follows from the fact $P_A(\bar{q}_A) \leq P_A(1) = v_h$, $p^d_A > P_A(1/2)$ and $w_l > 0$, and the second inequality follows from the assumption $H(v_h - P_A(1/2)) \leq w_l/w_h$ and $w_l > 0$. Given that $p^d_A \leq P_A(1/2)$, we get that $\pi'_A < \pi^*_A$ since $H(P_A(\bar{q}_A) - p^d_A) < 1$ and $\bar{q}_A \geq 1/2$.

D.2 Firms set unobservable commissions (naive beliefs)

Given the proposed equilibrium in Section 5.2.1, we can show firm i does not want to set $p^m_i > P_i(q^e_i)$ so as to steer consumers to buy directly even if the informative equilibrium is selected in the stage-2 subgame.

If firm A deviates in this way, then the cutoff used by M is again determined by (D.1) with q^e_A replaced by q^* given naive beliefs. The optimal deviation profit of firm A is

$$\pi'_A = \max_{p^d_A}[p^d_A - c_A]H(P_A(q^*) - p^d_A)(1 - G(\bar{q}_A)),$$

which is lower than the equilibrium profit of firm A, since

$$\begin{aligned}
& \max_{p^d_A}[p^d_A - c_A]H(P_A(q^*) - p^d_A)(1 - G(\bar{q}_A)) \\
\leq\ & \max_{p^d_A}[p^d_A - c_A]H\left(P_A\left(q^*\right) - p^d_A\right)\left(1 - G\left(\frac{1}{2} + \frac{f^*_B}{2w}\right)\right) \\
<\ & [P_A\left(q^*\right) - c_A]\left(1 - G\left(\frac{1}{2} + \frac{f^*_B}{2w}\right)\right) \\
\leq\ & [P_A\left(q^*\right) - f^*_A - c_A]\left(1 - G\left(\frac{1}{2} + \frac{f^*_B - f^*_A}{2w}\right)\right).
\end{aligned}$$

The first inequality follows from $\bar{q}_A \geq \frac{1}{2} + \frac{f_B^*}{2w}$, the second inequality holds since the optimal deviation must have $p_A^d < P_A(q^*)$ otherwise no consumers will switch to buy directly from firm A, and the third inequality holds since in equilibrium $f_A = f_A^*$ maximizes firm A's profit given f_B^* and naive beliefs imply $q^e = q^* = \frac{1}{2} + \frac{f_B^* - f_A^*}{2w}$.

D.3 Firms set unobservable commissions (sophisticated beliefs)

Given the proposed equilibrium in Section 5.2.2, we provide conditions under which firm i does not want to set $p_i^m > P_i(q_i^e)$ so as to steer consumers to buy directly even if the informative equilibrium is selected in the stage-2 subgame.

In the proposed equilibrium in Section 5.2.2, the equilibrium commissions are

$$f_A^* = -\frac{c_A + c_B}{2} + \frac{3v_h + v_l}{4} - w + \frac{(c_B - c_A)w}{6w + \Delta v}, \tag{D.4}$$

$$f_B^* = -\frac{c_A + c_B}{2} + \frac{3v_h + v_l}{4} - w - \frac{(c_B - c_A)w}{6w + \Delta v}, \tag{D.5}$$

when both f_A^* and f_B^* are positive.

Suppose firm A deviates in this way from the proposed equilibrium. Since M will get no commission from firm A by definition and consumers expect this under sophisticated beliefs, the expected cutoff will be the same as the actual cutoff, i.e. $q_A^e = \bar{q}_A$. Therefore, the cutoff used by M is again given by (D.1) with q_A^e replaced by $\bar{q}_A$. Then firm A's optimal deviation profit can be written as (D.3), where $\bar{q}_A$ is determined as above. Then again we have $\frac{1}{2} + \frac{f_B^*}{2w} \leq \bar{q}_A \leq \frac{f_B^* + w + w_l}{w}$, where f_B^* could be positive.

Whenever $f_B^* \geq w$, we must always have $\bar{q}_A = 1$, which implies that $0 = \pi'_A < \pi_A^*$. A similar argument applies for firm B. Thus, a sufficient condition to ensure no firm can profitably deviate by setting $p_i^m > P_i(q_i^e)$ is that both commissions in equilibrium are no smaller than w, which holds if and only if

$$\frac{3v_h + v_l}{4} - \frac{c_A + c_B}{2} - \frac{(c_B - c_A)w}{2(6w + \Delta v)} - 2w \geq 0, \tag{D.6}$$

given that (D.4) and (D.5). Note (D.6) holds whenever w is sufficiently small.

E More general beliefs

In the main paper, for the case with unobservable commissions, we focus on naive beliefs and sophisticated beliefs in the case firms set commissions, and naive beliefs in the case M sets commissions (since "sophisticated beliefs" did not make sense in that context). In this section, we show our main result stills holds under more general beliefs.

E.1 Firms set commissions

When firms set unobservable commissions, suppose consumers' beliefs about the cutoff used by firm i ($\bar{q}_i$) is q_i^e, which does not depend on the actual commissions, but could depend on the actual prices. When a consumer observes firm A sets a different intermediated price p_A^m and/or a different discount Δp_A than expected, let the expected cutoff be denoted by $q_A^e(\Delta p_A)$, while taking into account that firm A optimally sets $p_A^m = P_A(q_A^e)$. Then firm A's profit can be written as

$$\pi_A = [P_A(q_A^e(\Delta p_A)) - (1 - H(\Delta p_A))f_A - c_A - \Delta p_A H(\Delta p_A)](1 - G(\bar{q}_A)).$$

Note that π_A only depends on the discount Δp_A and the effective commission F_A, rather than the actual commission f_A. For any discount Δp_A, firm A can always adjust f_A accordingly to keep F_A unchanged. Therefore, differentiating π_A with respect to Δp_A, and taking into account the F_A that is optimally set, we obtain

$$\frac{\partial \pi_A}{\partial \Delta p_A} = \left(\frac{dP_A(q_A^e(\Delta p_A))}{dq_A^e(\Delta p_A)} \frac{dq_A^e(\Delta p_A)}{d\Delta p_A} - \Delta p_A h(\Delta p_A) - H(\Delta p_A) \right) (1 - G(\bar{q}_A)).$$

Thus, it is optimal for firm A to set $\Delta p_A = 0$ if

$$\frac{dP_A(q_A^e(\Delta p_A))}{dq_A^e(\Delta p_A)} \frac{dq_A^e(\Delta p_A)}{d\Delta p_A} - \Delta p_A h(\Delta p_A) - H(\Delta p_A) \leq 0, \tag{E.1}$$

for any $\Delta p_A \geq 0$.

When G is uniformly distributed over $[0, 1]$, we have $\frac{dP_A(q_A^e(\Delta p_A))}{dq_A^e(\Delta p_A)} = \frac{v_h - v_l}{2}$. Thus, (E.1) holds if and only if

$$\frac{dq_A^e(\Delta p_A)}{d\Delta p_A} \leq \frac{2(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{v_h - v_l}. \tag{E.2}$$

Under (E.2), firm A does not want to offer any discount in equilibrium. Note that $\frac{dq_A^e(\Delta p_A)}{d\Delta p_A} = 0$ under naive beliefs and $\frac{dq_A^e(\Delta p_A)}{d\Delta p_A} = \frac{2(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{8w + v_h - v_l}$ under sophisticated beliefs, so (E.2) holds in both cases. In addition, for any beliefs that are between naive beliefs and sophisticated beliefs in the sense that $0 \leq \frac{dq_A^e(\Delta p_A)}{d\Delta p_A} \leq \frac{2(\Delta p_A h(\Delta p_A) + H(\Delta p_A))}{8w + v_h - v_l}$), (E.2) holds as well. A similar argument holds for firm B. Thus, we can conclude that whenever (E.2) holds so that consumers' beliefs about the cutoff are not sufficiently responsive to the change of discount (i.e., an increase in a discount is not a very strong signal that the firm is offering a suitable product), firms do not offer any discount in equilibrium.

E.2 M sets commissions

Suppose now M sets unobservable commissions. Consider general beliefs which satisfy $\frac{dq_i^e}{d\Delta p_i} \geq 0$. This implies that consumers interpret a higher discount to mean any firm that is still recommended is more likely to have a suitable product. We show below that firms would still want to offer positive discounts under certain reasonable conditions.

Consider the problem of firm A. Note that the expected cutoff by q_A^e would only depend on Δp_A, while taking into account firm A optimally sets $p_A^m = P_A(q_A^e)$. Then firm A's profit is

$$\pi_A = [P_A(q_A^e) - c_A - (1 - H(\Delta p_A))f_A - \Delta p_A H(\Delta p_A)](1 - G(\hat{q}_A)).$$

Firm A's choice of Δp_A is given by the following first-order condition

$$\begin{aligned} \frac{d\pi_A}{d\Delta p_A} &= \left[\frac{dP_A(q_A^e)}{dq_A^e}\frac{dq_A^e}{d\Delta p_A} + (f_A - \Delta p_A)h(\Delta p_A) - H(\Delta p_A)\right](1 - G(\hat{q}_A)) \\ &\quad -[P_A(q_A^e) - c_A - (1 - H(\Delta p_A))f_A - \Delta p_A H(\Delta p_A)]g(\hat{q}_A)\frac{h(\Delta p_A)f_A}{2w}, \end{aligned} \tag{E.3}$$

and similarly for firm B. Define $\Theta_A(\Delta p_A) \equiv \frac{dP_A(q_A^e)}{dq_A^e}\frac{dq_A^e}{d\Delta p_A}$, and we have $\Theta_A(\Delta p_A) \geq 0$ for all Δp_A. A similar argument applies to firm B.

We provide a sufficient condition below under which in equilibrium both firms do offer discounts. Suppose to the contrary, there exists an equilibrium in which firm A does not offer any discount, i.e. $f_A^*, f_B^* > 0$, $\Delta p_A^* = 0$ and $\Delta p_B^* \geq 0$. For any commission f_A, the optimal choice of Δp_A by firm A is given by equating (E.3) to zero, which can be written as

$$\begin{aligned} \Psi(f_A, \Delta p_A) &\equiv P_A(q_A^e) - c_A - (1 - H(\Delta p_A))f_A - \Delta p_A H(\Delta p_A) \\ &- \left(\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A)f_A} + 1 - \frac{1}{f_A}\left(\Delta p_A + \frac{H(\Delta p_A)}{h(\Delta p_A)}\right)\right)\frac{2w(1 - G(\hat{q}_A))}{g(\hat{q}_A)} = 0. \end{aligned}$$

Given that $\Psi(f_A, \Delta p_A) = 0$, totally differentiating Ψ with respect to f_A and Δp_A yields

$$\frac{\partial \Psi}{\partial f_A}df_A + \frac{\partial \Psi}{\partial \Delta p_A}d\Delta p_A = 0,$$

where

$$
\begin{aligned}
\frac{\partial \Psi}{\partial f_A} &= -(1-H(\Delta p_A)) - \left(\frac{1}{f_A^2}\left(\Delta p_A + \frac{H(\Delta p_A)}{h(\Delta p_A)}\right) - \frac{\Theta_A(\Delta p_A)}{h(\Delta p_A) f_A^2}\right)\frac{2w(1-G(\hat{q}_A))}{g(\hat{q}_A)} \\
&\quad + \left(\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A) f_A} + 1 - \frac{1}{f_A}\left(\Delta p_A + \frac{H(\Delta p_A)}{h(\Delta p_A)}\right)\right)(1-H(\Delta p_A))\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right), \\
\frac{\partial \Psi}{\partial \Delta p_A} &= \Theta_A(\Delta p_A) h(\Delta p_A) f_A - (H(\Delta p_A) + \Delta p_A h(\Delta p_A)) \\
&+ \left[\frac{1}{f_A}\left(1 + \left(\frac{d}{d\Delta p_A}\left(\frac{H(\Delta p_A)}{h(\Delta p_A)}\right)\right)\right) - \frac{d}{d\Delta p_A}\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A)}\frac{1}{f_A}\right]\frac{2w(1-G(\hat{q}_A))}{g(\hat{q}_A)} \\
&- \left(\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A) f_A} + 1 - \frac{1}{f_A}\left(\Delta p_A + \frac{H(\Delta p_A)}{h(\Delta p_A)}\right)\right) h(\Delta p_A) f_A \left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right).
\end{aligned}
$$

Evaluating $\frac{\partial \Psi}{\partial f_A}$ and $\frac{\partial \Psi}{\partial \Delta p_A}$ at the proposed equilibrium in which $\Delta p_A = \Delta p_A^* = 0$ yields

$$
\begin{aligned}
\frac{\partial \Psi}{\partial f_A} &= -1 + \left(\frac{\Theta_A(0)}{h(0) f_A^*} + 1\right)\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\Bigg|_{\hat{q}_A = q^*} + \frac{\Theta_A(0) 2w(1-G(q^*))}{h(0) f_A^* g(q^*)}, \\
\frac{\partial \Psi}{\partial \Delta p_A} &= h(0) f_A^* \left(\begin{array}{c} 1 + \frac{\Theta_A(0)}{h(0) f_A^*} - \left(\frac{\Theta_A(0)}{h(0) f_A^*} + 1\right)\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\big|_{\hat{q}_A = q^*} \\ + \left(2 + \frac{d}{d\Delta p_A}\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A)}\big|_{\Delta p_A = 0}\right)\frac{2w(1-G(q^*))}{(f_A^*)^2 h(0) g(q^*)} \end{array}\right).
\end{aligned}
$$

Thus,

$$
\frac{d\Delta p_A}{df_A} = -\frac{\frac{\partial \Psi}{\partial f_A}}{\frac{\partial \Psi}{\partial \Delta p_A}} =
$$

$$
\frac{1 - \left(\frac{\Theta_A(0)}{h(0) f_A^*} + 1\right)\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\big|_{\hat{q}_A = q^*} - \frac{\Theta_A(0) 2w(1-G(q^*))}{h(0) f_A^* g(q^*)}}{1 + \frac{\Theta_A(0)}{h(0) f_A^*} - \left(\frac{\Theta_A(0)}{h(0) f_A^*} + 1\right)\left(\frac{d}{d\hat{q}_A}\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}\right)\big|_{\hat{q}_A = q^*} + \left(2 + \frac{d}{d\Delta p_A}\frac{\Theta_A(\Delta p_A)}{h(\Delta p_A)}\big|_{\Delta p_A = 0}\right)\frac{2w(1-G(q^*))}{(f_A^*)^2 h(0) g(q^*)}}\frac{1}{h(0) f_A^*}.
$$

Suppose the following condition holds

$$
2 + \frac{d}{d\Delta p_i}\frac{\Theta_i(\Delta p_i)}{h(\Delta p_i)}\Bigg|_{\Delta p_i = 0} \geq 0, \tag{E.4}
$$

for $i = A, B$. Then we know the denominator of the expression for $d\Delta p_A / df_A$ is positive, given that $\Theta_A(0) \geq 0$, and $\frac{(1-G(\hat{q}_A))}{g(\hat{q}_A)}$ is decreasing in $\hat{q}_A$ following the increasing hazard rate of $G(q)$. It is straightforward to show that $d\Delta p_A / df_A$ is smaller than 1. Thus, f_A^* cannot maximize M's profit. This is because M is always better off setting f_A above f_A^* reflecting that

$$\left.\frac{d[(1-H(\Delta p_A))f_A]}{df_A}\right|_{f_A=f_A^*} = (1-H(\Delta p_A^*)) - h(\Delta p_A^*)f_A^*\frac{d\Delta p_A}{df_A}$$
$$= 1-h(0)f_A^*\frac{d\Delta p_A}{df_A}$$
$$> 0,$$

where the inequality follows from $d\Delta p_A/df_A < 1$. Therefore, from the proposed equilibrium, M would always want to set $f_A > f_A^*$ which would induce a positive discount by firm A. A similar argument implies that the discount set by firm B also cannot be zero in equilibrium.

Therefore, (E.4) is a sufficient condition to ensure that in equilibrium firms want to offer positive discounts. Note that under naive beliefs $\Theta_i(\Delta p_i) = 0$ so that (E.4) is trivially satisfied. When G is uniform and consumers hold beliefs over commissions that are fixed at the equilibrium level (i.e., $\frac{dq_i^e}{d\Delta p_i} = \frac{h(\Delta p_i)f_i^*}{2w}$), we have $\frac{d}{d\Delta p_i}\frac{\Theta_i(\Delta p_i)}{h(\Delta p_i)} = 0$ so that (E.4) is also satisfied.

More generally, whenever $\frac{dq_i^e}{d\Delta p_i} \geq 0$, a higher discount would lead to consumers being willing to pay weakly more for the firm's product. If (E.4) holds so that this positive effect of offering a discount is stronger, or at least not too much weaker, when the discount is higher, we find that firms would always want to offer positive discounts.