Maps and Meaning

Just as English speakers are able to understand an infinite number of English sentences, map readers are able to understand an infinite number of maps. This isn't only because the meaning of a novel map, like the meaning of a novel word, may be stipulated or ostended when it's introduced, but also because the ability to understand maps is productive: it's possible to understand an infinite number of maps unaided.

So just as the ability of English speakers to understand an infinite number of English sentences is explained by their tacit knowledge of a compositional theory of meaning for English, it seems likely that the ability of map readers to understand an infinite number of maps, without having to learn what each represents, is explained by their tacit knowledge of a compositional theory of representation for maps.

According to the compositional theory of representation for maps proposed by Achille Varzi and Roberto Casati, what a map represents depends compositionally on what is represented by its atomic stages. A map stage, according to Casati and Varzi (1999: 192), is any colouring of a map's regions. A map stage is atomic if and only if it colours all and only the regions of a single shade (Casati & Varzi 1999: 192).

If, for example, the whole of the region representing France is coloured purple, and nowhere else on the map is purple, then the colouring of that region is an atomic map stage. In contrast, if the region representing Vichy France is coloured purple, and the region representing occupied France is coloured red, then the red and purple colouring of the region representing France is not an atomic map stage.

Since atomic map stages are colourings of all and only the regions of a single shade, they may be colourings of disconnected regions: if the whole of the region representing the British empire is coloured pink, for example, then this colouring is an atomic map stage, even though its parts are scattered and disconnected.

Just as what a map represents, according to Casati and Varzi, depends compositionally on what is represented by its atomic stages, what its atomic stages represent depends compositionally on what the region it colours and its colour represent. Despite being a compositional theory, I will argue that Casati and Varzi's theory still can't explain our ability to understand an infinite number of maps.

Colours, according to Casati and Varzi (1999: 191), are like predicates: they represent properties. Regions, according to Casati and Varzi (1999: 191) are like names: they represent objects. Thus, atomic map stages are analogous to atomic sentences. The problem I will raise with Casati and Varzi's semantics arises from the analogy between map regions and names (Rescorla (forthcoming) objects sympathetically to the analogy between colours and predicates), because the former are infinite in number.

An atomic map stage is true, according to Casati and Varzi's (1999: 194) semantics, if and only if (a) it colours a region of the map which represents a region of the world which has the property represented by its colour and (b) the region of the world represented by the rest of the map does not have that property (Rescorla (forthcoming) defends (b)). So the maximal blue colouring of the map, for example, is true if and only if the region it colours represents a region covered by ocean, and the rest of the map doesn't.

Whole maps are colourings composed of atomic map stages so, according to Casati and Varzi, the truth-conditions of whole maps can be specified recursively in terms of the truth-conditions of their atomic stages. All other map stages – including whole maps – are true, according to Casati and Varzi (1999: 195), if and only if all their atomic stages are true. The world map, for example, is true if and only if its maximal green and blue colourings are both true.

The problem with Casati and Varzi's semantics is that although it can explain how maps are understood in terms of the understanding of atomic maps stages, and it can explain how atomic maps stages are understood in terms of the understanding of colours and regions, it can't explain how an infinite number of maps can be understood, because the number of regions it would be necessary to understand is also infinite.

To understand an atomic map stage, one has to understand what is represented by the map region which that stage colours. But since every difference in shape, size and location produces a different region, there is an infinite number of such regions, all of which have to be understood in order to understand the atomic map stages which colour them, and thus whole maps. If slightly more of the world map was coloured blue, for example, then its atomic map stages would be colourings of different regions, and what those regions represent would have to be antecedently understood.

It might be objected that the mere fact that there is an infinite number of atomic map stages and the regions they colour does not show that our ability to understand an infinite number of maps is not explained in terms of our ability to understand those stages and regions, since perhaps understanding only a finite number of stages or regions may suffice for understanding an infinite number of maps.

To grasp this objection, consider the fact that a person doesn't have to understand the entire English lexicon to be able to understand an infinite number of sentences: even a person who understands, for example, only one name, one predicate and one connective is able to understand the infinitely many sentences which those expressions compose. This would still be possible, even if the English lexicon were infinite.

However, understanding what is represented by a finite number of atomic map stages never suffices for being able to understand an infinite number of maps. Understanding the maximal green and blue colourings of the world map, for example, suffices for understanding only four maps: the map composed of only the blue colouring, the map composed of only the green colouring, the map composed of both, and – debatably – the empty map, which is composed of no colourings.

In general, the number of maps one understands is equal to two to the power of the number of atomic map stages one understands, because this is the number of maps composed by each number of atomic map stages. Since two to the power of a finite number is always a finite number, only the ability to understand a finite number of maps follows from the ability to understand a finite number of atomic map stages.

Likewise, understanding what is represented by a finite number of map regions doesn't suffice for understanding what is represented by an infinite number of atomic map stages, because the number of atomic map stages one has the ability to understand is always equal to the number of regions one understands multiplied by the number of colours one understands, which is finite whenever they are finite.

So the mere fact that it's impossible to understand every map region, because of their infinite number, is no more problematic than the fact that it's impractical to understand every word in English, because of their impractically large number. What is problematic is that, unlike understanding a fairly small number of English words,

understanding a finite number of atomic map stages or of the regions they colour doesn't suffice for understanding an infinite number of maps.

To complete their explanation of how it is possible to understand an infinite number of maps, Casati and Varzi need to add an explanation of how it is possible to understand an infinite number of map regions. Since it is possible to understand an infinite number of map regions, this must be possible. But, as I will now argue, it need not be possible to do so in a way which is consonant with Casati and Varzi's analogy between map regions and linguistic names.

Casati and Varzi themselves suggest that "Once a map region gets assigned a world region, its subregions are associated with subregions of the world in a tight way. We can somehow expect that the tightness of the assignment compensates for the compositional looseness of maps" (Casati & Varzi 1999: 191). So although the number of map regions is infinite, Casati and Varzi may argue that their constraints on which map regions represent which world regions explain our ability to understand them all.

Casati and Varzi suggest two constraints on which regions of the map represent which regions of the world. The first constraint is that one region of a map is part of another if and only if the region of the world represented by the former is part of the region of the world represented by the latter (Casati & Varzi 1999: 194). This ensures that the part of the map representing France, for example, must be a part of the part of the map that represents Europe.

The second constraint is that one region of a map is connected to another if and only if the region of the world represented by the former is connected to the region of the world represented by the latter (Casati & Varzi 1999: 194). This ensures that the part of the map representing Italy, for example, is adjacent to the part representing France.

Unfortunately, these constraints don't escape the problem, because they don't fully specify which regions of the map represent which regions of the world, but only which regions of the map represent which regions of the world given which other regions of the map represent which other regions of the world. The first constraint, for example, only specifies that the whole map represents the whole world *if* its halves represent the hemispheres.

To complete their specification of which parts of a map represent which parts of the world, Casati and Varzi could add direct non-conditional specifications of which regions some parts of the map represent. This could be done in two ways: by directly specifying what is represented by whole map regions, or by directly specifying what is represented by the map regions which atomic map stages colour.

However, since both kinds of region are infinite in number, both choices would require an infinite number of specifications, not all of which could be learned. Moreover, directly specifying what whole map regions represent would specify what subregions represent only incompletely: directly specifying that a whole map region represents the world, for example, determines that its halves represents hemispheres, but not which ones.

So to complete their theory, Casati and Varzi need to specify what is represented by map regions in some other way. One possibility is to specify what is represented by map regions using a coordinate system. But once such a system were introduced, it would be possible to specify what whole maps represent directly, without the intermediary of atomic map stages. Coordinates would replace, not complete, the compositional theory.

The problem in principle is that, according to Casati and Varzi's theory, the map regions coloured by atomic map stages are just like words. But if this is so, then what atomic map stages represent should be specified directly, just as the meaning of words are specified directly by the theory of meaning for a language.

But since the ability to understand infinitely many maps requires the ability to understand an infinite number of the regions coloured by atomic map stages, Casati and Varzi's theory would require an infinite number of such specifications. Since these specifications could not in principle be learned, nor could the theory.

To paraphrase Davidson (1967: 9), we ought to be far more puzzled than we are by maps. We understand maps very well, at least in this, that we always know what a map represents. Since there are infinitely many maps, our knowledge apparently enshrines a rule. The puzzle comes when we try to express this rule as a theory of meaning for maps.[1]

References

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[1] Thanks to Daniel Friedrich and Uriah Kriegel.